Optimum Design - Sheet 3 - Solution
Optimum Design By Matlab

1. Solve Consider the cantilever beam-mass system shown in Fig.1. Formulate and solve the minimum weight design problem for the rectangular cross section so that the fundamental vibration frequency is larger than 8 rad/s and the cross-sectional dimensions satisfy the limitations

$$0.5 \le b \le 1.0, \text{ in.}$$
$$0.2 \le h \le 2.0, \text{ in.}$$

Use a Matlab Toolbox to solve the problem. Verify the solution graphically and trace the history of the iterative process on the graph of the problem.
Let the starting point be (0.5,0.2). The data and various equations for the problem are as shown in the following.
Cantilever beam with spring-mass at the free end.
Fundamental vibration frequency

$$0.5 \le b \le 1.0, \text{ in.}$$
$$0.2 \le h \le 2.0, \text{ in.}$$

Equivalent spring constant k_e

$$0.5 \le b \le 1.0, \text{ in.}$$
$$0.2 \le h \le 2.0, \text{ in.}$$

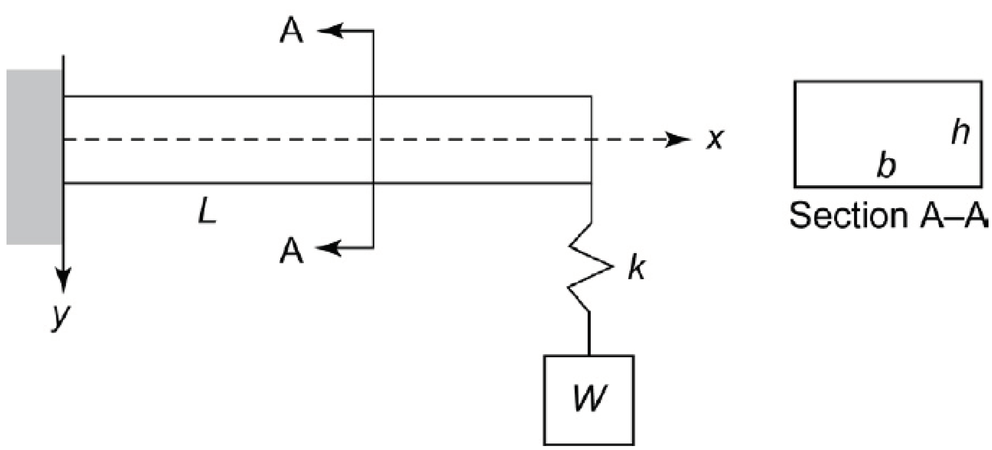

Figure 1. Cantilever beam with spring-mass at the free end

Mass attached to the spring

$$m = W/g$$

Weight attached to the spring

$$W = 50 \text{ lb}$$

Length of the beam

$$L = 12 \text{ in.}$$

Modulus of elasticity

$$E = (3 \times 10^7) \text{ psi}$$

Spring constant

$$k = 10 \text{ lb/in.}$$

Moment of inertia

$$I, \text{ in.}^4$$

Gravitational constant

$$g, \text{ in./s}^2$$

Solution:

Problem formulation: Minimize $f = bh$;
subject to $g_1 = 1.0 - [3gkEI/(3WEI + kWL^3)]^{1/2}/8.0 \leq 0$, where $I = bh^3/12$,
and $0.5 \leq b \leq 1.0$, $0.2 \leq h \leq 2.0$.
Solution: Initial design: $b = 0.5$, $h = 0.2$, optimum solution; $b^* = 0.5$ in, $h^* = 0.28107$ in, $f^* = 0.140536$ in^2 , active constraints (Lagrange multiplier); $g_1(0.54523)$, lower limit on $b(0.0936936)$.

2. Solve a prismatic steel beam with symmetric I cross-section is shown in Fig. 2. Formulate and solve the minimum weight design problem subject to the following constraints:
   1. The maximum axial stress due to combined bending and axial load effects should not exceed 100 MPa.
   2. The maximum shear stress should not exceed 60 MPa.
   3. The maximum deflection should not exceed 15 mm.
   4. The beam should be guarded against lateral buckling.
   5. Design variables should satisfy the limitations b ≥ 100 mm, t1 ≤ 10 mm, t2 ≤ 15 mm, h ≤ 150 mm.

