

Optimum Design - Sheet 5
Optimality Conditions - Constrained

Find points satisfying KKT necessary conditions for the following problems; check if they are optimum points using the graphical method for two variable problems.

1. Maximize $F(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8$
subject to $x_1 + x_2 \leq 4$
2. Minimize $f(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8$
subject to $x_1 + x_2 \leq 4$
3. Maximize $F(x_1, x_2) = 4x_1^2 + 3x_2^2 - 5x_1x_2 - 8x_1$
subject to $x_1 + x_2 \leq 4$
4. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$
subject to $x_1 + x_2 \geq 4$
 $x_1 - x_2 - 2 = 0$
5. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$
subject to $x_1 + x_2 = 4$
 $x_1 - x_2 - 2 \geq 0$
6. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$
subject to $x_1 + x_2 \geq 4$
 $x_1 - x_2 \geq 2$
7. Minimize $f(x_1, x_2) = 2x_1^2 - 6x_1x_2 + 9x_2^2 - 18x_1 + 9x_2$
subject to $x_1 + 2x_2 \leq 10$
 $4x_1 - 3x_2 \leq 20; x_i \geq 0; i = 1, 2$
8. Minimize $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 1)^2$
subject to $x_1 + x_2 - 4 \leq 0$
 $x_1 - x_2 - 2 \leq 0$
9. Minimize $f(x_1, x_2) = 9x_1^2 - 18x_1x_2 + 13x_2^2 - 4$
subject to $x_1^2 + x_2^2 + 2x_1 \geq 16$
10. Minimize $f(x, y) = (x - 4)^2 + (y - 6)^2$
subject to $x + y \leq 12$
 $x \leq 6$
 $x, y \geq 0$

11.

$$\begin{aligned} &\text{Maximize } F(x, y) = (x - 4)^2 + (y - 6)^2 \\ &\text{subject to } x + y \leq 12 \\ &\quad 6 \geq x \\ &\quad x, y \geq 0 \end{aligned}$$

12.

$$\begin{aligned} &\text{Maximize } F(r, t) = (r - 3)^2 + (t - 2)^2 \\ &\text{subject to } 10 \geq r + t \\ &\quad t \leq 5 \\ &\quad r, t \geq 0 \end{aligned}$$

13.

An engineering design problem is formulated as

$$\text{Minimize } f(x_1, x_2) = x_1^2 + 320x_1x_2$$

$$\text{subject to } \frac{1}{100}(x_1 - 60x_2) \leq 0$$

$$1 - \frac{1}{3600}x_1(x_1 - x_2) \leq 0$$

$$x_1, x_2 \geq 0$$

Write KKT necessary conditions and solve for the candidate minimum designs. Verify the solutions graphically. Interpret the KKT conditions on the graph for the problem.