REE 307
Fluid Dynamics II

Lecture 3

Dr./ Ahmed Nagib Elmekawy

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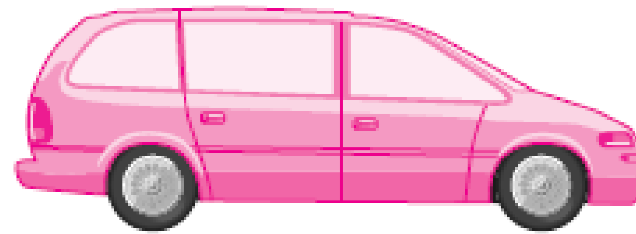
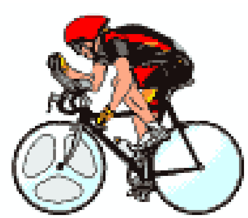
External Flow

- Bodies and vehicles in motion, or with flow over them, experience fluid-dynamic forces and moments.
- Examples include: aircraft, automobiles, buildings, ships, submarines, turbomachines.
- These problems are often classified as ***External Flows.***
- Fuel economy, speed, acceleration, maneuverability, stability, and control are directly related to the aerodynamic/hydrodynamic forces and moments.

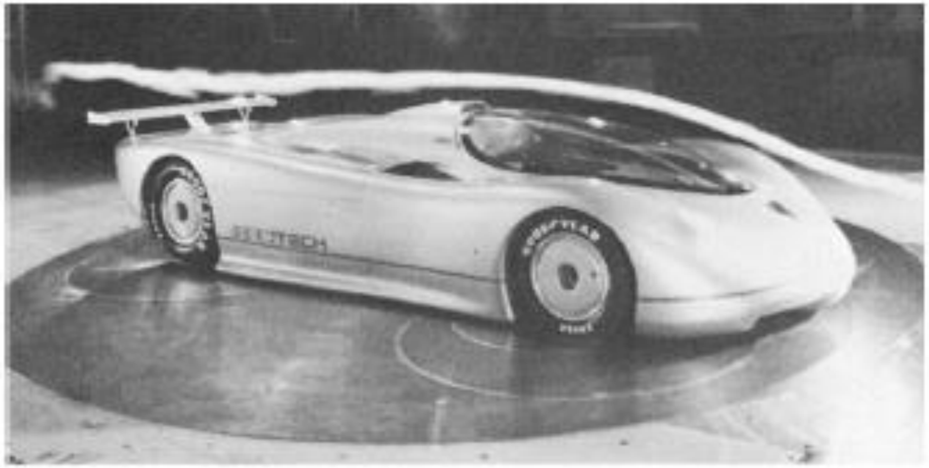




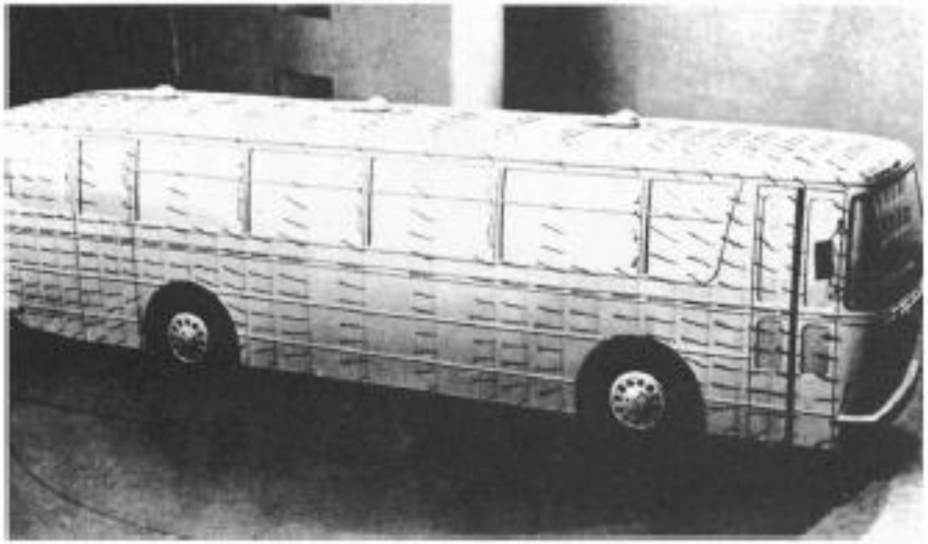
External Flow

(a) Flow past a full-sized streamlined vehicle in the GM aerodynamics laboratory wind tunnel, and 18-ft by 34-ft test section facility driven by a 4000-hp, 43-ft-diameter fan.

(*b*) Surface flow on a model vehicle as indicated by tufts attached to the surface.

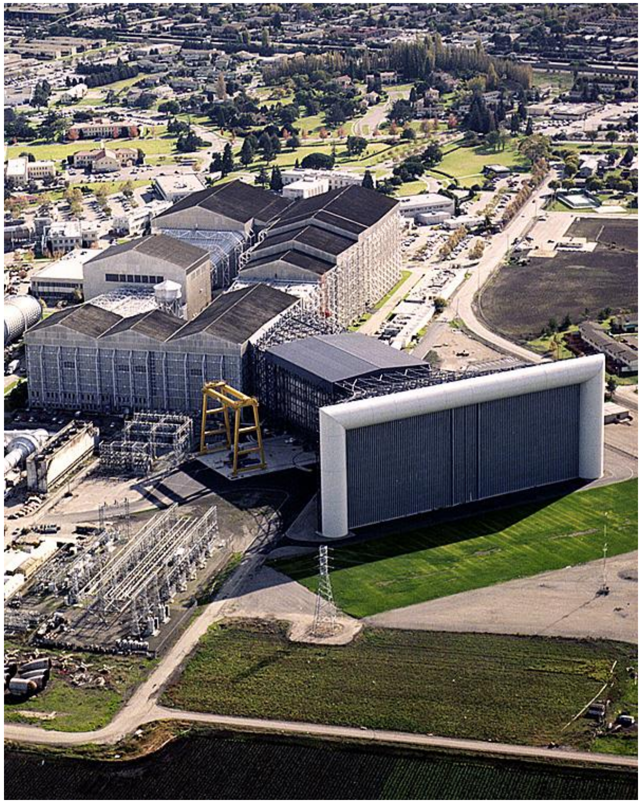


(*a*)



(*b*)

Two of NASA's Wind Tunnels



Ames 80' x 120'

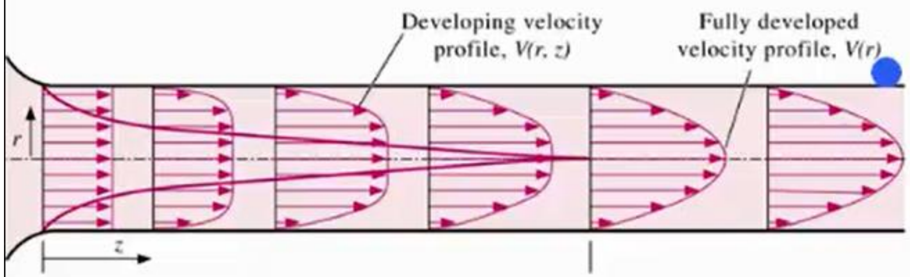
Langley

External Flow over bodies

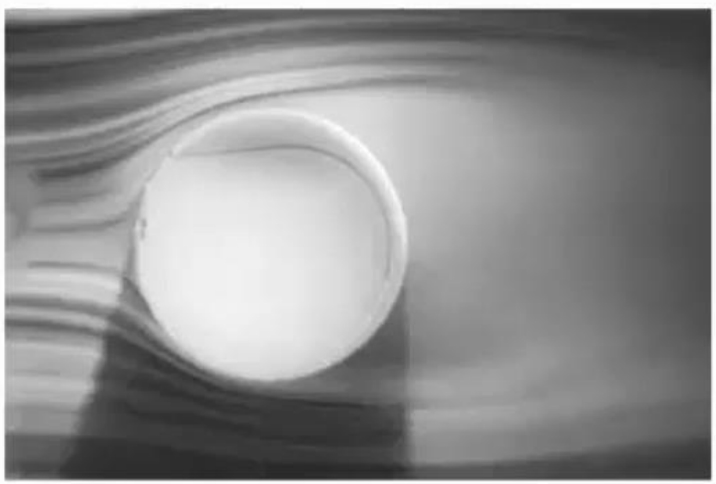
General Characteristics

- External flow is characterized by **a freely growing viscous boundary layer surrounded by an outer flow region that involves small velocity and temperature gradients.**
- In internal flows, the entire flow field is dominated by viscous effects, while in external flow; **the viscous effects are confined to a portion of the flow field such as the boundary layers and wakes.**
- When a fluid moves over a solid body, it **exerts pressure forces normal to the surface and shear forces parallel to the surface along the outer surface of the body.**
- The component of the resultant pressure and shear forces that acts **in the flow direction is called the *drag force*** (or just *drag*), and the component that acts **normal to the flow direction is called the *lift force*** (or just *lift*).

Internal vs. External Flow

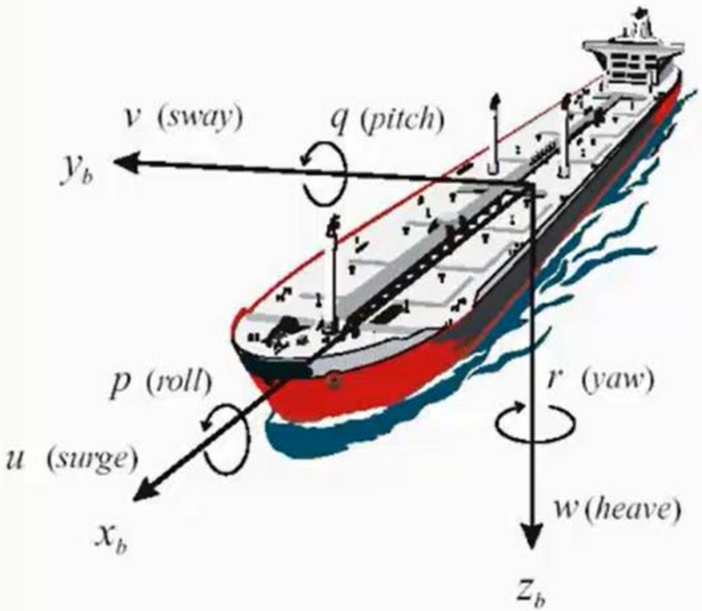


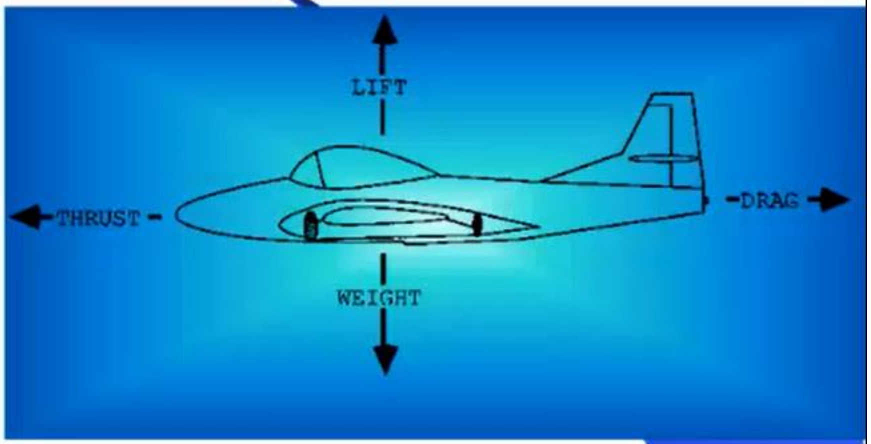
- Internal flows are dominated by the influence of viscosity throughout the flow field



- For external flows, viscous effects are limited to the boundary layer and wake.

Fluid Dynamic Forces and Moments





Ships in waves present one of the most difficult 6DOF problems.

Airplane in level steady flight: drag = thrust and lift = weight.

External Flow over bodies

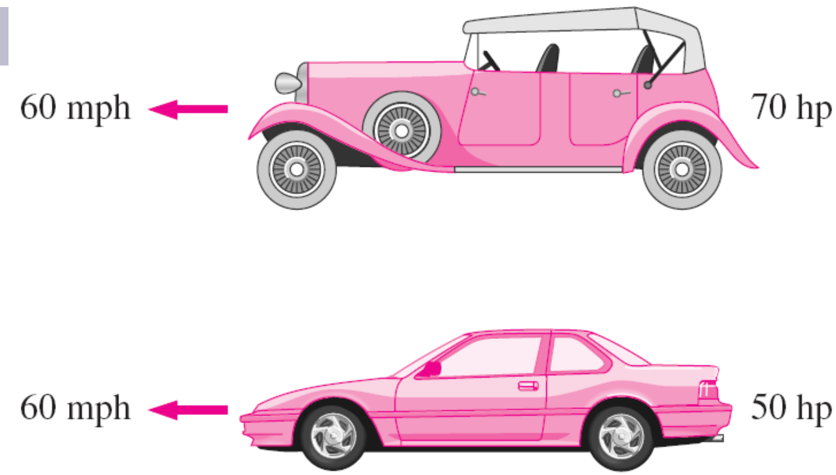
General Characteristics

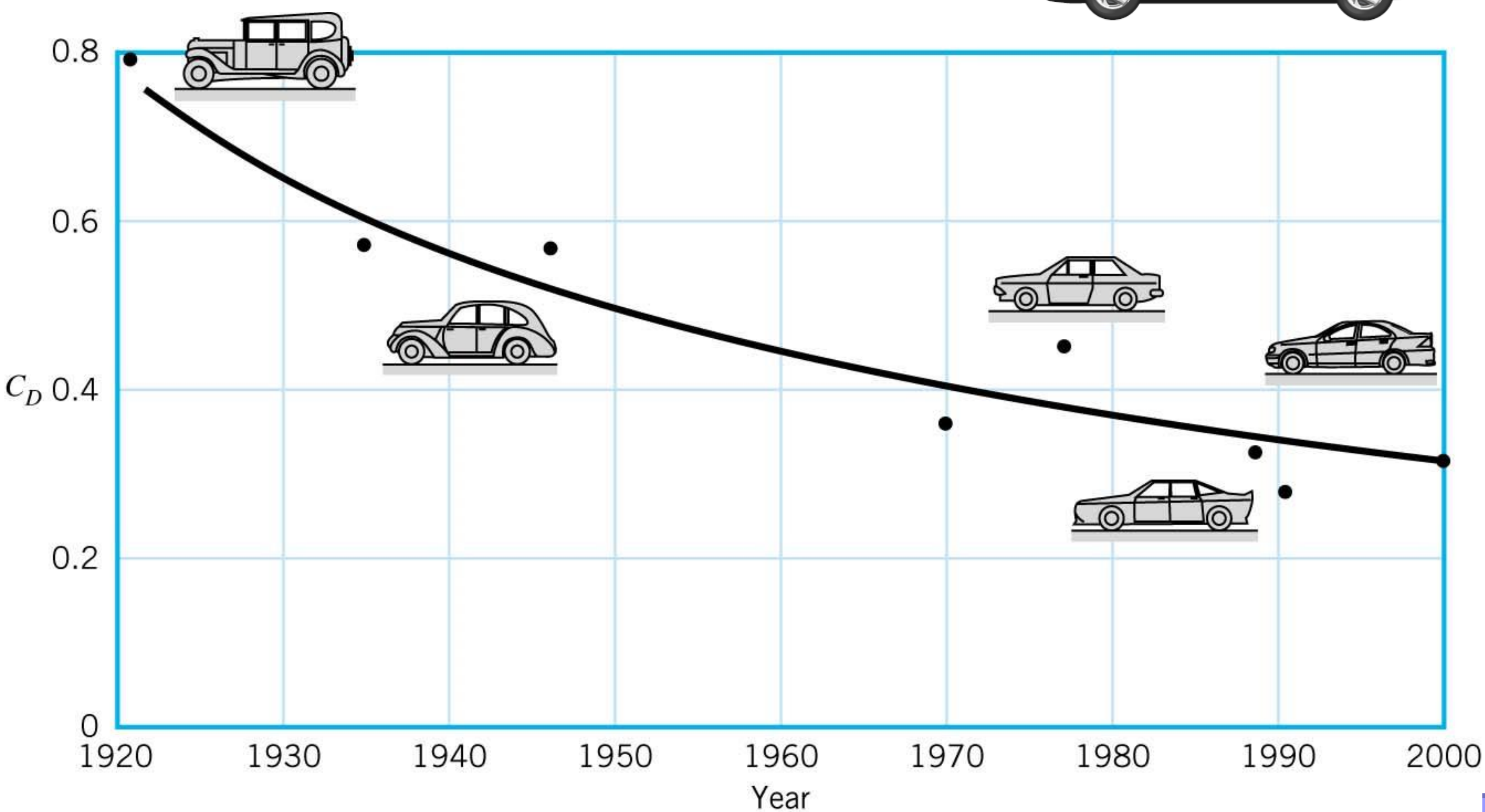
- For a given-shaped object, the characteristics of the flow depend very strongly on various parameters such as size, orientation, speed, and fluid properties.
- **Fuel economy, speed, acceleration, maneuverability, stability, and control are directly related to the aerodynamic / hydrodynamic forces and moments**

FIGURE 2-3

Flow over bodies is commonly encountered in practice.