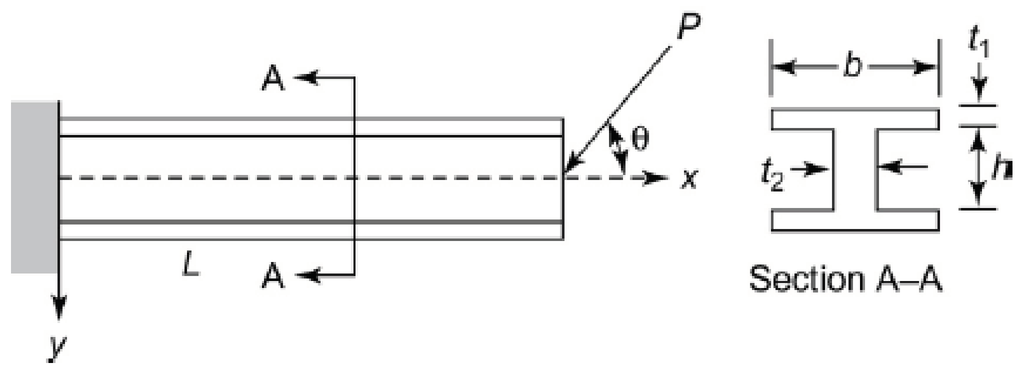

Fig.2 Cantilever I beam. Design variables b, t_1, t_2, and h.

Solve the problem using a numerical optimization method, for the following data:

Modulus of elasticity, E = 200 GPa
Shear modulus, G = 70 GPa
Load, P = 70 kN
Load angle, θ = 45 degree
Beam length, L = 1.5 m

Solution:

<u>Formulation:</u> Units of N and cm are used

1. Design variables: $x_1 = b$, $x_2 = t_1$, $x_3 = t_2$, $x_4 = h$
2. Cost function: $f = L\,(2x_1x_2 + x_3x_4) = 150(2x_1x_2 + x_3x_4)$
3. Constraints:

<axial stress> $g_1 = (Mc/I + P\cos\theta/A)/\sigma_a - 1.0 \le 0$,

where $M = PL\sin\theta$, $c = x_2 + x_4/2$, $I = [x_1(2x_2 + x_4)^3 - (x_1 - x_3)\,x_4^3]/12$, $A = 2x_1x_2 + x_3x_4$,

$P = 70000$, $L = 150$, $\theta = 45°$, $\sigma_a = 10000$;

<shear stress> $g_2 = (VQ/Ix_3)/\tau_a - 1.0 \le 0$,

where $V = P\sin\theta$, $Q = x_1x_2\,(x_2 + x_4)/2 + x_3x_4^2/8$, $\tau_a = 6000$;

<deflection> $g_3 = [(P\sin\theta)L^3/(3EI)]/\Delta - 1.0 \le 0$, where $\Delta = 1.5$;

<buckling> $g_4 = 1.0 - \pi^2 EI\,/\,(4L^2\,P\cos\theta) \le 0$, $g_5 = 1.0 - \pi^2 EI'/(4L^2\,P\cos\theta) \le 0$,

where $I' = x_1^3x_2/6 + x_3^3x_4/12$;

<design limits> $x_1 \ge 10$, $x_2 \le 1$, $x_3 \le 1.5$, $x_4 \le 15$.

Initial design; $x_1 = 60$, $x_2 = 0.9$, $x_3 = 0.9$, $x_4 = 14$,

Optimum; $x_1^* = 50.4437$ cm, $x_2^* = 1.0$ cm, $x_3^* = 0.52181$ cm, $x_4^* = 15.0$ cm, $f^* = 16307.2$ cm^3,
active constraints (Lagrange multipliers); g_1(15502.0), g_2(805.224), upper limit of x_2(154.797), upper limit of x_4(14641.4).