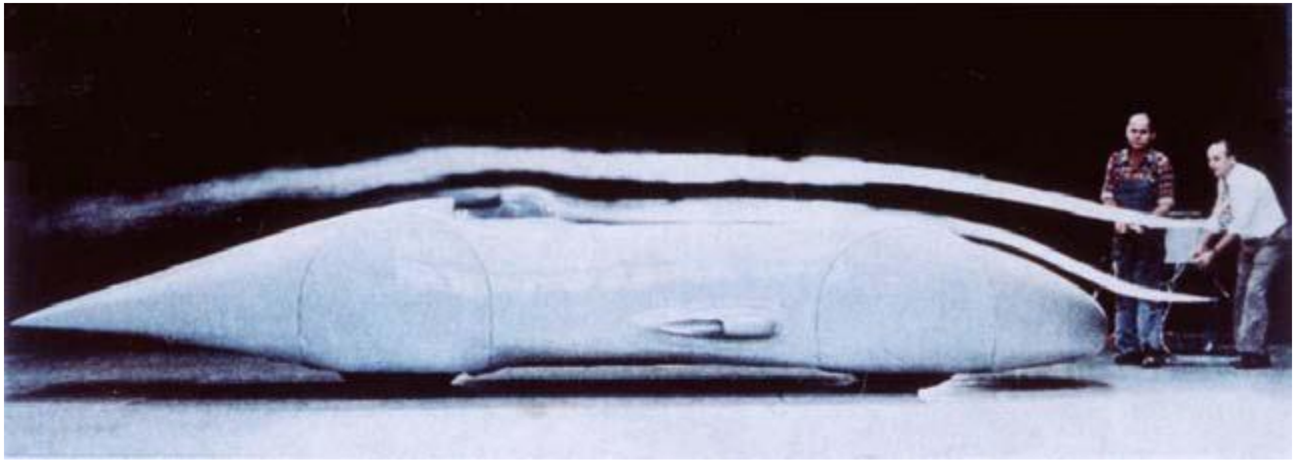
History of Automobiles





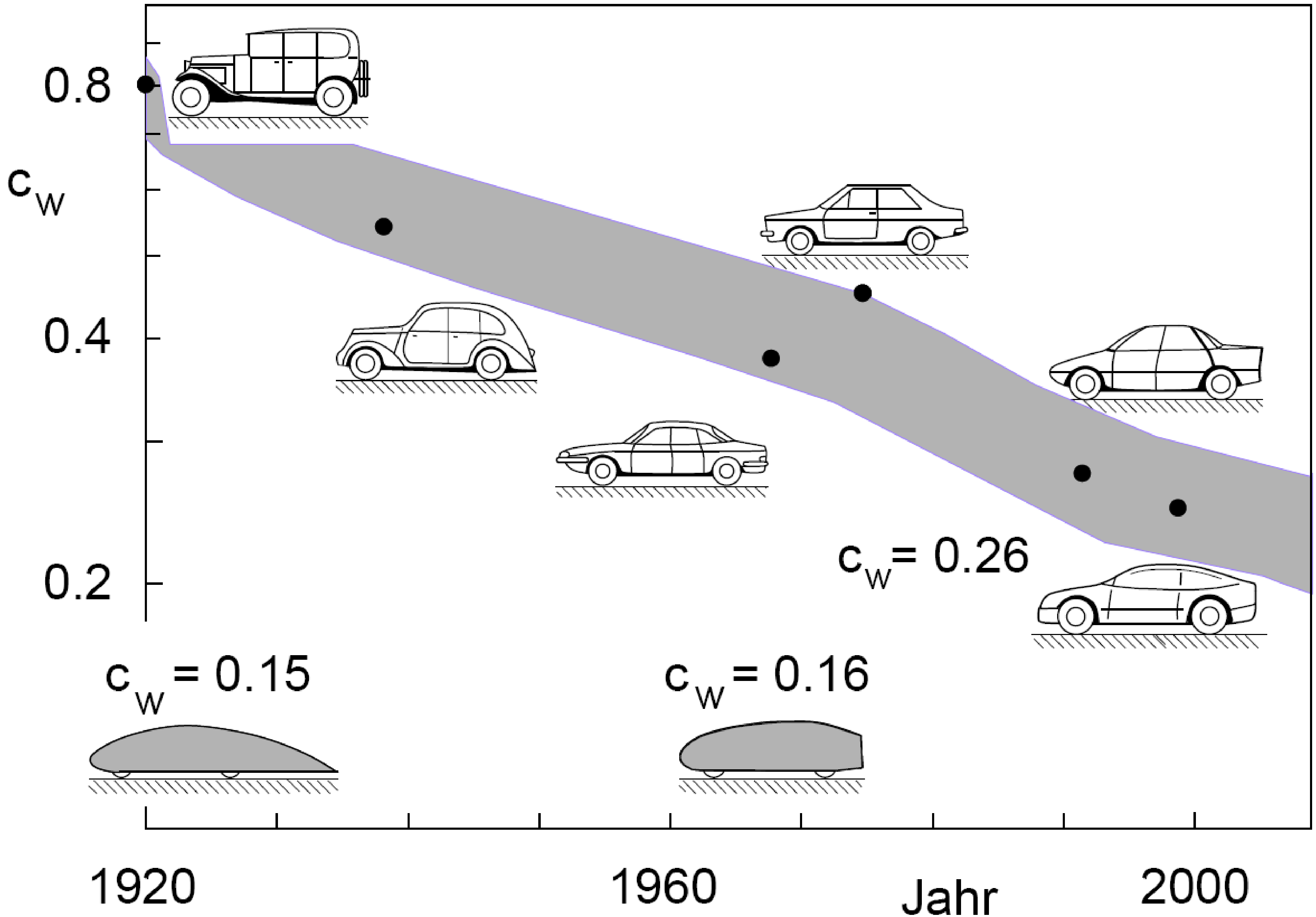
History of Automobiles

$C_D = 0.365$ 1937

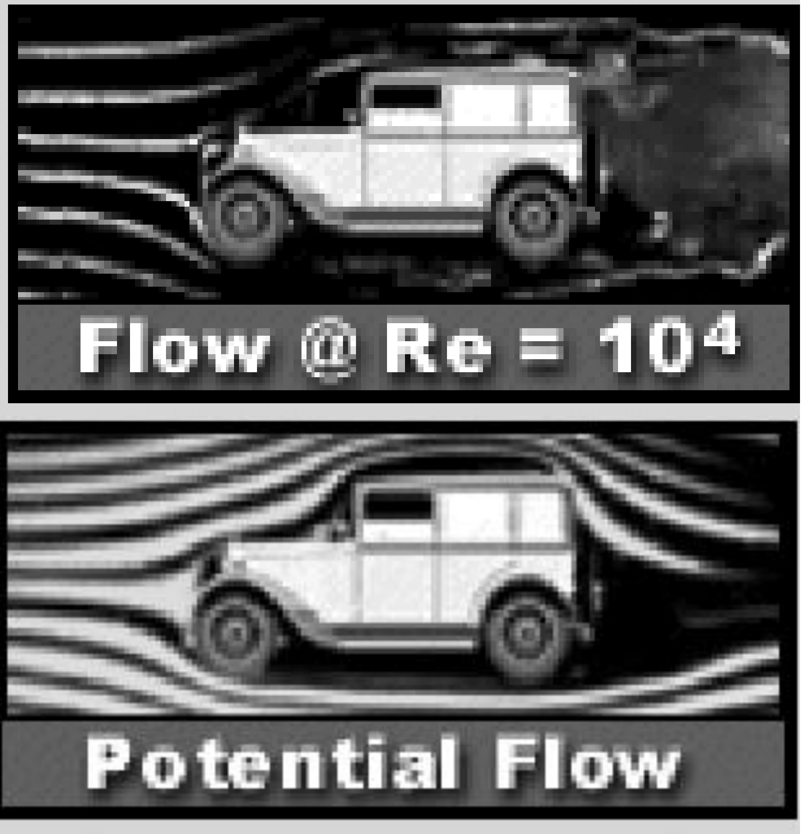


$C_D = 0.170$ 1938

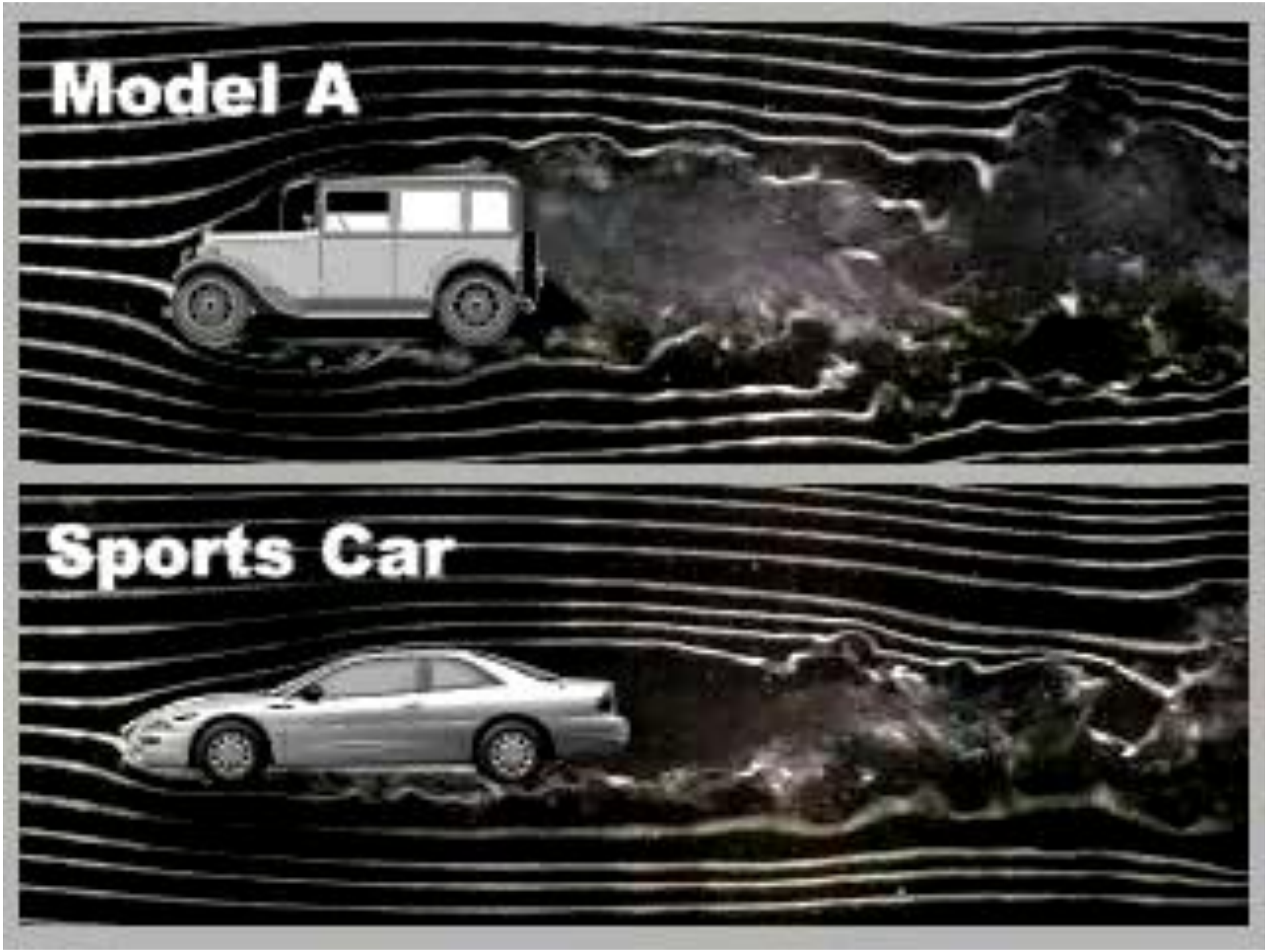
Mercedes-Benz W125 in a wind tunnel


0.8
0.4
0.2
c_w
$c_w = 0.26$
$c_w = 0.15$
$c_w = 0.16$
1920
1960
Jahr
2000

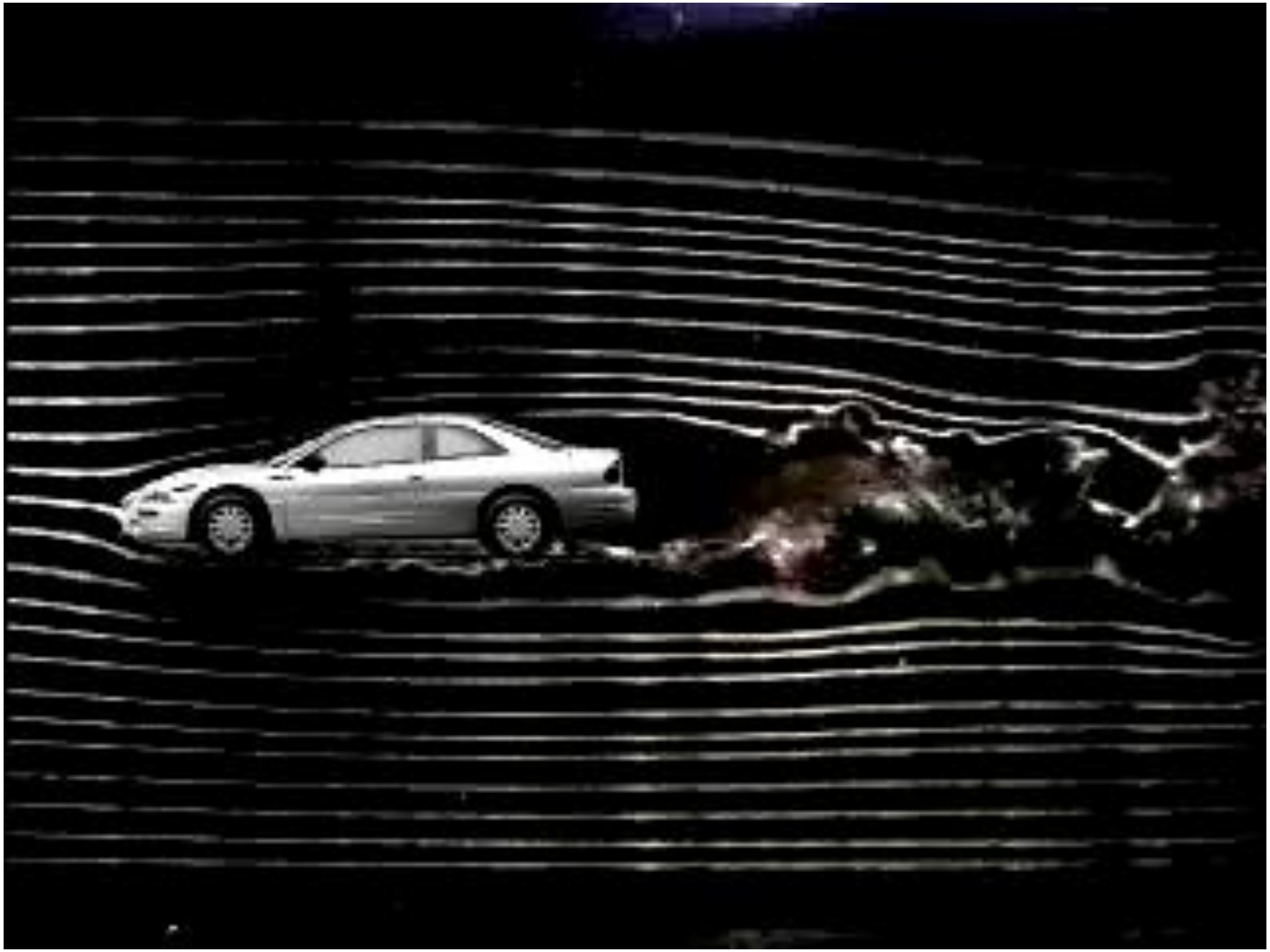
History of Automobiles



History of Cars



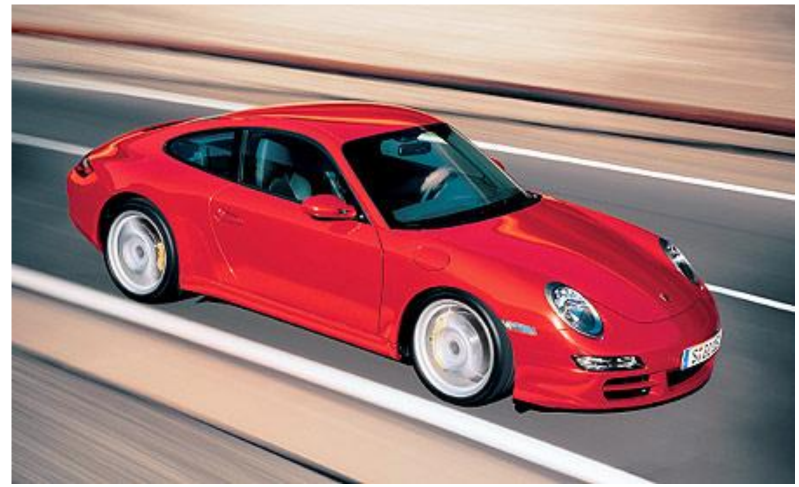
History of Cars



Automobile Drag

Scion XB

Porsche 911



$C_D = 1.0$, $A = 25\ ft^2$, $C_DA = 25ft^2$

$C_D = 0.28$, $A = 10\ ft^2$, $C_DA = 2.8ft^2$

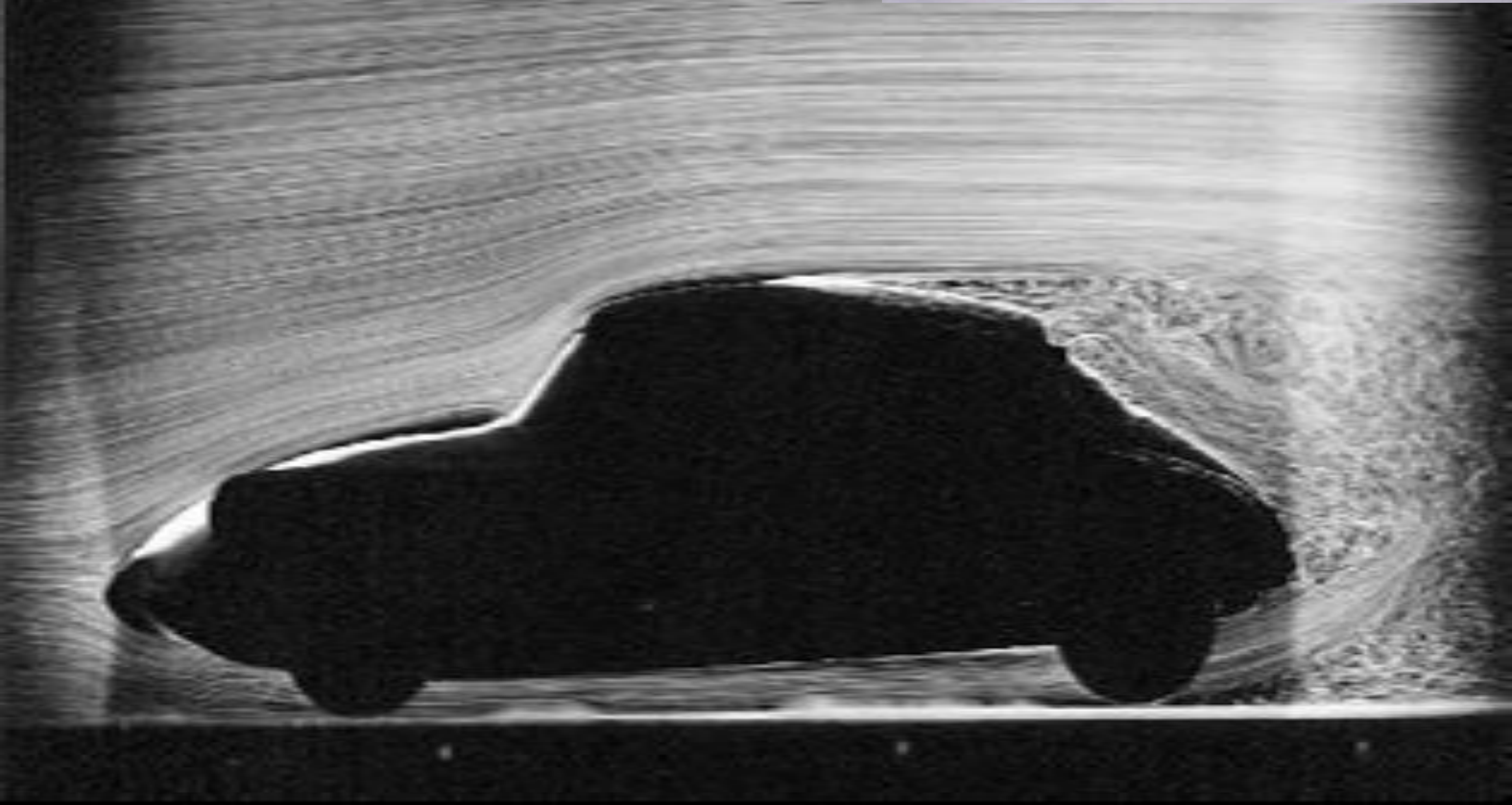
- Drag force $F_D=1/2\rho V^2(C_DA)$ will be ~ 10 times larger for Scion XB
- Source is large C_D and large projected area
- Power consumption $P = F_DV = 1/2\rho V^3(C_DA)$ for both scales with V^3!

Electric Vehicles

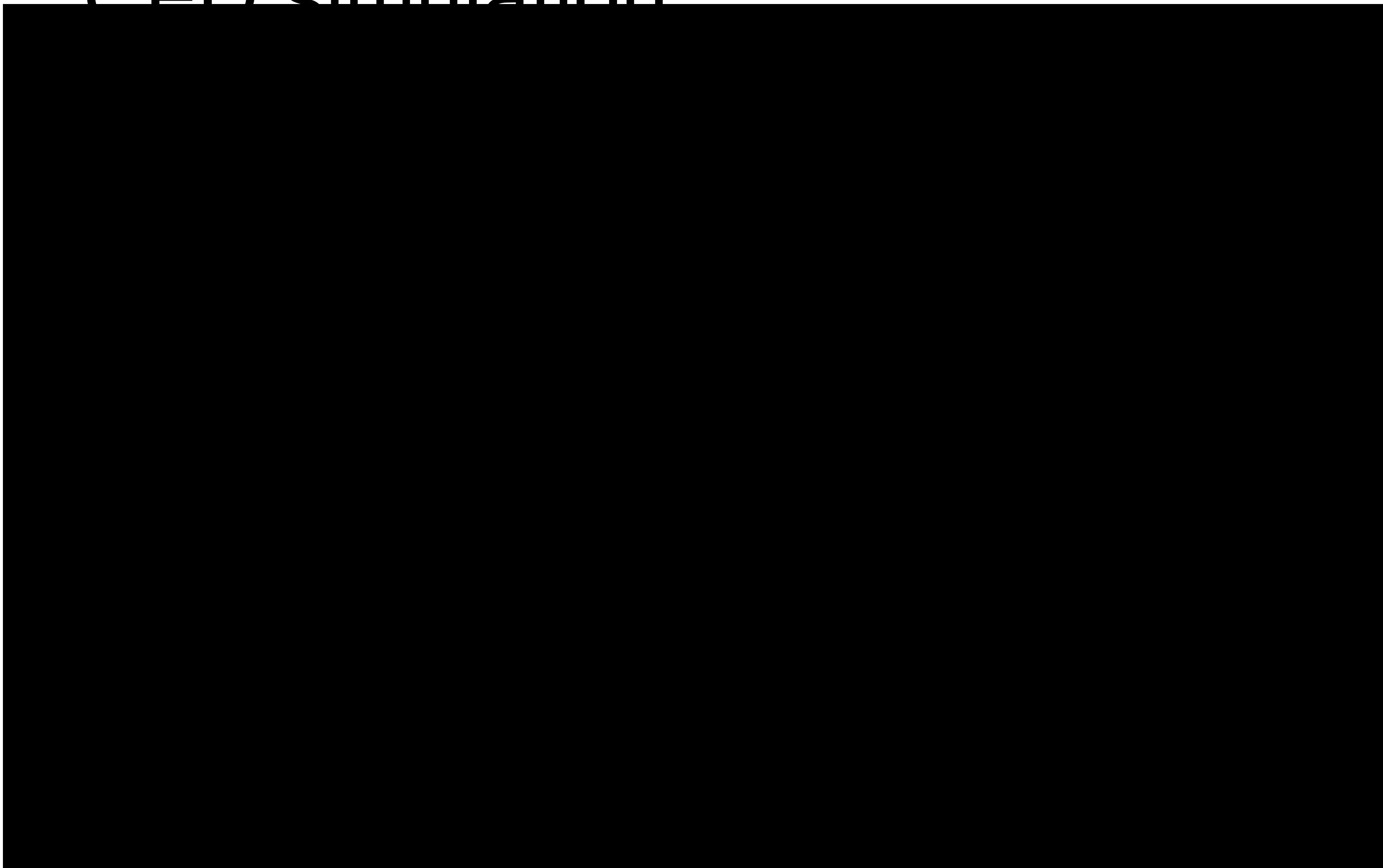
- Electric vehicles are designed to minimize drag.
- Typical cars have a coefficient drag of 0.30-0.40.
- The EV1 has a drag coefficient of 0.19.

Smooth connection to windshield

Automobile Drag



CFD simulation



Challenge

$$Drag = \frac{C_d \rho U^2 A}{2}$$

- You are going on vacation and you can't back all of your luggage in your Matrix. Should you put it on the roof rack or on the hitch?

Categories of Bodies

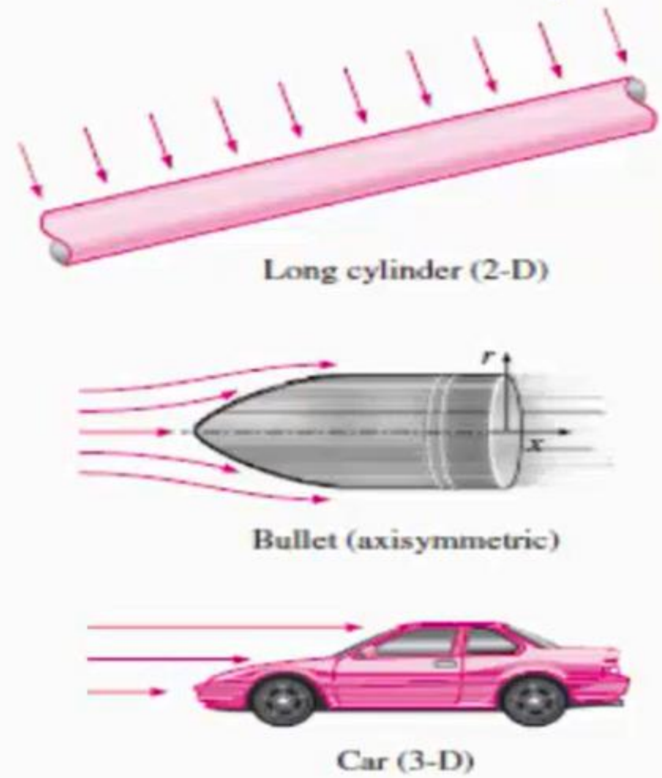
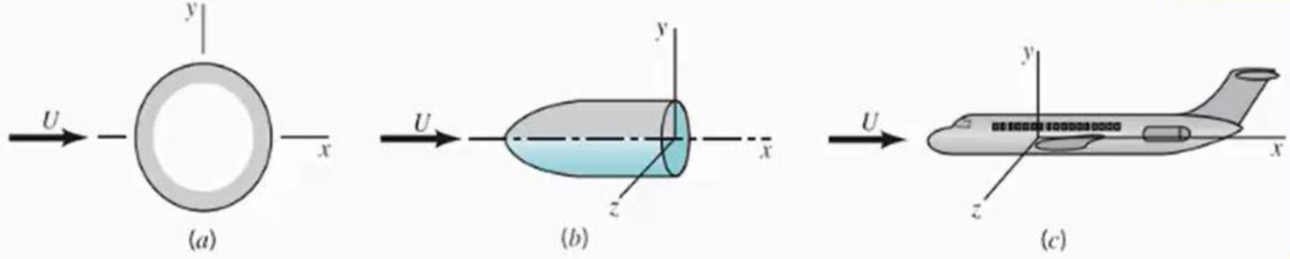


FIGURE 11–2
Two-dimensional, axisymmetric, and three-dimensional flows.

Categories of Bodies

- The structure of an external flow and the ease with which the flow can be described and analyzed often depend on the nature of the body in the flow.
- Three general categories of bodies include (a) two-dimensional objects, (b) axisymmetric bodies, and (c) three-dimensional bodies.



- Another classification of body shape can be made depending on whether the body is **streamlined or blunt**.

Categories of Bodies

- Another classification of body shape can be made depending on whether the body is **streamlined or blunt**.

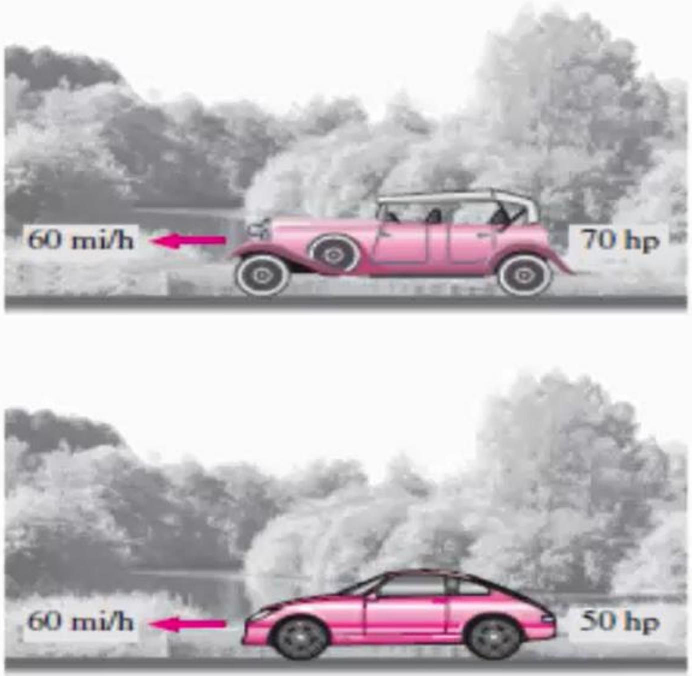
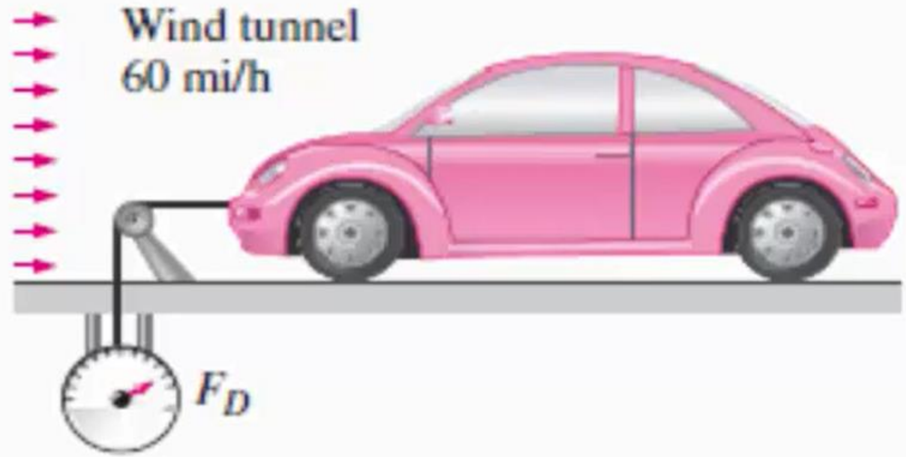
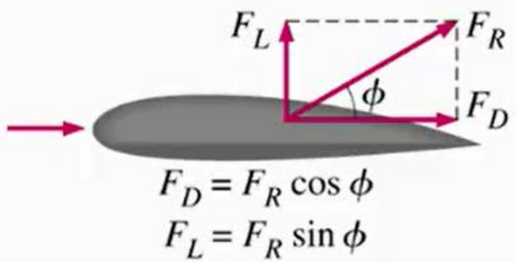


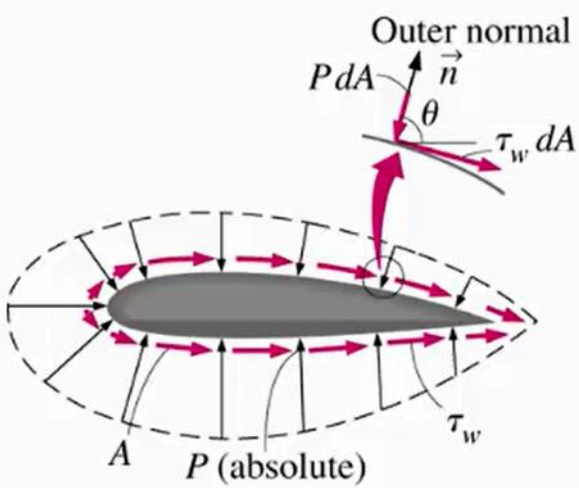
FIGURE 11–3
It is much easier to force a streamlined body than a blunt body through a fluid.


Wind tunnel
60 mi/h
F_D

Lift and Drag



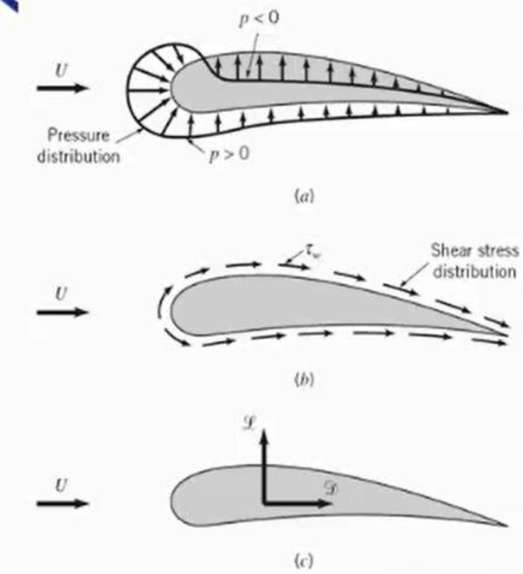
- Fluid dynamic forces are due to pressure and viscous forces acting on the body surface.



- Drag: component parallel to flow direction.
- Lift: component normal to flow direction.

Lift and Drag Concepts 1/3

- The interaction between the body and the fluid:
  - ⇨ Stresses-wall shear stresses, τ_w, due to viscous effects.
  - ⇨ Normal stresses, due to the pressure p.
- Both τ_w and p vary in magnitude and direction along the surface.
- The detailed distribution of τ_w and p is **difficult** to obtain.
- However, only the integrated or resultant effects of these distributions are needed.

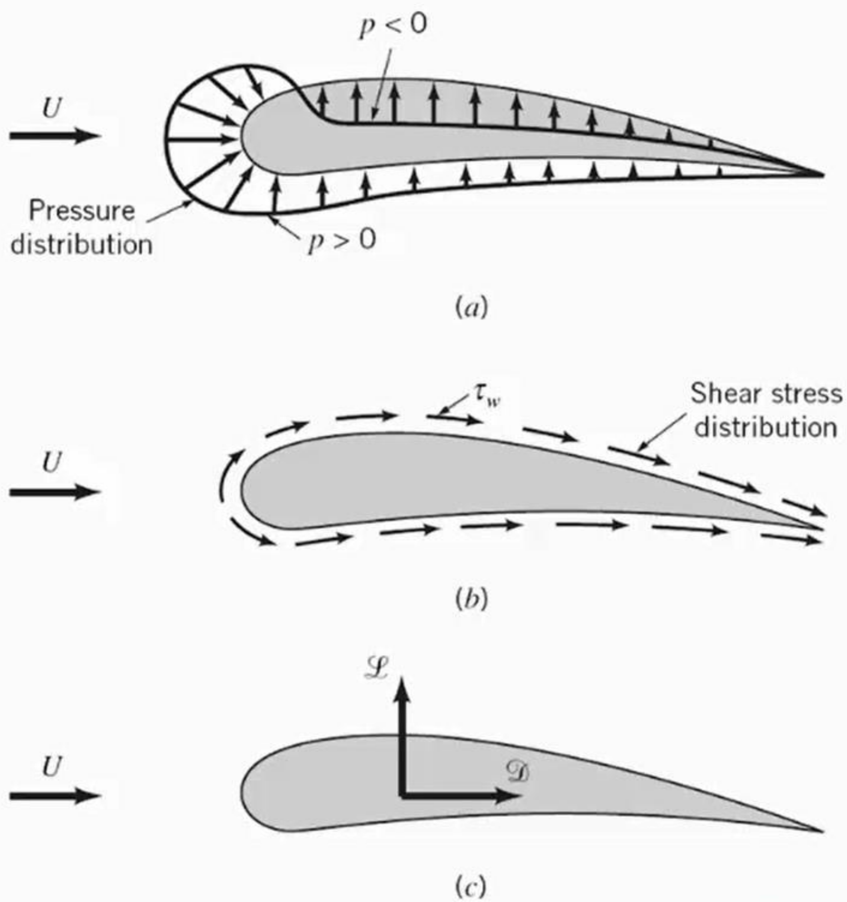


Forces from the surrounding fluid on a two-dimensional object:

(*a*) pressure force,

(*b*) viscous force,

(*c*) Resultant force (lift and drag).



Drag and Lift Concepts

Forces from the surrounding fluid on a two-dimensional object:

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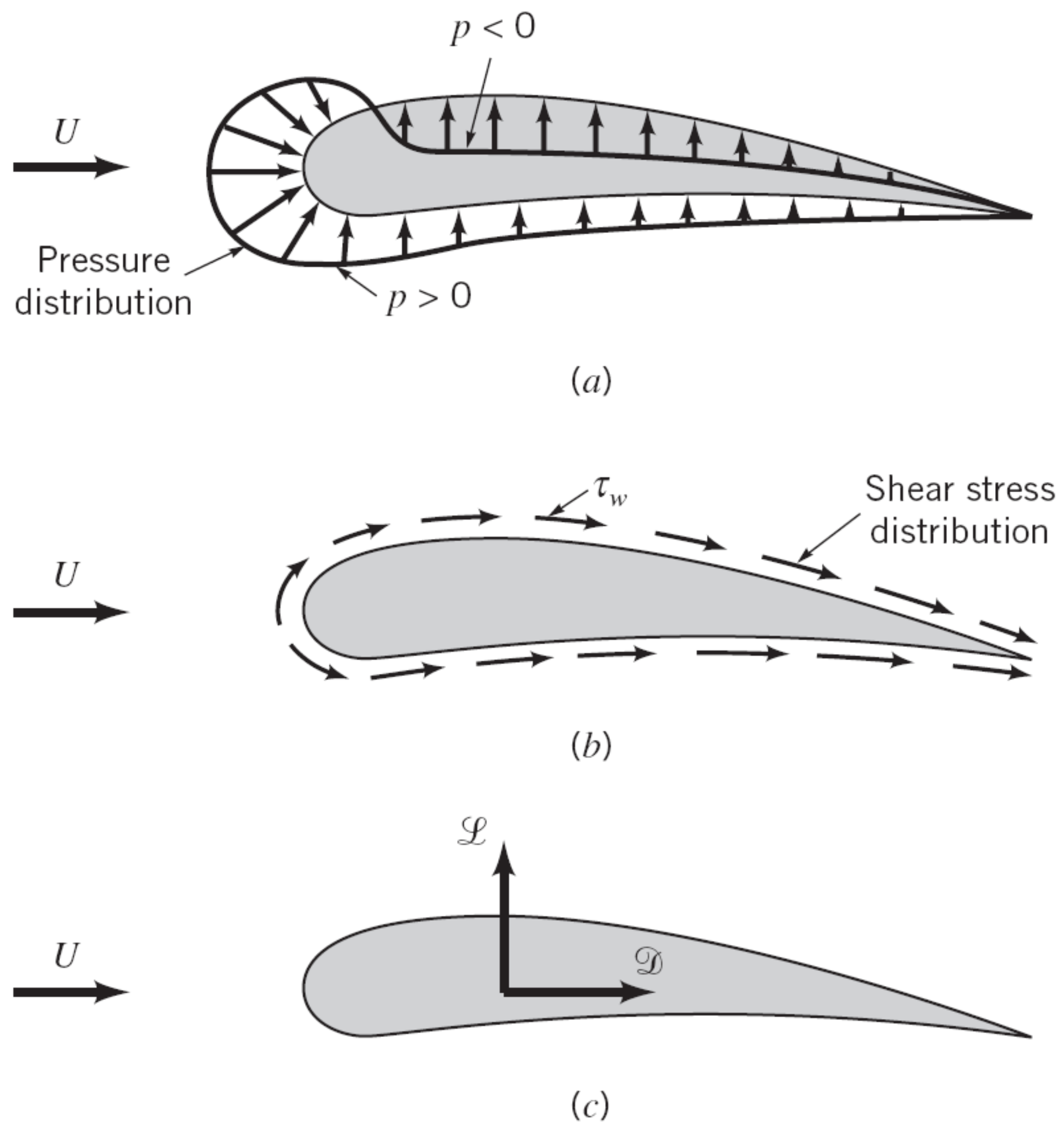
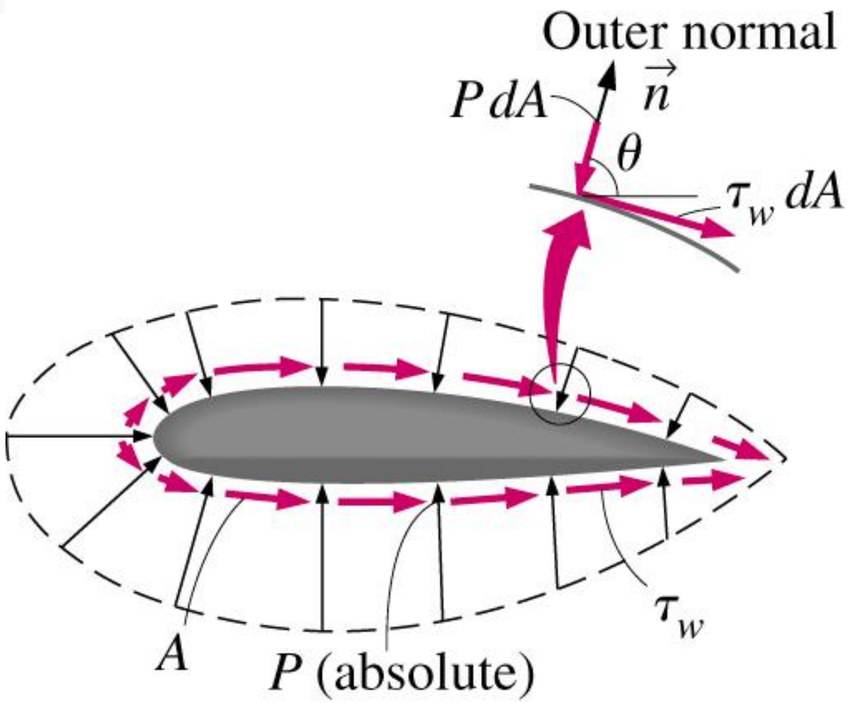
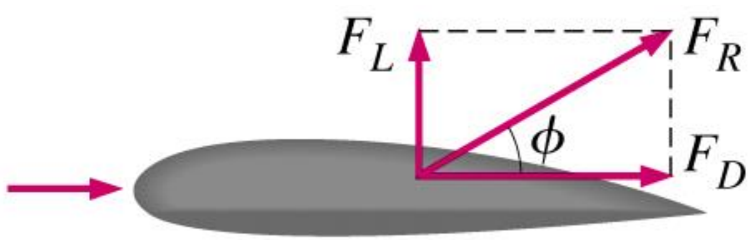


Drag and Lift



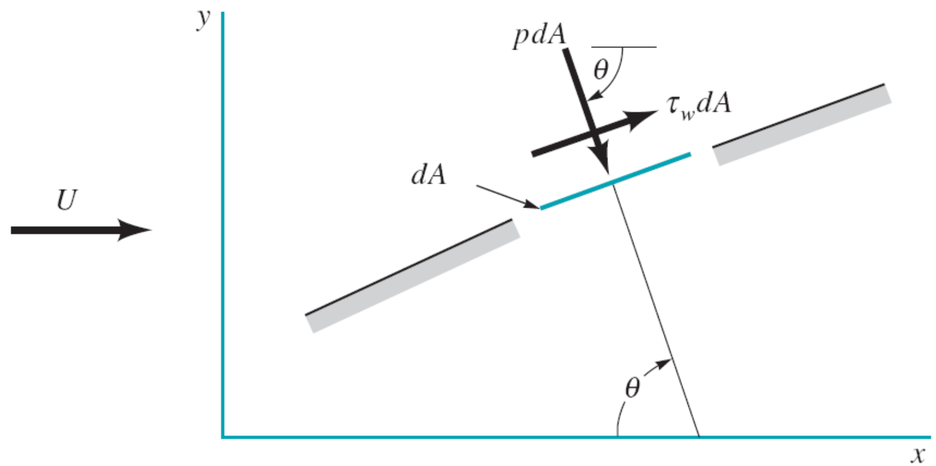


$$F_D = F_R \cos\phi$$
$$F_L = F_R \sin\phi$$

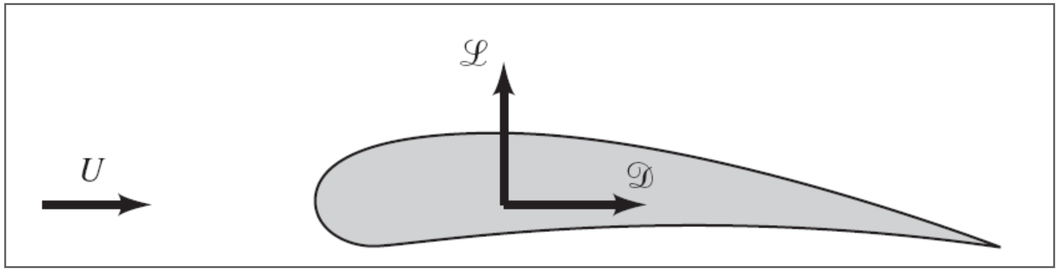
- Fluid dynamic forces are due to pressure (normal) and viscous (shear) forces acting on the body surface.
- Drag: Force component parallel to flow direction.
- Lift: Force component normal to flow direction.

Drag Forces

- Drag forces can be found by integrating pressure and wall-shear stresses.



$$dF_x = (p\,dA)\cos\theta + (\tau_w\,dA)\sin\theta$$



$$dF_y = -(p\,dA)\sin\theta + (\tau_w\,dA)\cos\theta$$

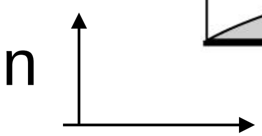
$$\mathscr{D} = \int dF_x = \int p\cos\theta\,dA + \int \tau_w \sin\theta\,dA$$

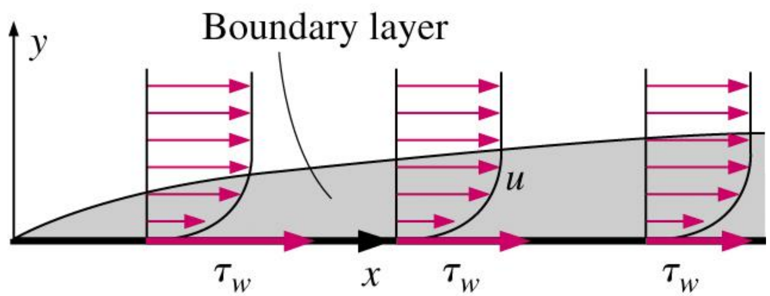
$$\mathscr{L} = \int dF_y = -\int p\sin\theta\,dA + \int \tau_w \cos\theta\,dA$$

θ is the angle between the normal vector and the direction of motion

Drag Forces

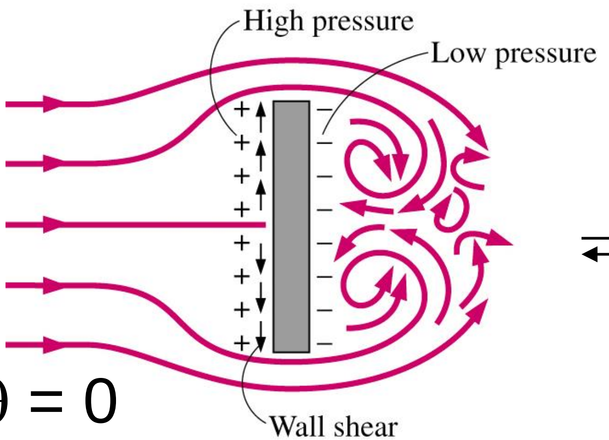
Special Cases

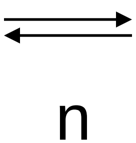




$\theta = 90$

Drag due to friction only





$\theta = 0$

Drag due to pressure only

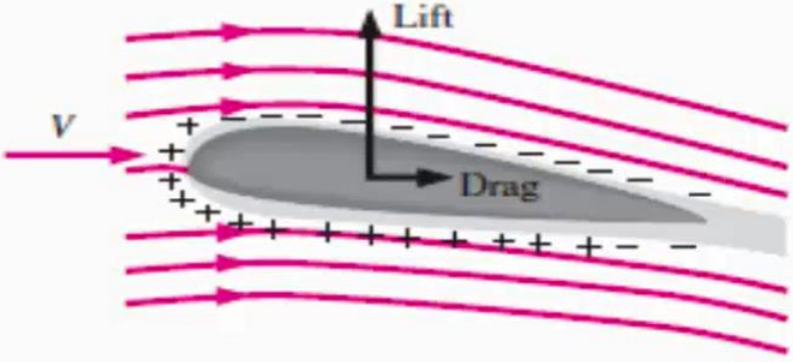
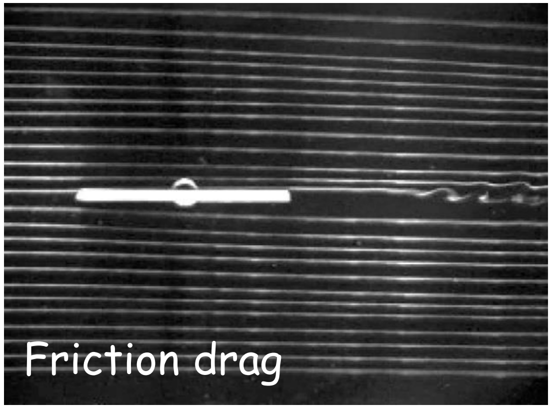
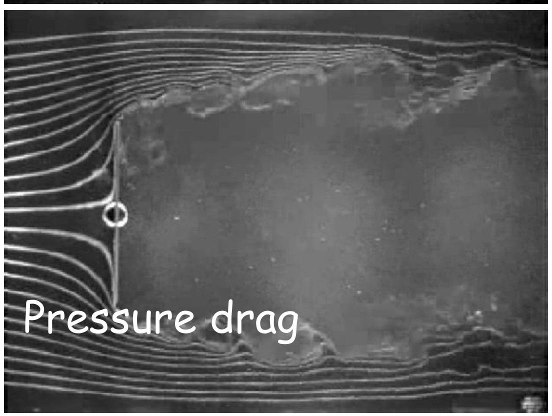


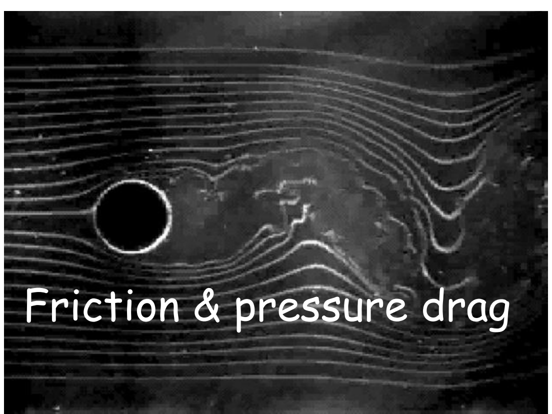
FIGURE 11–7

Airplane wings are shaped and positioned to generate sufficient lift during flight while keeping drag at a minimum. Pressures above and below atmospheric pressure are indicated by plus and minus signs, respectively.