3. Design synthesis of a nine-speed gear drive. The arrangement of a nine-speed gear train is shown in Fig. E 7.12. The objective of the synthesis is to find the size of all gears from the mesh and speed ratio equations such that the sizes of the largest gears are kept to a minimum (Osman et al., 1978). Because of the mesh and speed ratio equations, it is found that only the following three independent parameters need to be selected:

$$x_1 = \text{gear ratio, } d/a$$
$$x_2 = \text{gear ratio, } e/a$$
$$x_3 = \text{gear ratio, } j/a$$

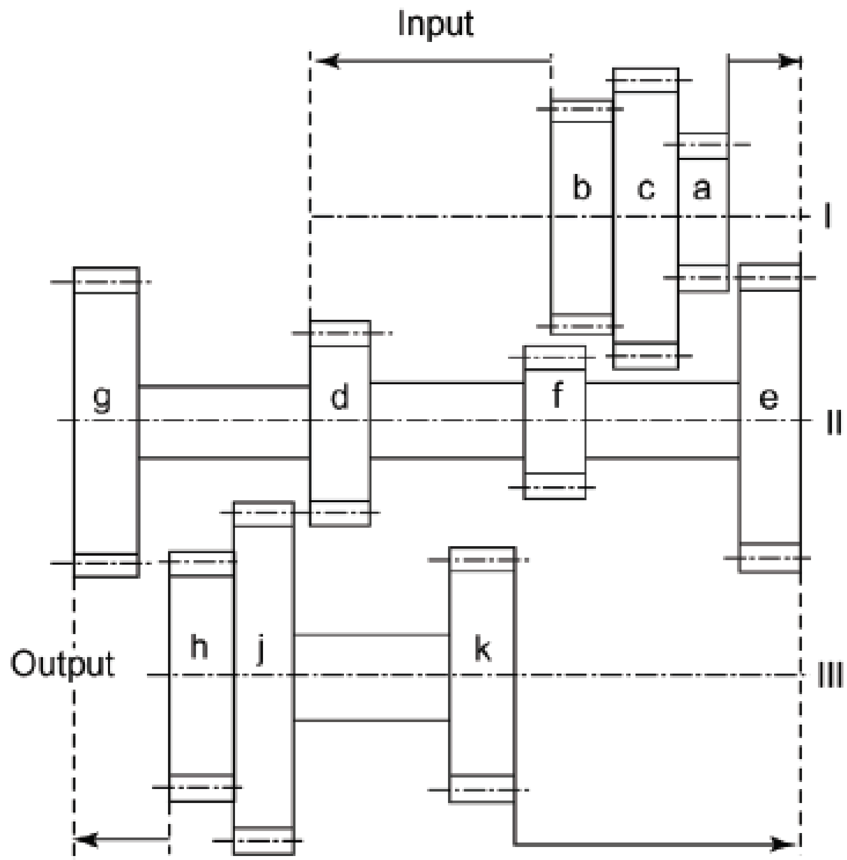

Fig.3 Schematic arrangement of a nine-speed gear train.

Because of practical considerations, it is found that the minimization of $|x_2 - x_3|$ results in the reduction of the cost of manufacturing the gear drive. The gear sizes must satisfy the following mesh equations:

$$\phi^2 x_1(x_1 + x_3 - x_2) - x_2 x_3 = 0$$
$$\phi^3 x_1 - x_2(1 + x_2 - x_1) = 0$$

where φ is the step ratio in speed. Find the optimum solution for the problem for two different values of φ as $\sqrt{2}$ and $(2)^{1/3}$.

Solution:

Formulation: Minimize $f = (x_2 - x_3)^2$;

subject to $h_1 = \phi^2 x_1(x_1 - x_2 + x_3)/x_2 x_3 - 1 = 0$, $h_2 = 1.0 - x_2(1 - x_1 + x_2)/\phi^3 x_1 = 0$,

and design bounds $1.0\text{E}{-10} \le x_1,\ x_2, x_3 \le 1000.0$

Solution for $\phi = \sqrt{2}$, $x^{(0)} = (1, 1, 1)$;

Optimum; $x_1^* = 2.4138$, $x_2^* = 3.4138$, $x_3^* = 3.4141$, $f^* = 1.2877\times10^{-7}$. Active constraints (Lagrange multipliers); $h_1(-0.007119)$, $h_2(0.003528)$. [Program used; IDESIGN; 8 iterations]

Solution for $\phi = 2^{1/3}$, $x^{(0)} = (1,1,1)$.

Optimum; $x_1^* = 2.2606$, $x_2^* = 2.8481$, $x_3^* = 2.8472$, $f^* = 8.03\times10^{-7}$.

Active constraints (Lagrange multipliers); $h_1(-5.47\times10^{-6})$, $h_2(1.6236\times10^{-6})$.