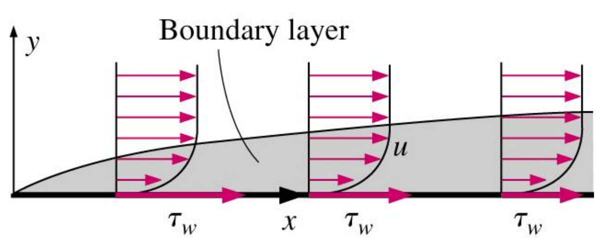
Friction Drag and Pressure Drag



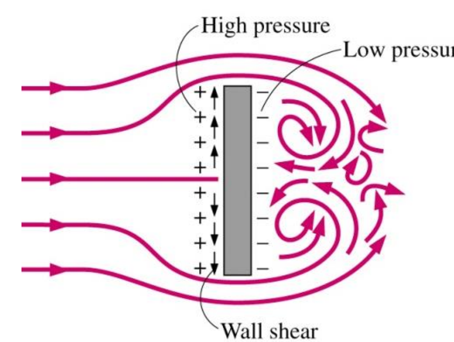




- Fluid dynamic forces are comprised of pressure (form) and friction effects.
- Often useful to decompose,
  - $F_D = F_{D,friction} + F_{D,pressure}$
  - $C_D = C_{D,friction} + C_{D,pressure}$
- This forms the basis of model testing.



$C_D = C_{D,friction}$

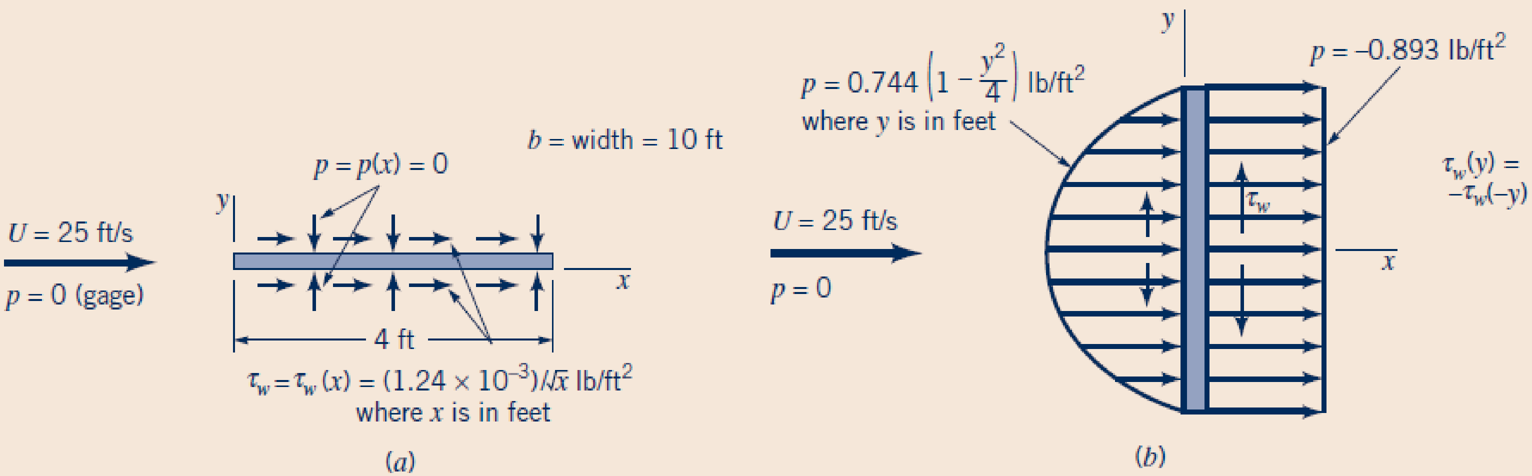


$C_D = C_{D,pressure}$

Example 9.1

GIVEN Air at standard conditions flows past a flat plate as is indicated in Fig. E9.1. In case (a) the plate is parallel to the upstream flow, and in case (b) it is perpendicular to the upstream flow. The pressure and shear stress distributions on the surface are as indicated (obtained either by experiment or theory).

FIND Determine the lift and drag on the plate.



Solution

For either orientation of the plate, the lift and drag are obtained from Eqs. 9.1 and 9.2. With the plate parallel to the upstream flow we have $\theta = 90°$ on the top surface and $\theta = 270°$ on the bottom surface so that the lift and drag are given by

$$\mathscr{L} = -\int_{\text{top}} p\, dA + \int_{\text{bottom}} p\, dA = 0$$

and

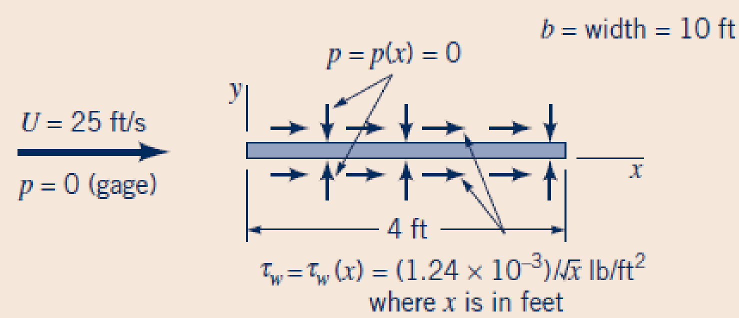
$$\mathscr{D} = \int_{\text{top}} \tau_w\, dA + \int_{\text{bottom}} \tau_w\, dA = 2\int_{\text{top}} \tau_w\, dA \qquad (1)$$

where we have used the fact that because of symmetry the shear stress distribution is the same on the top and the bottom surfaces, as is the pressure also [whether we use gage ($p = 0$) or absolute ($p = p_{\text{atm}}$) pressure]. There is no lift generated—the plate does not know up from down. With the given shear stress distribution, Eq. 1 gives

$$\mathscr{D} = 2\int_{x=0}^{4\text{ ft}} \left(\frac{1.24 \times 10^{-3}}{x^{1/2}}\ \text{lb/ft}^2\right)(10\text{ ft})\, dx$$

or

$$\mathscr{D} = 0.0992\ \text{lb} \qquad \textbf{(Ans)}$$



(a)

With the plate perpendicular to the upstream flow, we have $\theta = 0°$ on the front and $\theta = 180°$ on the back. Thus, from Eqs. 9.1 and 9.2

$$\mathscr{L} = \int_{\text{front}} \tau_w \, dA - \int_{\text{back}} \tau_w \, dA = 0$$

and

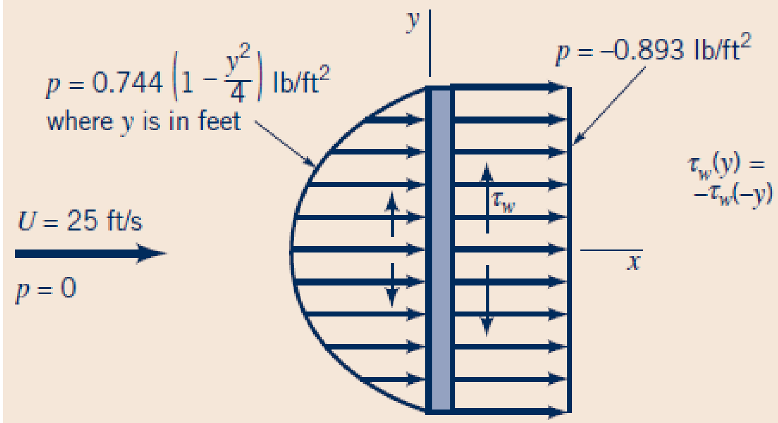
$$\mathscr{D} = \int_{\text{front}} p \, dA - \int_{\text{back}} p \, dA$$

Again there is no lift because the pressure forces act parallel to the upstream flow (in the direction of $\mathscr{D}$ not $\mathscr{L}$) and the shear stress is symmetrical about the center of the plate. With the given relatively large pressure on the front of the plate (the center of the plate is a stagnation point) and the negative pressure (less than the upstream pressure) on the back of the plate, we obtain the following drag

$$\mathscr{D} = \int_{y=-2}^{2\text{ ft}} \left[0.744 \left(1 - \frac{y^2}{4}\right) \text{lb/ft}^2 - (-0.893) \text{ lb/ft}^2 \right] (10 \text{ ft}) \, dy$$

or

$$\mathscr{D} = 55.6 \text{ lb} \qquad \textbf{(Ans)}$$

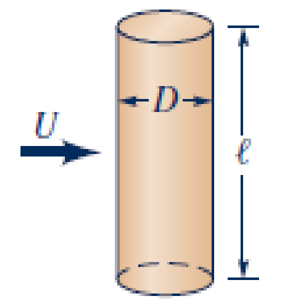


Lift Coefficient, C_L.

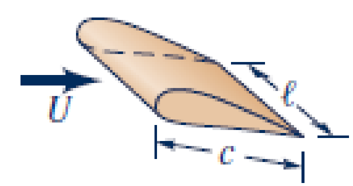
- In addition to geometry, Lift force F_L is a function of density ρ and velocity U.
- drag coefficient, C_L:

$$C_L = \frac{F_L}{\frac{1}{2}\rho U^2 A}$$

- Area *A* is a reference area: can be frontal area (the area projected on a plane normal to the direction of flow) (drag applications), plan-form area (wing aerodynamics), or wetted-surface area (ship hydrodynamics).



A = Dℓ



A = cℓ

Drag Coefficient, C_D.

- In addition to geometry, drag force F_D is a function of density ρ and velocity V.
- drag coefficient, C_D:

$$\boxed{C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}}$$

- Area A is a reference area: can be frontal area (the area projected on a plane normal to the direction of flow) (drag applications), plan-form area (wing aerodynamics), or wetted-surface area (ship hydrodynamics).
- For applications such as tapered wings, C_D may be a function of span location. For these applications, a local $C_{D,x}$ is introduced and the total drag is determined by integration over the span L

$$\boxed{C_D = \frac{1}{L}\int_0^L C_{D,x}\, dx}$$

Pressure Coefficient, C_p.

$$\boxed{C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2}}$$

- Where ρ density, p_0 reference pressure and U velocity.

Reynolds and Mach Numbers.

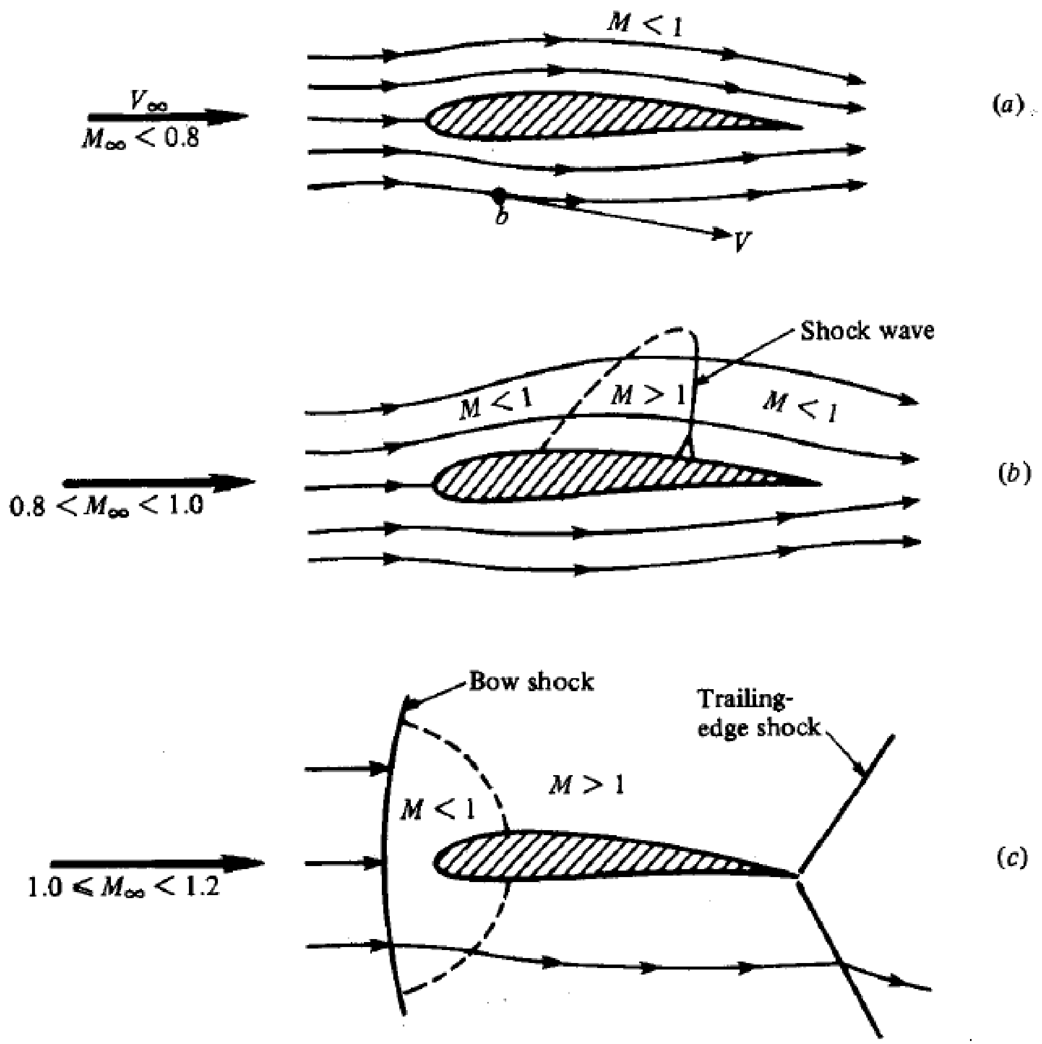
$$\boxed{Re = \frac{\rho V L}{\mu} = \frac{Inertia\ Effect}{VIscosity\ Effect}}$$

- Where ρ density, μ viscosity and V velocity.
- Area L is the characteristic length:
  for a flat plate: Plate Length
  For a circle or a sphere is Diameter

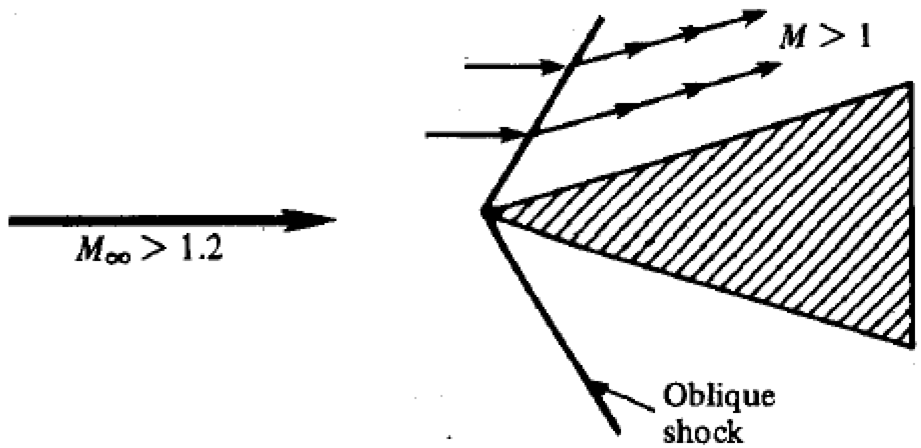
$$\boxed{M = \frac{V}{C}}$$

- Where C =speed of sound and V velocity.

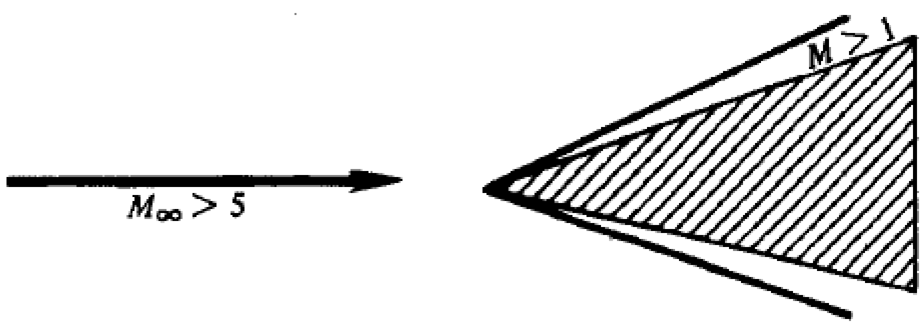
Flow Regimes



Flow Regimes



(d)



(e)

- This is the maximum velocity a falling body can attain and is called the **terminal velocity** (Fig. 2–15). The forces acting on a falling body are usually the drag force, the buoyant force, and the weight of the body.

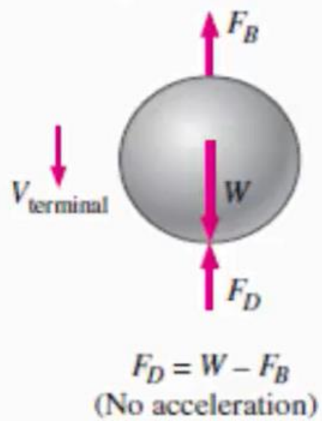


FIGURE 11–8
During a free fall, a body reaches its *terminal velocity* when the drag force equals the weight of the body minus the buoyant force.

Example: Automobile Drag

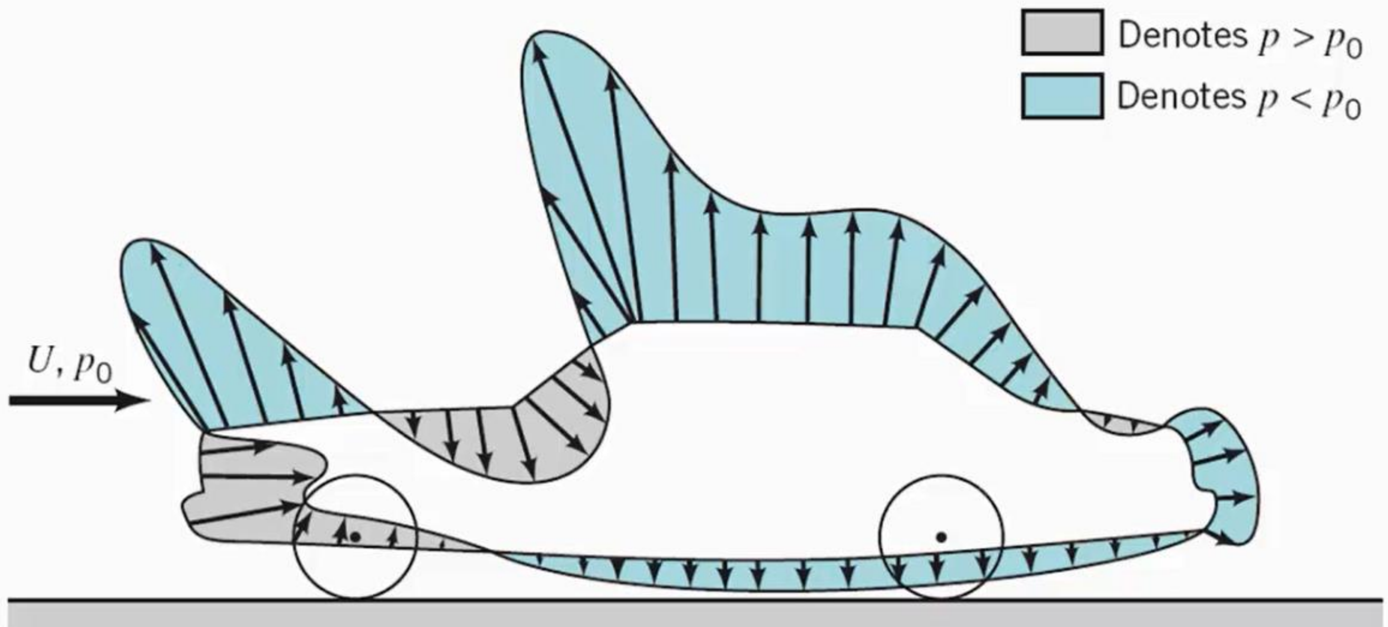
Scion XB

$C_D = 1.0$, $A = 25\ ft^2$, $C_DA = 25ft^2$

Porsche 911

$C_D = 0.28$, $A = 10\ ft^2$, $C_DA = 2.8ft^2$

- Drag force F_D=1/2$\rho V^2(C_DA)$ will be ~ 10 times larger for Scion XB
    - Source is large C_D and large projected area
- Power consumption $P = F_DV$ =1/2$\rho V^3(C_DA)$ for both scales with V^3!



Denotes $p > p_0$
Denotes $p < p_0$
U, p_0

Friction Drag

- Friction drag is due directly to the shear stress on the object

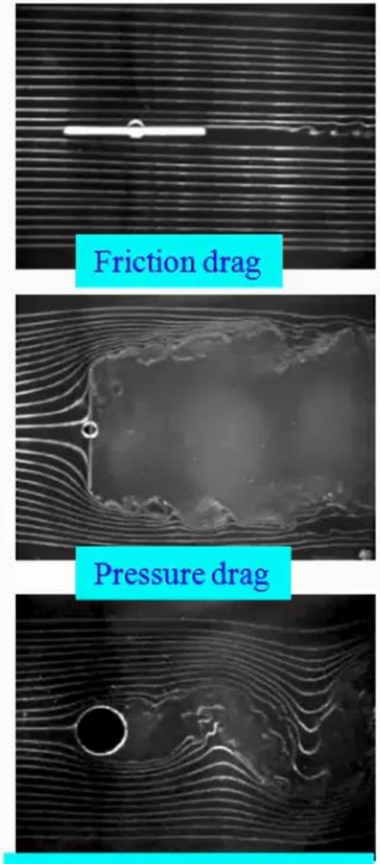
$$D_f = \frac{1}{2}\rho U^2 b \ell C_{Df}$$

C_{Df}=f (shear stress, orientation of the surface on which it acts)

$$C_{Df} = \frac{D_f}{\frac{1}{2}\rho U^2 A}$$ is the friction drag coefficient.

$$\Rightarrow C_{Df} = f\left(Re_\ell, \varepsilon / \ell\right) \quad Re_\ell = \frac{\rho U \ell}{\mu}$$

Friction and Pressure Drag



Friction drag

Pressure drag

Friction & pressure drag

- Fluid dynamic forces are comprised of pressure and friction effects.
- Often useful to decompose,

$$F_D = F_{D,friction} + F_{D,pressure}$$
$$C_D = C_{D,friction} + C_{D,pressure}$$

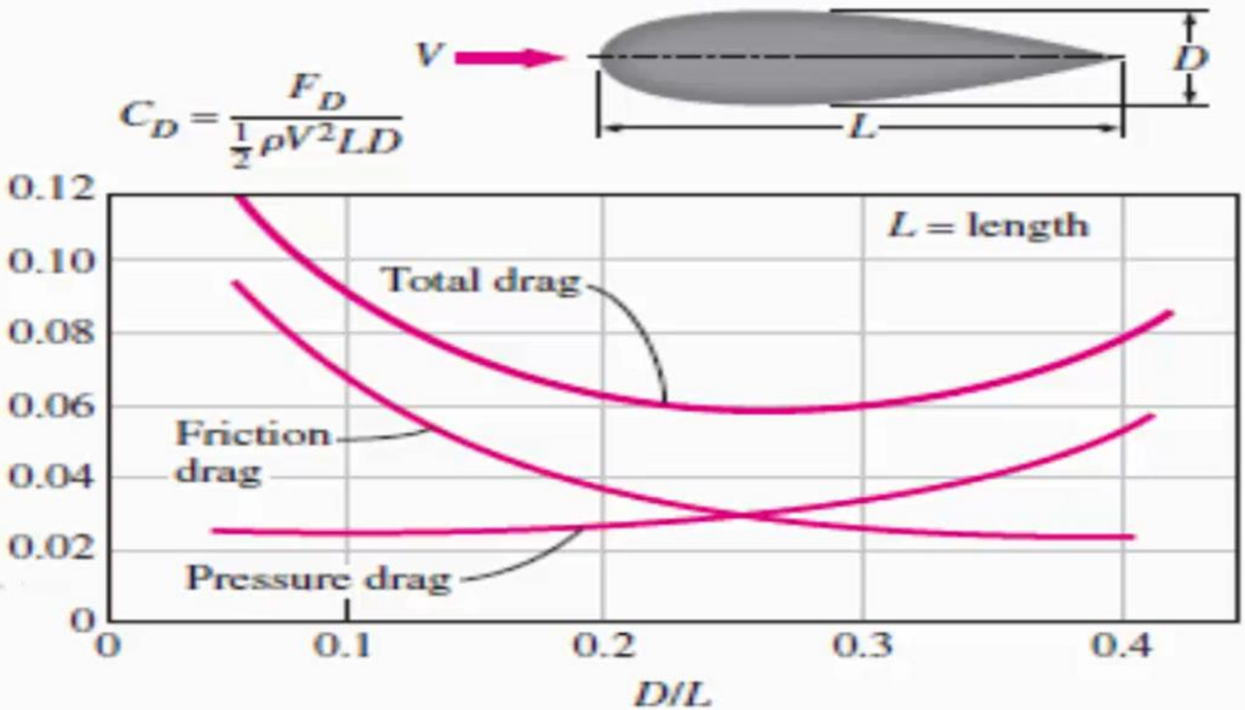


FIGURE 11–11
The variation of friction, pressure, and total drag coefficients of a streamlined strut with thickness-to-chord length ratio for Re $= 4 \times 10^4$. Note that C_D for airfoils and other thin bodies is based on *planform* area rather than frontal area.

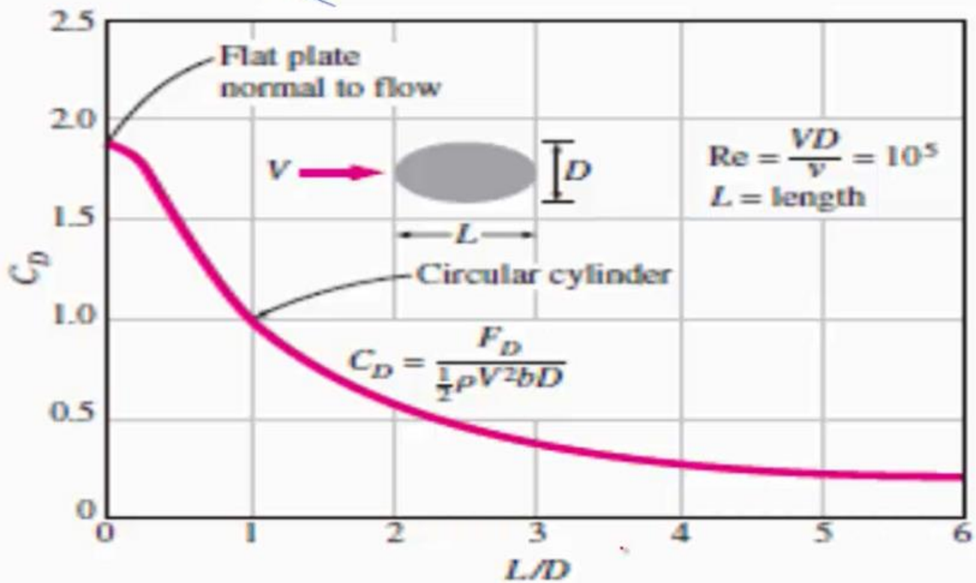
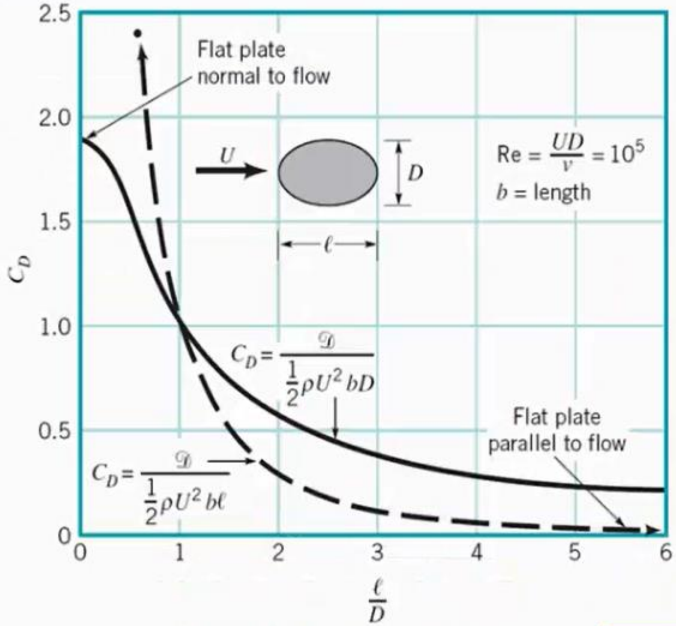


FIGURE 11–12

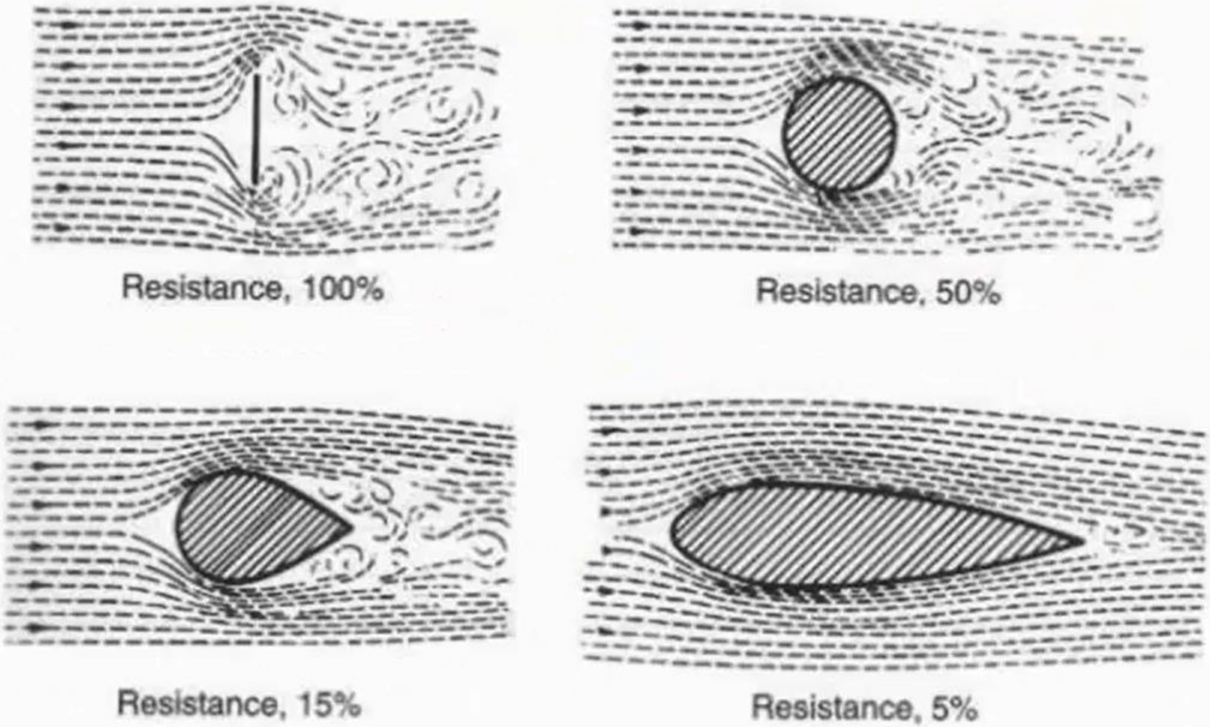
The variation of the drag coefficient of a long elliptical cylinder with aspect ratio. Here C_D is based on the frontal area bD where b is the width of the body.

C_D – Shape Dependence

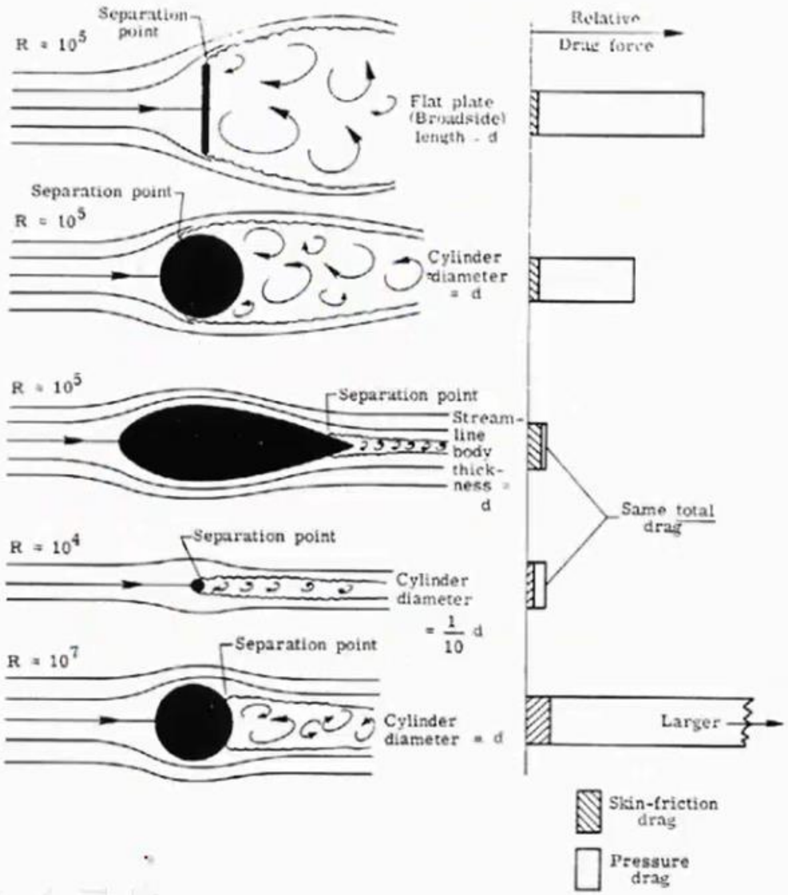
- The drag coefficient for an object depends on the shape of the object, with shapes ranging from those that are streamlined to those that are blunt.
- Drag coefficient for an ellipse with the characteristic area either the frontal area, A=bD, or the planform area, A=bℓ.



Streamlining



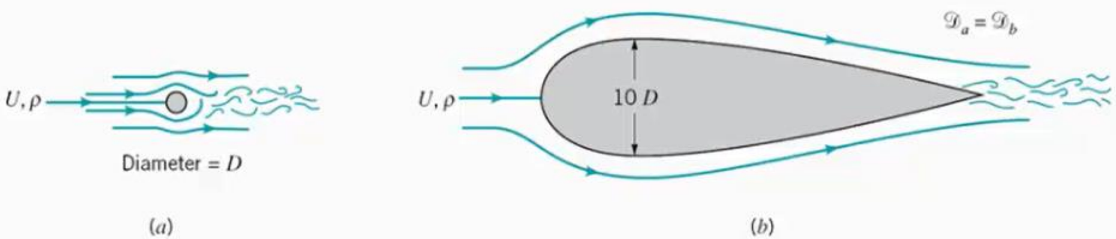
C_D – Shape Dependence Streamlining



- Streamlining reduces drag by reducing $F_{D,pressure}$, at the cost of increasing wetted surface area and $F_{D,friction}$.
- Goal is to eliminate flow separation and minimize total drag F_D
- Also improves structural acoustics since separation and vortex shedding can excite structural modes.

C_D – Shape Dependence

- Two objects of considerably different size that have the same drag force: (a) circular cylinder C_D=1.2, (b) streamlined strut C_D=0.12



Flow Separation

At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called **flow separation** (Fig. 2–23). Flow can separate from a surface even if it is fully submerged in a liquid or immersed in a gas (Fig. 2–24).

The location of the separation point depends on several factors such as the **Reynolds number, the surface roughness, and the level of fluctuations in the free stream,** and it is usually difficult to predict exactly where separation will occur **unless there are sharp corners or abrupt changes in the shape of the solid surface.**

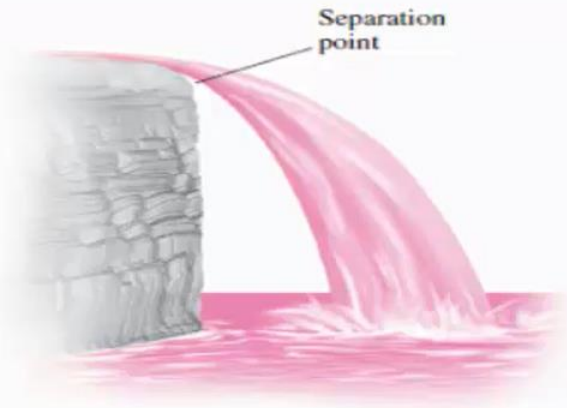


FIGURE 11–13
Flow separation in a waterfall.

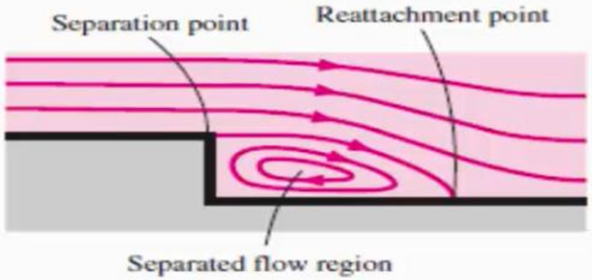


FIGURE 11–14
Flow separation over a backward-facing step along a wall.

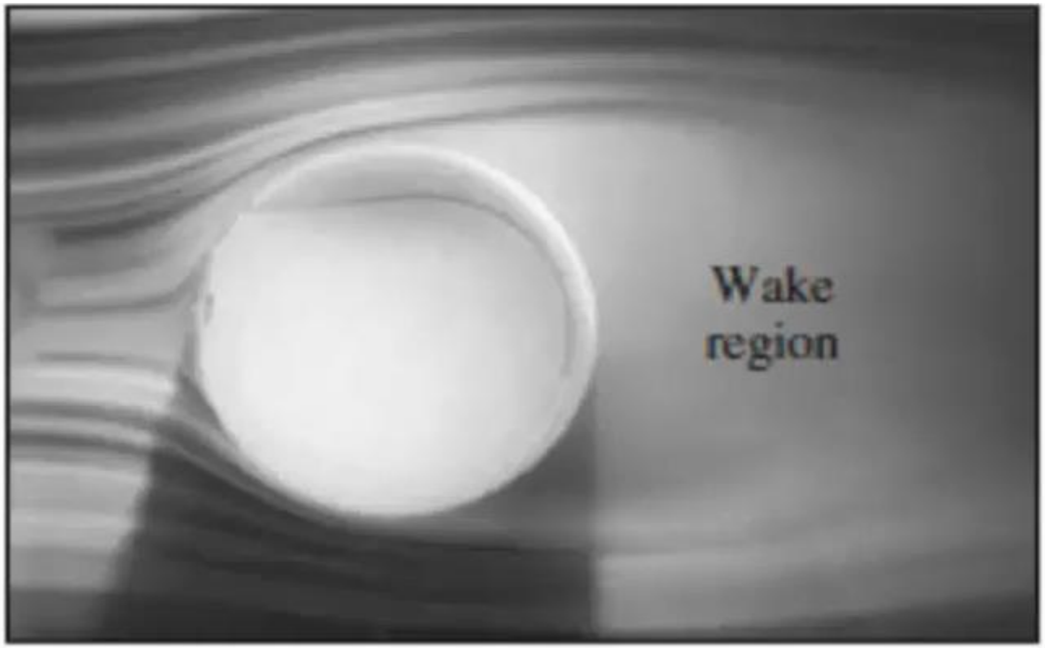
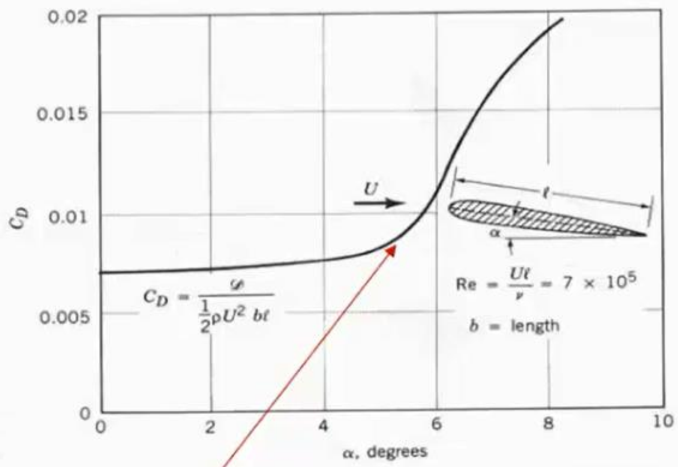


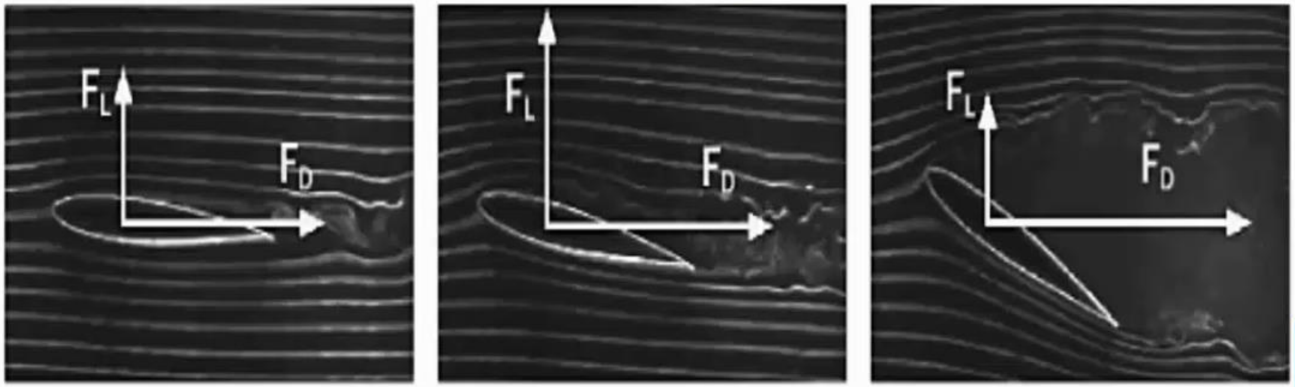
FIGURE 11–15
Flow separation during flow over a tennis ball and the wake region.

C_D – Shape Dependence Fox

- The variation of drag coefficient as a function of angle of attack for an airfoil.
- The angle of attack is small, the boundary layer remain attached to the airfoil, and the drag is relatively small.
- For angles larger than critical angle the body appears to the flow as if it were a blunt body, and the drag increases greatly.



Critical angle of attack



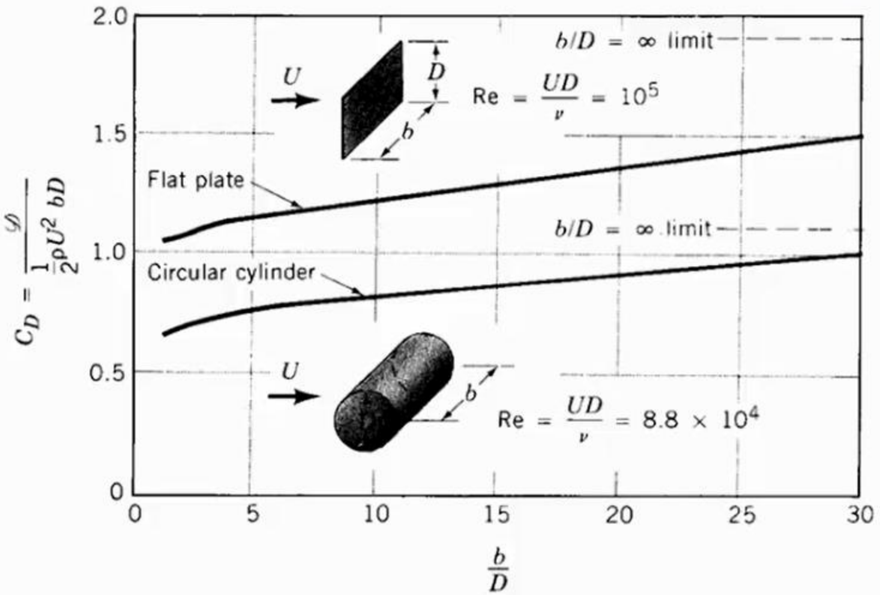
(*a*) 5°   (*b*) 15°   (*c*) 30°

FIGURE 11–16

At large angles of attack (usually larger than 15°), flow may separate completely from the top surface of an airfoil, reducing lift drastically and causing the airfoil to stall.

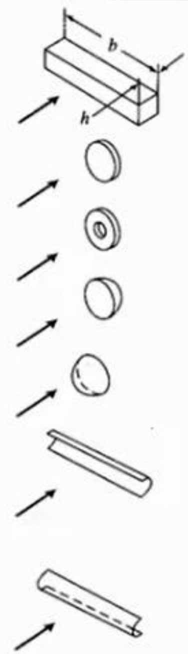
C_D – Shape Dependence Fox

- The variation of drag coefficient as a function of aspect ratio for a flat plate normal to the upstream flow and a circular cylinder.



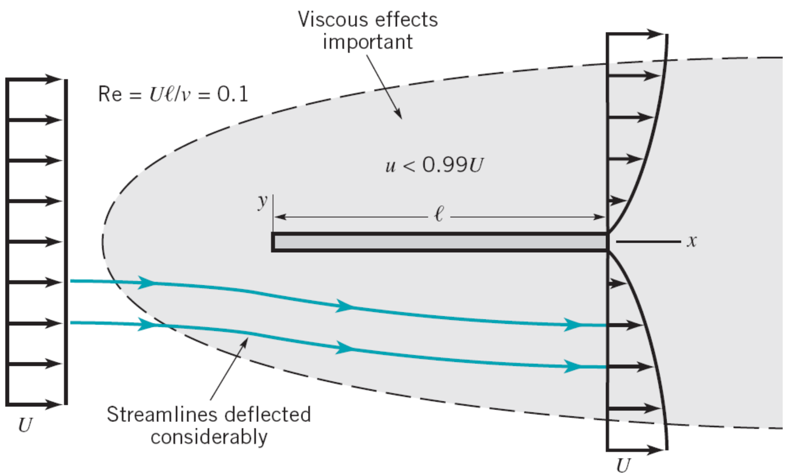
C_D – Shape Dependence Fox

- Drag coefficient for flow past a variety of objects.
- Re>1000

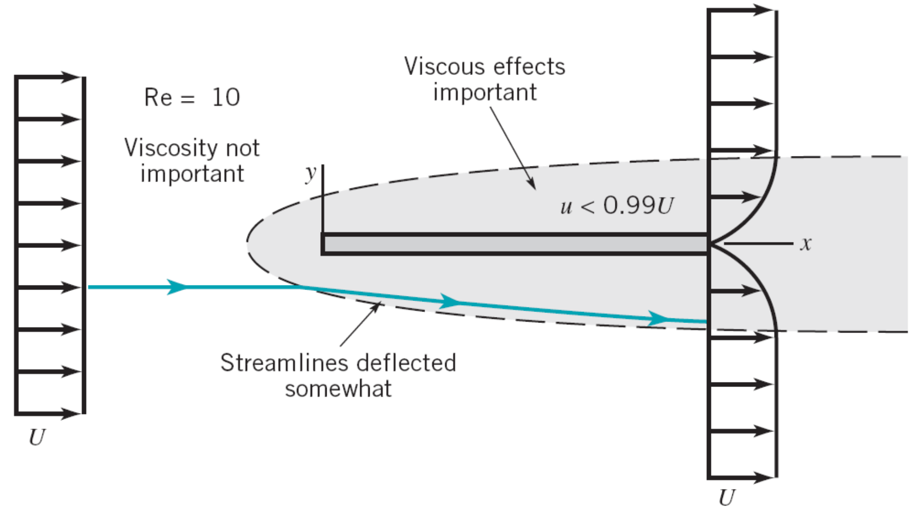
Object	Diagram		$C_D(Re \gtrsim 10^3)$
Square prism		$b/h = \infty$ $b/h = 1$	2.05 1.05
Disk			1.17
Ring			1.20[b]
Hemisphere (open end facing flow)			1.42
Hemisphere (open end facing downstream)			0.38
C-section (open side facing flow)			2.30
C-section (open side facing downstream)			1.20

Source: Fox et al.

Character of the steady, viscous flow past a flat plate parallel to the upstream velocity



(a)



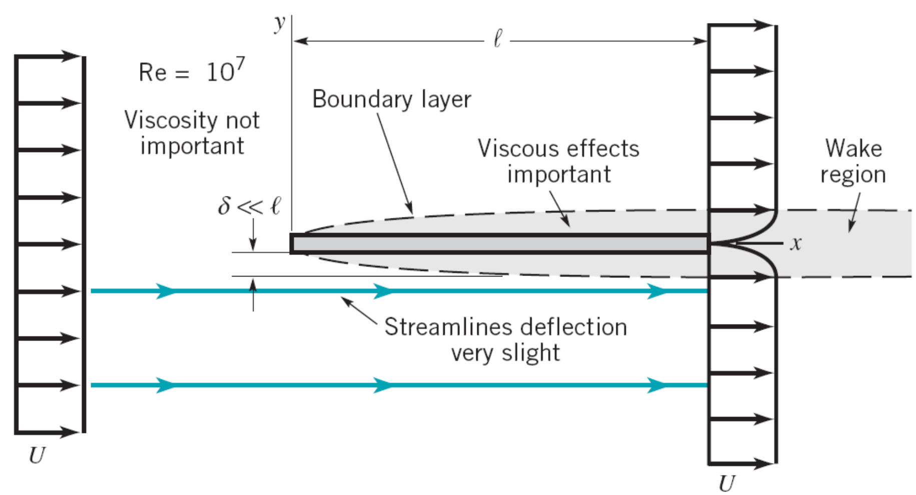
(b)

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{\mathit{Inertia\ Force}}{\mathit{Viscous\ Force}}$$

(a) low Reynolds number flow,

(b) moderate Reynolds number flow,

(c) large Reynolds number flow.



(c)

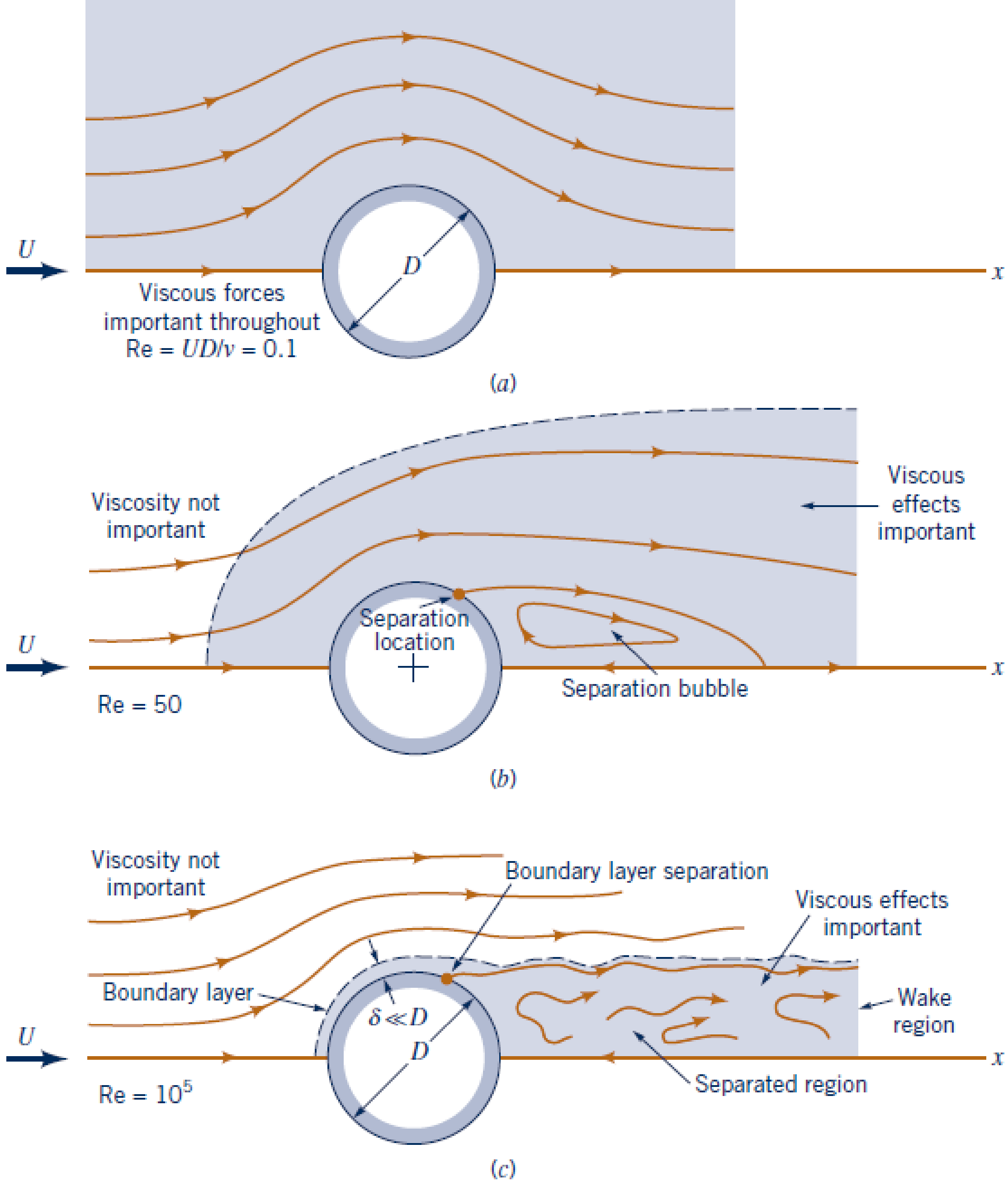
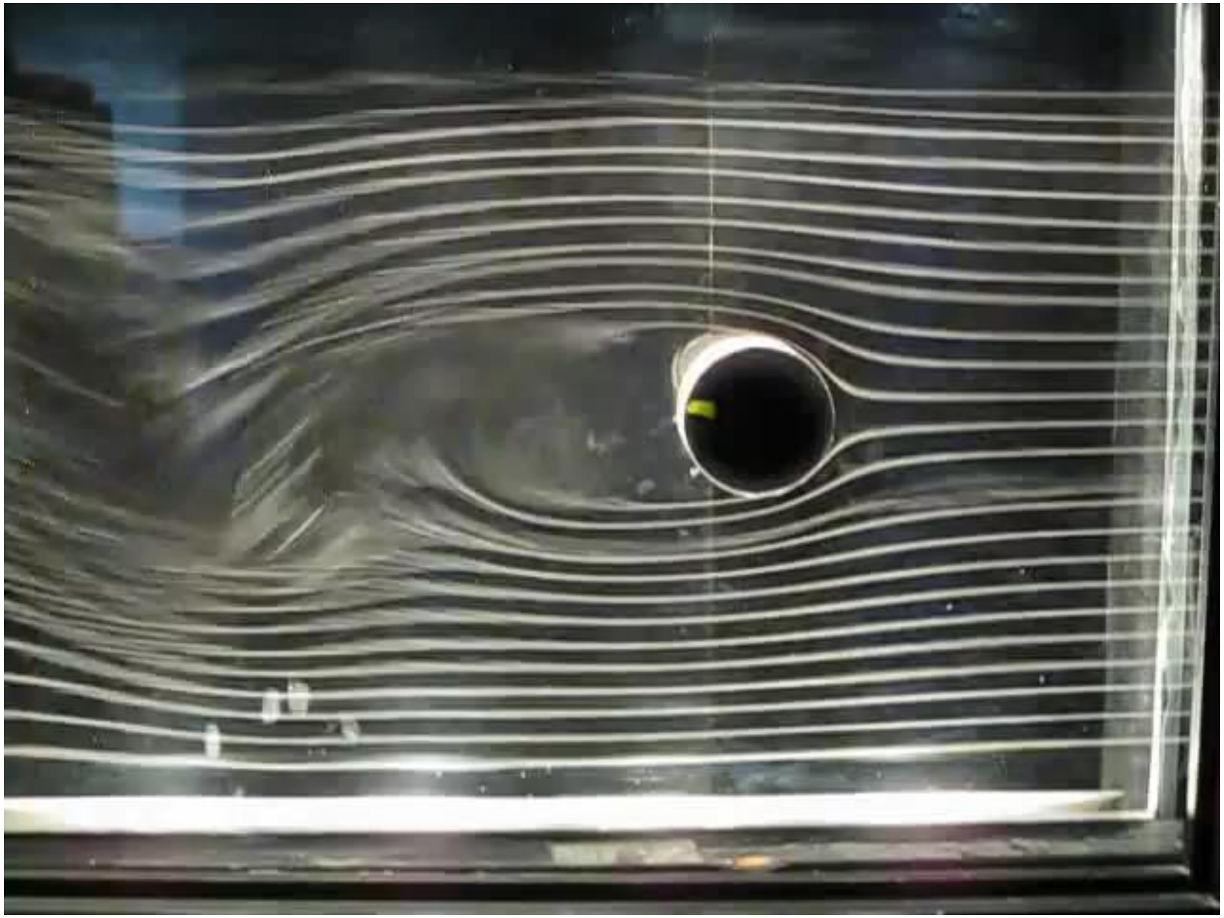
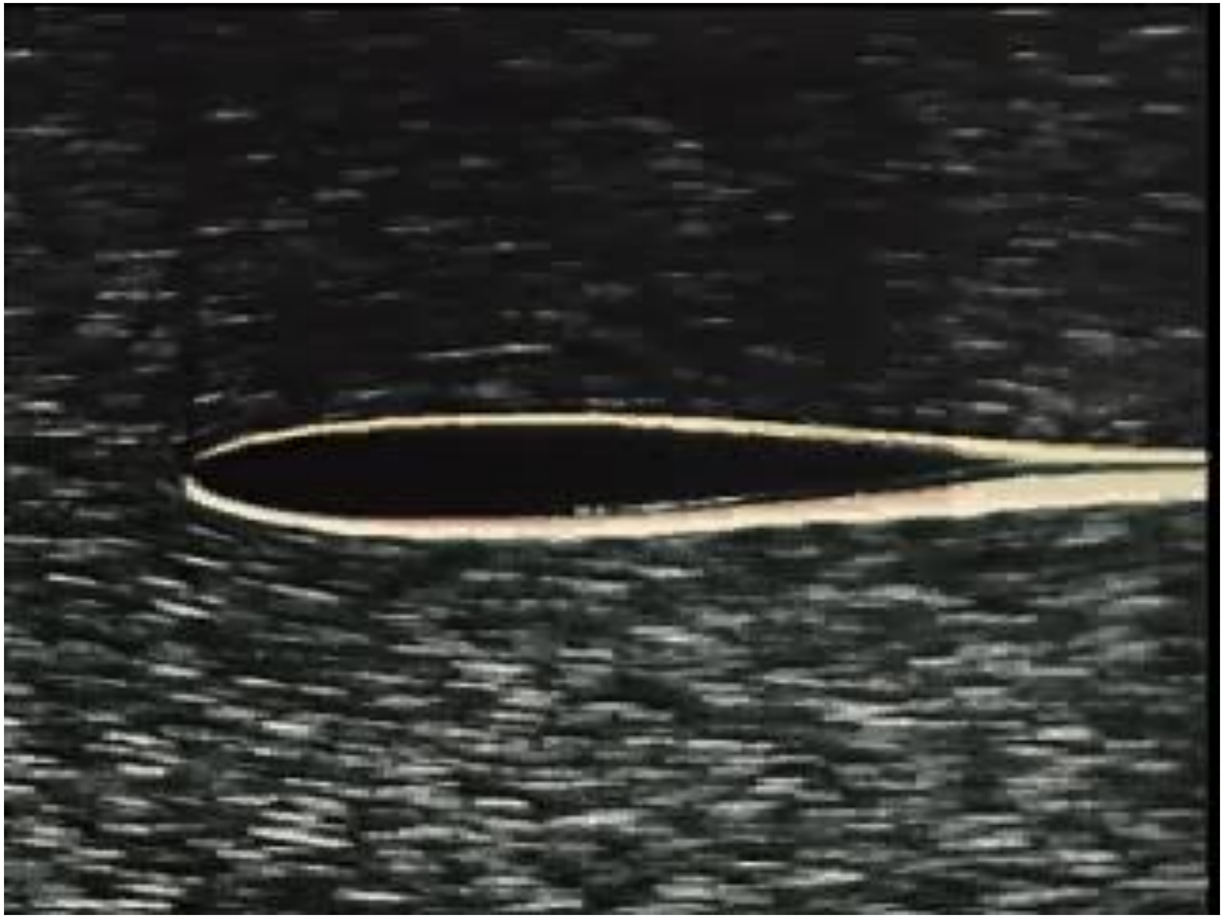


Figure 9.6 Character of the steady, viscous flow past a circular cylinder: (*a*) low Reynolds number flow, (*b*) moderate Reynolds number flow, (*c*) large Reynolds number flow.

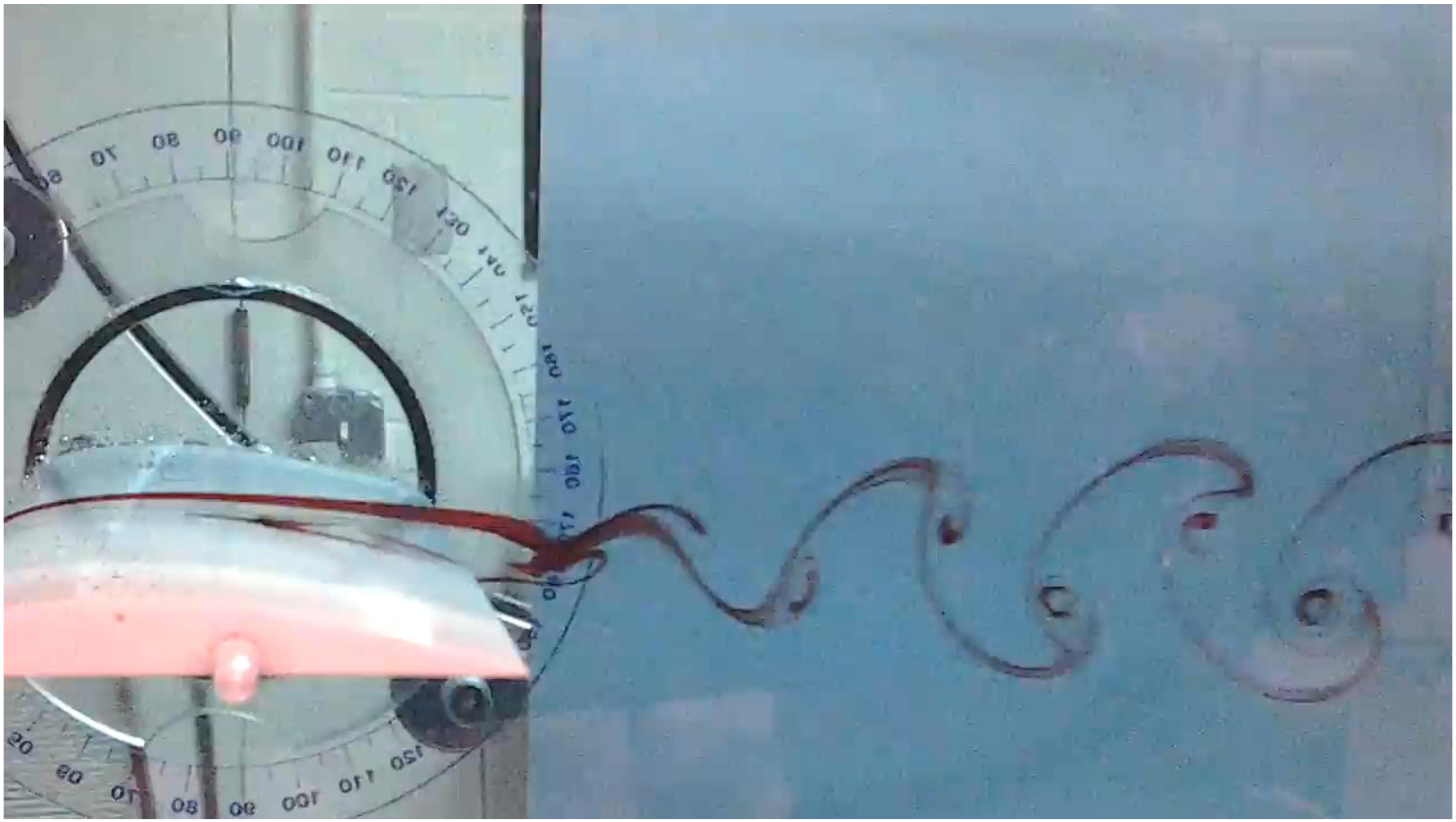
Karman Vortex Wake



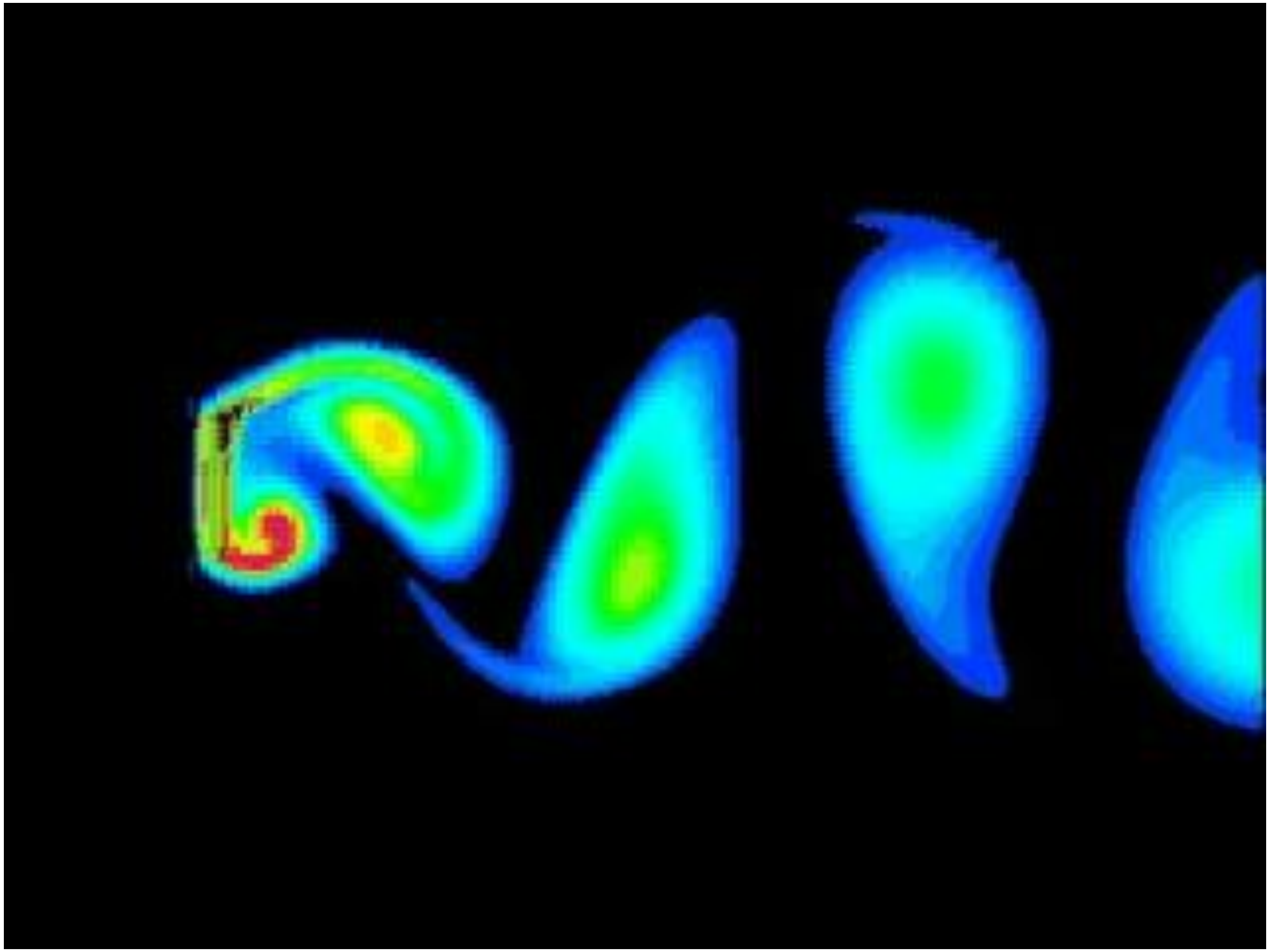
Karman Vortex Wake



Karman Vortex Wake

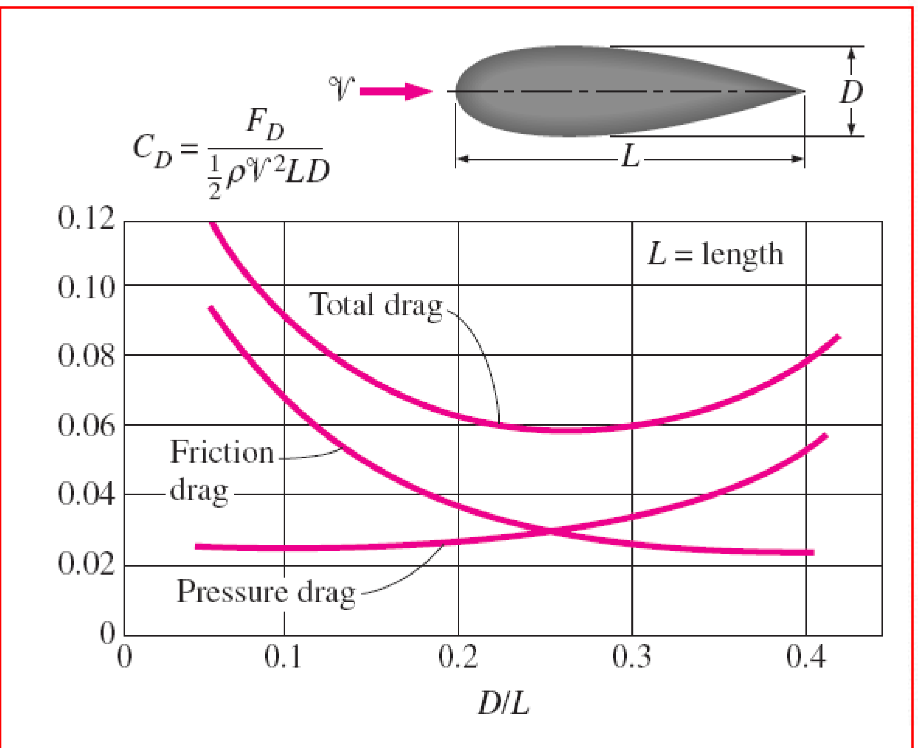


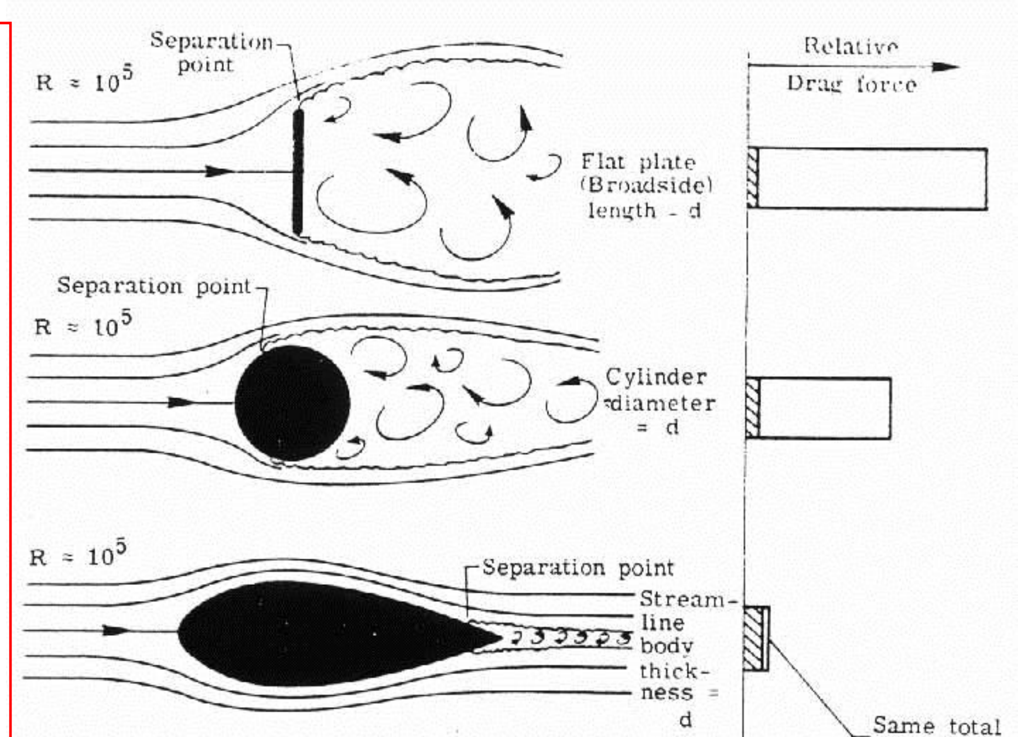
Pressure Drag on a Flat Plate



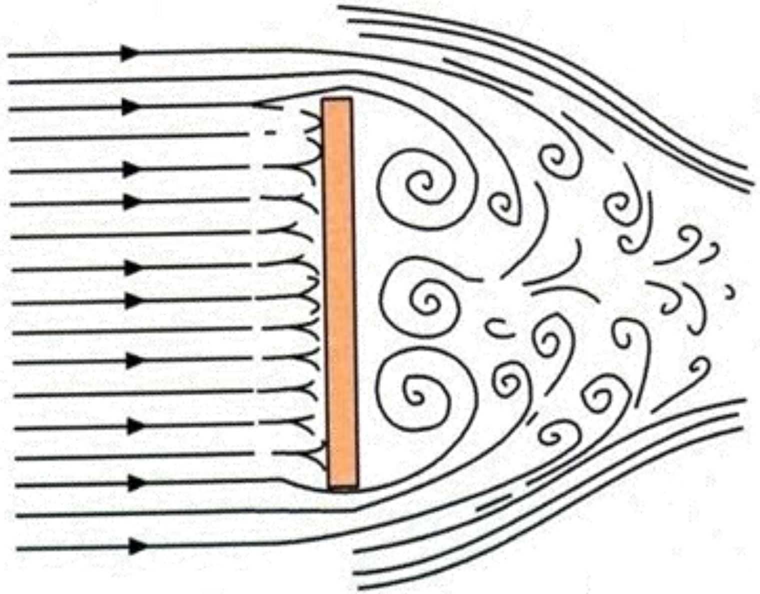
Streamlining

- Streamlining reduces drag by reducing $F_{D,pressure}$, at the cost of increasing wetted surface area and $F_{D,friction}$.
- Goal is to eliminate flow separation and minimize total drag F_D

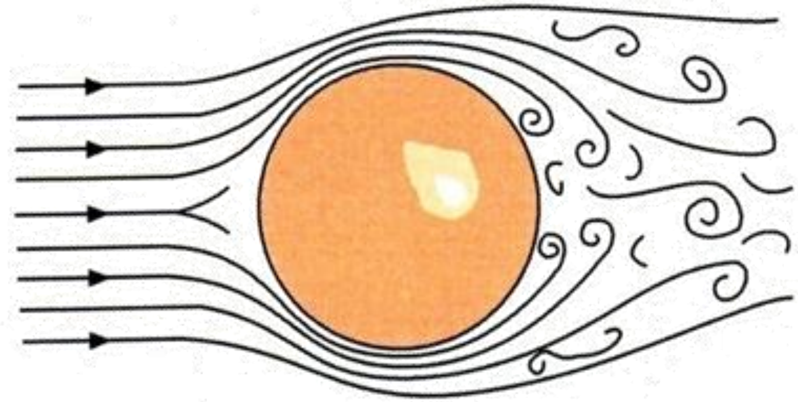




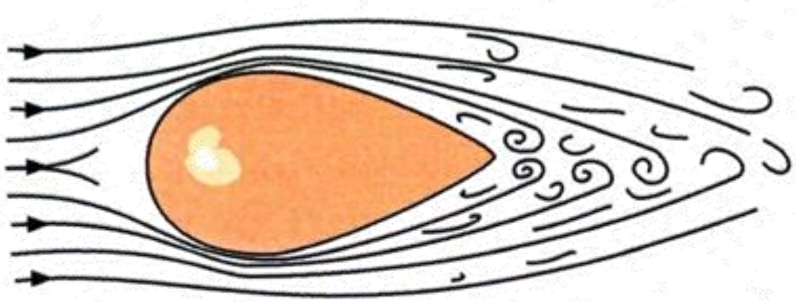
Streamlining to reduce pressure drag



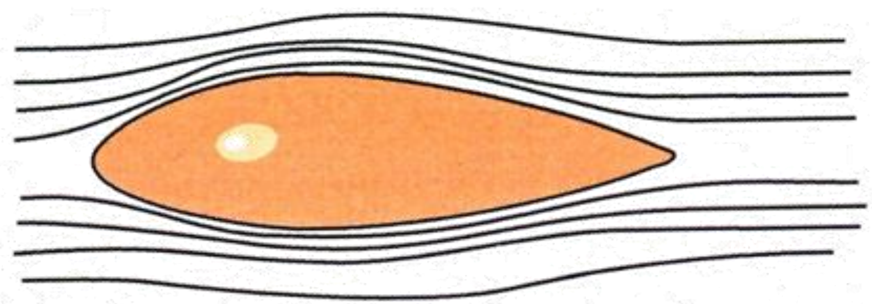
(a) Flat Plate 100% Resistance



(b) Sphere 50% Resistance



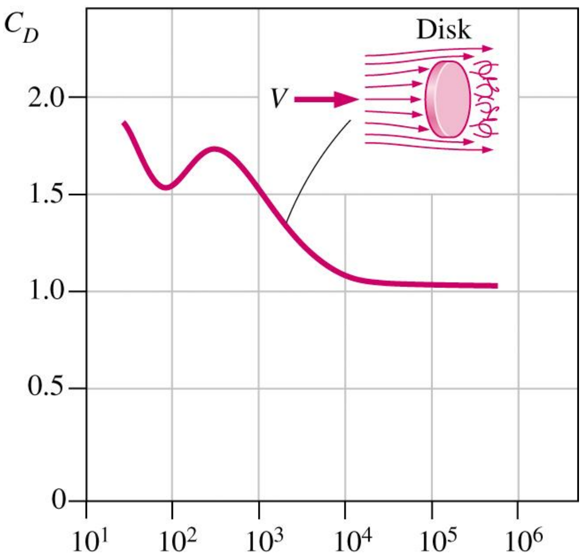
(c) Ovoid 15% Resistance



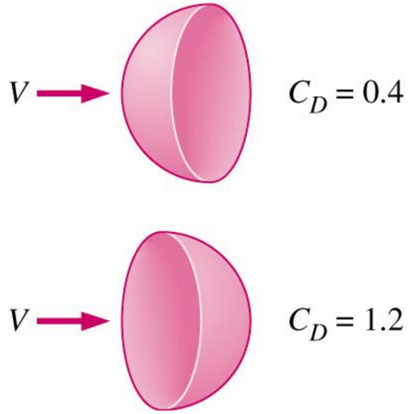
(d) Streamlined 5% Resistance

STREAMLINED

C_D of Common Geometries



A hemisphere at two different orientations for Re > 10^4



- For many geometries, total drag C_D is constant for $Re > 10^4$
- C_D can be very dependent upon orientation of body.
- As a crude approximation, superposition can be used to add C_D from various components of a system to obtain overall drag.

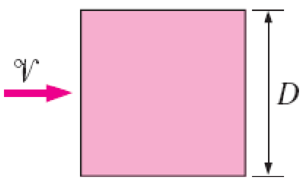
$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

$$F_D = C_D \frac{1}{2}\rho V^2 A = C_D \frac{1}{2}\rho V^2 \frac{\pi}{4} D^2$$

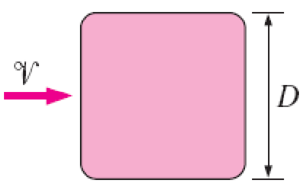
C_D of Common Geometries

Drag coefficients C_D of various two-dimensional bodies for Re $> 10^4$ based on the frontal area $A = bD$, where b is the length in direction normal to paper (for use in the drag force relation $F_D = C_D A \rho \mathcal{V}^2/2$ where $\mathcal{V}$ is the upstream velocity)

Square rod

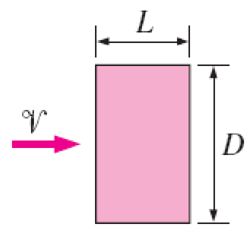


Sharp corners:
$C_D = 2.2$



Round corners
$(r/D = 0.2)$:
$C_D = 1.2$

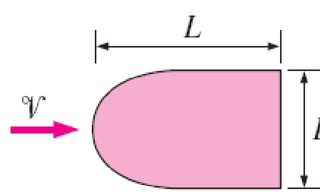
Rectangular rod



Sharp corners:

L/D	C_D
0.0*	1.9
0.1	1.9
0.5	2.5
1.0	2.2
2.0	1.7
3.0	1.3

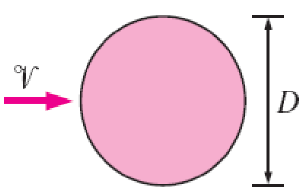
*Corresponds to thin plate



Round front edge:

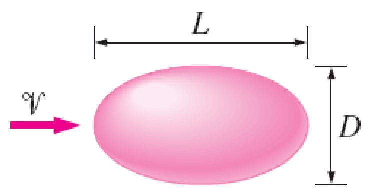
L/D	C_D
0.5	1.2
1.0	0.9
2.0	0.7
4.0	0.7

Circular rod (cylinder)



Laminar:
$C_D = 1.2$
Turbulent:
$C_D = 0.3$

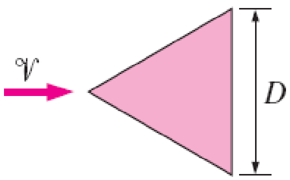
Elliptical rod



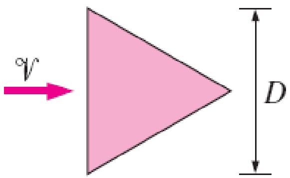
	C_D	
L/D	Laminar	Turbulent
2	0.60	0.20
4	0.35	0.15
8	0.25	0.10

Drag coefficients C_D of various two-dimensional bodies for Re > 10^4 based on the frontal area $A = bD$, where b is the length in direction normal to paper (for use in the drag force relation $F_D = C_D A \rho \mathcal{V}^2/2$ where $\mathcal{V}$ is the upstream velocity)

Equilateral triangular rod

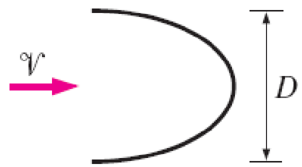


$C_D = 1.5$

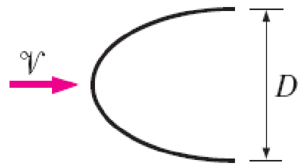


$C_D = 2.0$

Semicircular shell

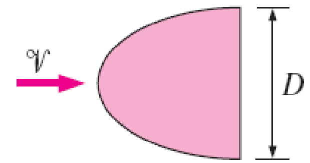


$C_D = 2.3$

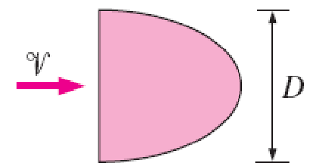


$C_D = 1.2$

Semicircular rod



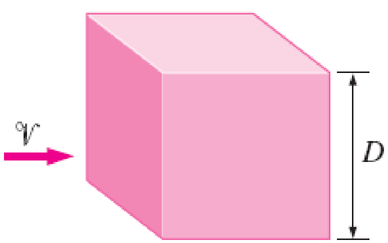
$C_D = 1.2$



$C_D = 1.7$

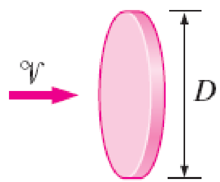
Representative drag coefficients C_D for various three-dimensional bodies for Re > 10^4 based on the frontal area (for use in the drag force relation $F_D = C_D A \rho \mathcal{V}^2/2$ where $\mathcal{V}$ is the upstream velocity)

Cube, $A = D^2$



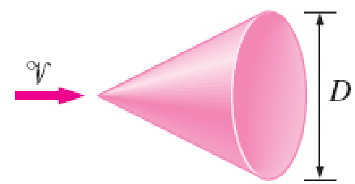
$C_D = 1.05$

Thin circular disk, $A = \pi D^2/4$



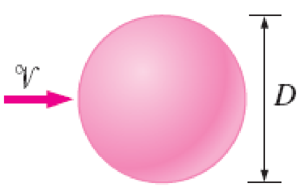
$C_D = 1.1$

Cone (for $\theta = 30°$), $A = \pi D^2/4$



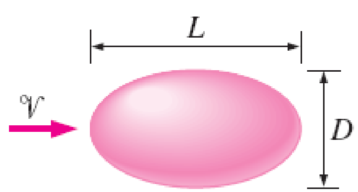
$C_D = 0.5$

Sphere, $A = \pi D^2/4$



Laminar: $C_D = 0.5$
Turbulent: $C_D = 0.2$

Ellipsoid, $A = \pi D^2/4$

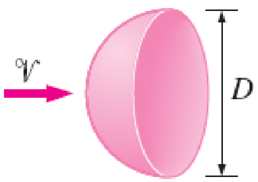


	C_D	
L/D	Laminar	Turbulent
0.75	0.5	0.2
1	0.5	0.2
2	0.3	0.1
4	0.3	0.1
8	0.2	0.1

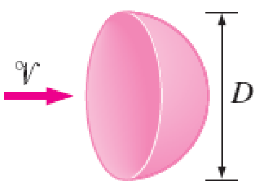
C_D of Common Geometries

Representative drag coefficients C_D for various three-dimensional bodies for Re $> 10^4$ based on the frontal area (for use in the drag force relation $F_D = C_D A\rho \mathscr{V}^2/2$ where $\mathscr{V}$ is the upstream velocity)

Hemisphere, $A = \pi D^2/4$

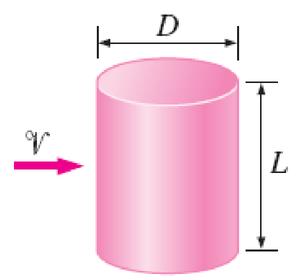


$C_D = 0.4$



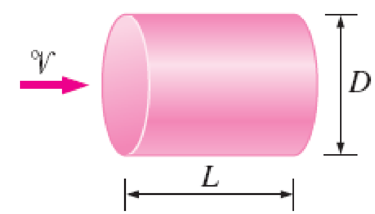
$C_D = 1.2$

Short cylinder, vertical, $A = L\ D$



L/D	C_D
1	0.6
2	0.7
5	0.8
10	0.9

Short cylinder, horizontal, $A = \pi D^2/4$

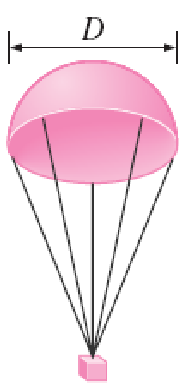


L/D	C_D
0.5	1.1
1	0.9
2	0.9
4	0.9
8	1.0

Streamlined body, $A = \pi D^2/4$

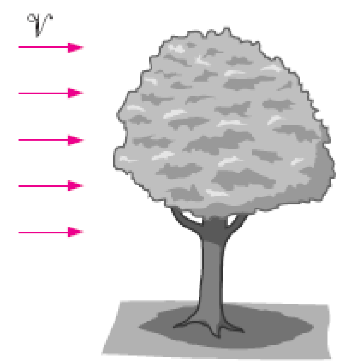
$C_D = 0.04$

Parachute, $A = \pi D^2/4$



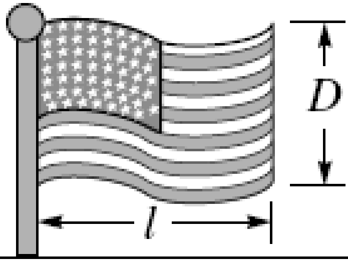
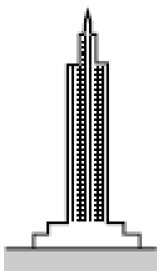
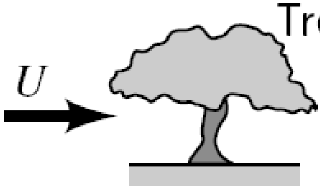
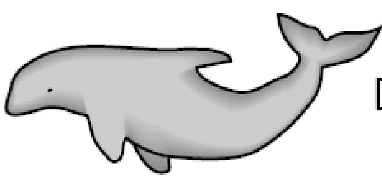
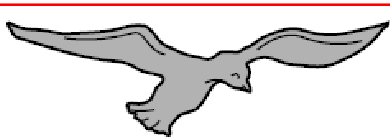
$C_D = 1.3$

Tree, A = frontal area



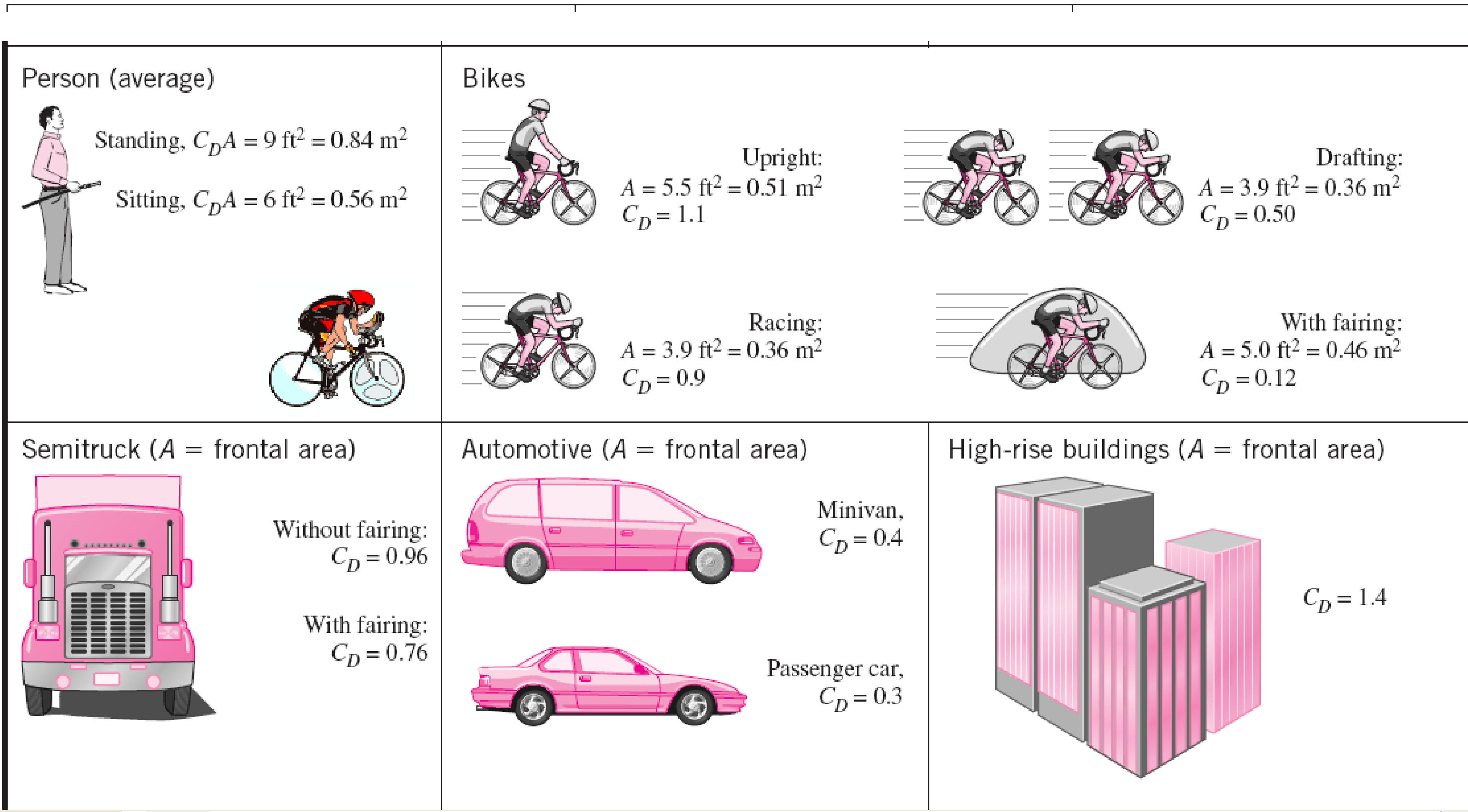
$\mathscr{V}$, m/s	C_D
10	0.4–1.2
20	0.3–1.0
30	0.2–0.7

Some more shapes

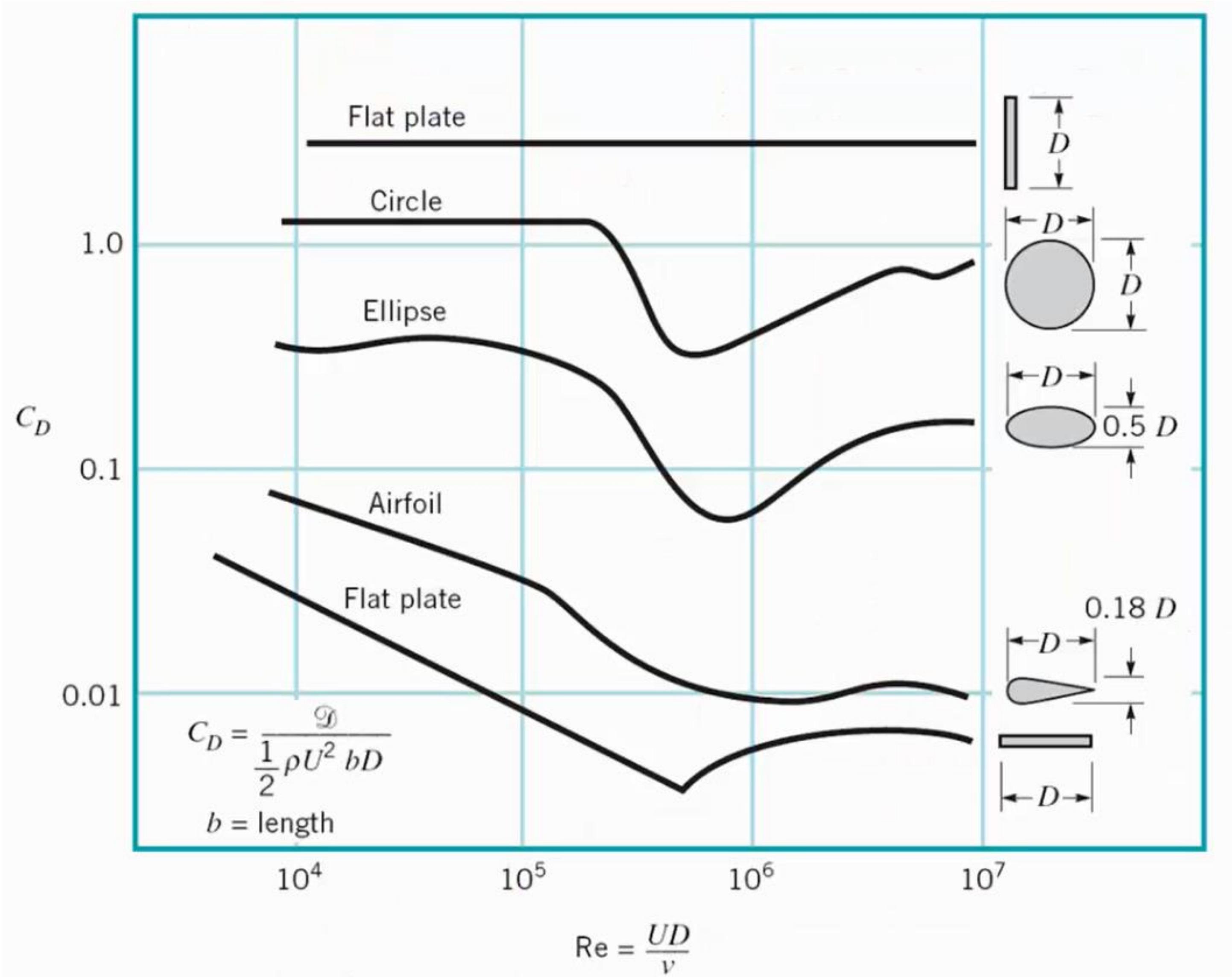
 Fluttering flag	$A = \ell D$	ℓ/D — C_D: 1 — 0.07 2 — 0.12 3 — 0.15
 Empire State Building	Frontal area	1.4
 Tree U = 10 m/s U = 20 m/s U = 30 m/s	Frontal area	0.43 0.26 0.20
 Dolphin	Wetted area	0.0036 at Re = 6×10^6 (flat plate has $C_{Df} = 0.0031$)
 Large birds	Frontal area	0.40

C_D of Common Geometries

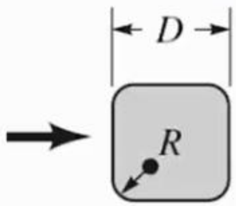
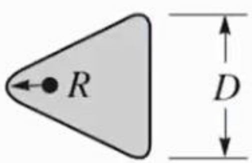
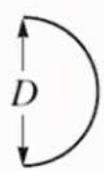
Representative drag coefficients C_D for various three-dimensional bodies for Re > 10^4 based on the frontal area (for use in the drag force relation $F_D = C_D A\rho\mathcal{V}^2/2$ where $\mathcal{V}$ is the upstream velocity)



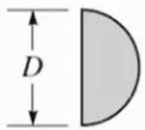
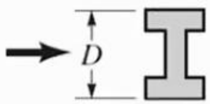
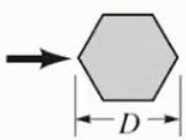
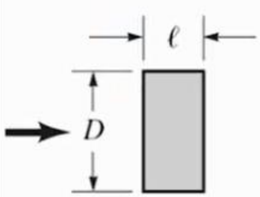
C_D of Common Geometries



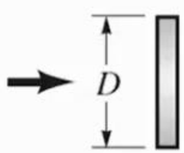
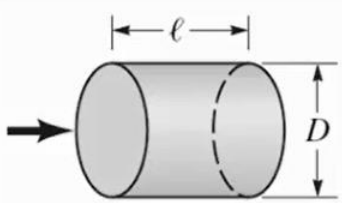
C_D of Common Geometries

Shape	Reference area A (b = length)	Drag coefficient $C_D = \dfrac{\mathcal{D}}{\frac{1}{2}\rho U^2 A}$	Reynolds number $\mathrm{Re} = \rho U D/\mu$
 Square rod with rounded corners	$A = bD$	R/D — C_D: 0 — 2.2 0.02 — 2.0 0.17 — 1.2 0.33 — 1.0	$\mathrm{Re} = 10^5$
 Rounded equilateral triangle	$A = bD$	R/D — C_D (→) — C_D (←): 0 — 1.4 — 2.1 0.02 — 1.2 — 2.0 0.08 — 1.3 — 1.9 0.25 — 1.1 — 1.3	$\mathrm{Re} = 10^5$
 Semicircular shell	$A = bD$	→ 2.3 ← 1.1	$\mathrm{Re} = 2 \times 10^4$

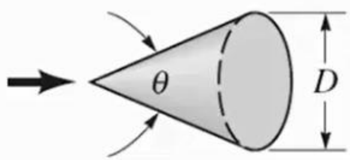
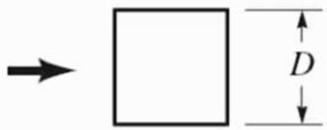
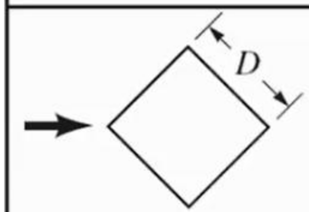
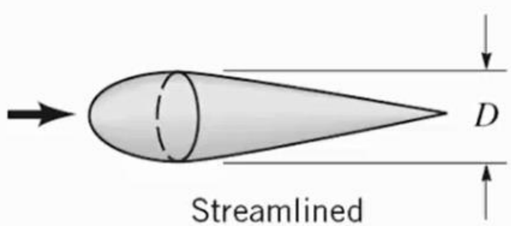
C_D of Common Geometries

	Semicircular cylinder	$A = bD$	2.15 1.15	$\mathrm{Re} > 10^4$
	T-beam	$A = bD$	1.80 1.65	$\mathrm{Re} > 10^4$
	I-beam	$A = bD$	2.05	$\mathrm{Re} > 10^4$
	Angle	$A = bD$	1.98 1.82	$\mathrm{Re} > 10^4$
	Hexagon	$A = bD$	1.0	$\mathrm{Re} > 10^4$
	Rectangle	$A = bD$	ℓ/D : C_D ≤ 0.1 : 1.9 0.5 : 2.5 0.65 : 2.9 1.0 : 2.2 2.0 : 1.6 3.0 : 1.3	$\mathrm{Re} = 10^5$

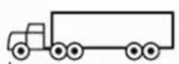
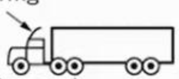
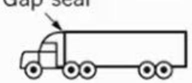
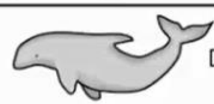
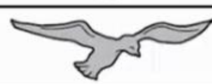
C_D of Common Geometries

Shape		Reference area A	Drag coefficient C_D	Reynolds number $Re = \rho UD/\mu$
	Solid hemisphere	$A = \frac{\pi}{4}D^2$	$\rightarrow$ 1.17 $\leftarrow$ 0.42	$Re > 10^4$
	Hollow hemisphere	$A = \frac{\pi}{4}D^2$	$\rightarrow$ 1.42 $\leftarrow$ 0.38	$Re > 10^4$
	Thin disk	$A = \frac{\pi}{4}D^2$	1.1	$Re > 10^3$
	Circular rod parallel to flow	$A = \frac{\pi}{4}D^2$	ℓ/D — C_D 0.5 — 1.1 1.0 — 0.93 2.0 — 0.83 4.0 — 0.85	$Re > 10^5$

C_D of Common Geometries

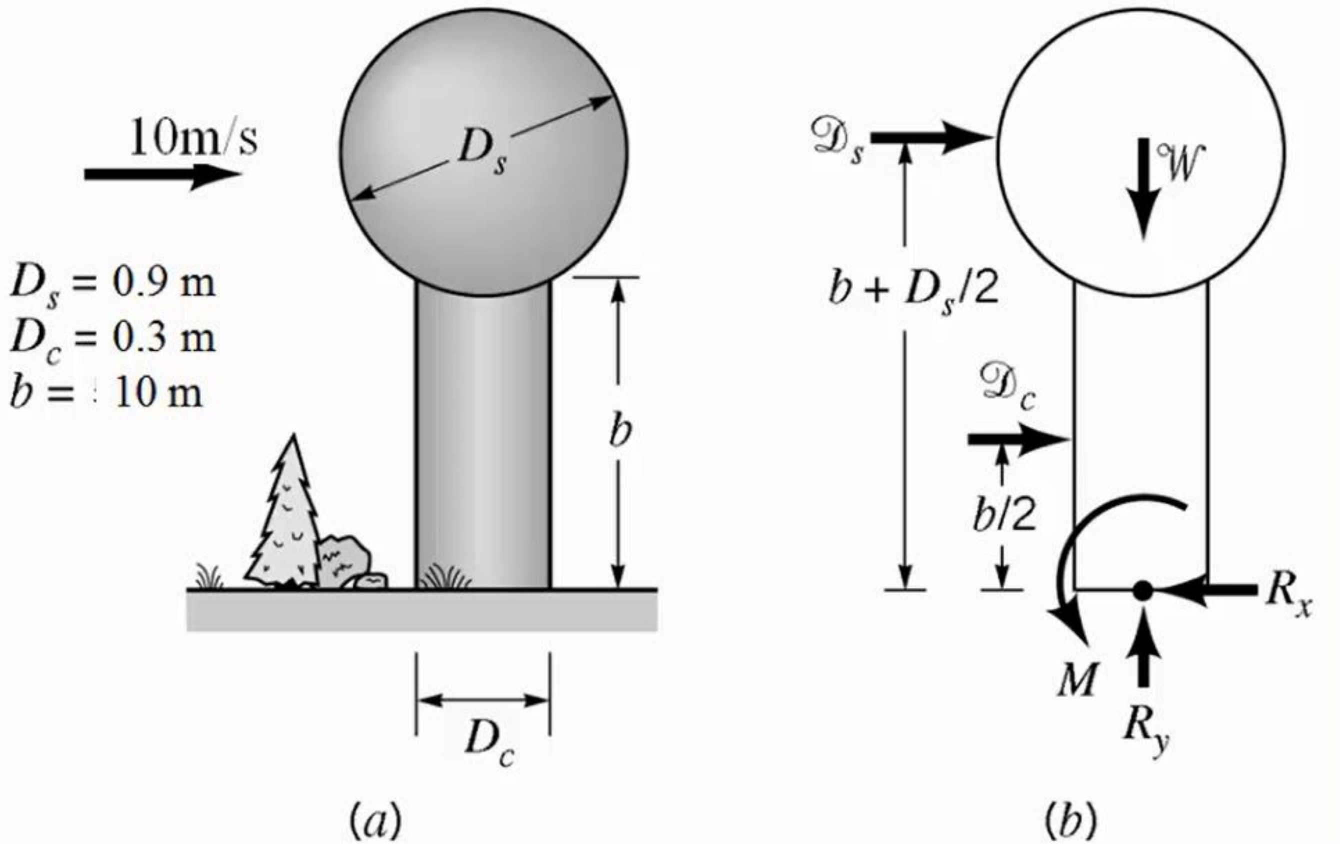
 Cone	$A = \frac{\pi}{4}D^2$	θ, degrees C_D 10 0.30 30 0.55 60 0.80 90 1.15	Re > 10^4
 Cube	$A = D^2$	1.05	Re > 10^4
 Cube	$A = D^2$	0.80	Re > 10^4
 Streamlined body	$A = \frac{\pi}{4}D^2$	0.04	Re > 10^5

C_D of Common Geometries

 Six-car passenger train	Frontal area	1.8
Bikes		
 Upright commuter	$A = 5.5\ ft^2$	1.1
 Racing	$A = 3.9\ ft^2$	0.88
 Drafting	$A = 3.9\ ft^2$	0.50
 Streamlined	$A = 5.0\ ft^2$	0.12
Tractor-trailer tucks		
 Standard	Frontal area	0.96
Fairing  With fairing	Frontal area	0.76
Gap seal  With fairing and gap seal	Frontal area	0.70
 Tree $U = 10$ m/s $U = 20$ m/s $U = 30$ m/s	Frontal area	0.43 0.26 0.20
 Dolphin	Wetted area	0.0036 at Re $= 6 \times 10^6$ (flat plate has $C_{Df} = 0.0031$)
 Large birds	Frontal area	0.40

Example

A 36 km/hr wind blows past the water tower shown in Fig. *Estimate the* moment, *M, needed at the base to keep the tower from tipping over.* ($\gamma_{air} = 1.2 * 10^{-3}$, $\upsilon_{air} = 1.5 * 10^{-5}\ m^2/s$)



- ***SOLUTION***
- We treat the water tower as a sphere resting on a circular cylinder and assume that the total drag is the sum of the drag from these parts. The free-body diagram of the tower *By summing moments about the base of the tower, we obtain:*

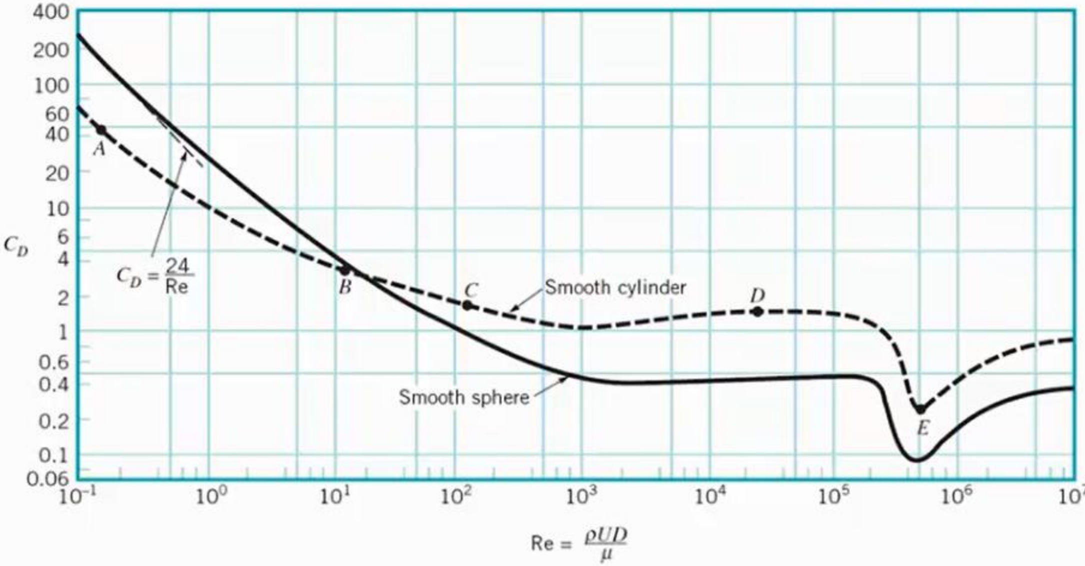
$$M = \mathscr{D}_s \left(b + \frac{D_s}{2} \right) + \mathscr{D}_c \left(\frac{b}{2} \right)$$

$$\mathscr{D}_s = \frac{1}{2} \rho U^2 \frac{\pi}{4} D_s^2 C_{Ds}$$

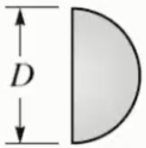
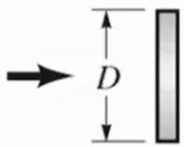
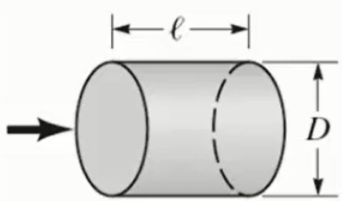
$$\mathscr{D}_c = \frac{1}{2} \rho U^2 b D_c C_{Dc}$$

$$Re_S = \frac{\rho U d_S}{\mu} = \frac{U d_S}{\upsilon} = \frac{10 * 0.9}{1.5 * 10^{-5}} = 6 * 10^5$$

$$Re_C = \frac{\rho U d_C}{\mu} = \frac{U d_C}{\upsilon} = \frac{10 * 0.3}{1.5 * 10^{-5}} = 2 * 10^5$$



(a)

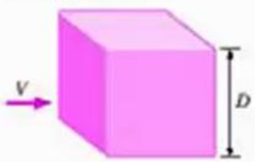
Shape		Reference area A	Drag coefficient C_D	Reynolds number $\mathrm{Re} = \rho UD/\mu$
	Solid hemisphere	$A = \frac{\pi}{4}D^2$	→ 1.17 ← 0.42	$\mathrm{Re} > 10^4$
	Hollow hemisphere	$A = \frac{\pi}{4}D^2$	→ 1.42 ← 0.38	$\mathrm{Re} > 10^4$
	Thin disk	$A = \frac{\pi}{4}D^2$	1.1	$\mathrm{Re} > 10^3$
	Circular rod parallel to flow	$A = \frac{\pi}{4}D^2$	ℓ/D | C_D 0.5 | 1.1 1.0 | 0.93 2.0 | 0.83 4.0 | 0.85	$\mathrm{Re} > 10^5$

C_D of Common Geometries

TABLE 11–2

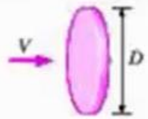
Representative drag coefficients C_D for various three-dimensional bodies for $Re > 10^4$ based on the frontal area (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream velocity)

Cube, $A = D^2$



$C_D = 1.05$

Thin circular disk, $A = \pi D^2/4$



$C_D = 1.1$

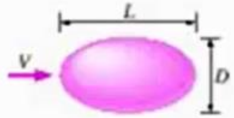
Cone (for $\theta = 30°$), $A = \pi D^2/4$

$C_D = 0.5$

Sphere, $A = \pi D^2/4$

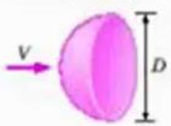
Laminar: $C_D = 0.5$
Turbulent: $C_D = 0.2$

Ellipsoid, $A = \pi D^2/4$

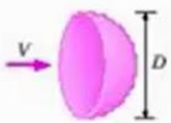


	C_D	
L/D	Laminar	Turbulent
0.75	0.5	0.2
1	0.5	0.2
2	0.3	0.1
4	0.3	0.1
8	0.2	0.1

Hemisphere, $A = \pi D^2/4$

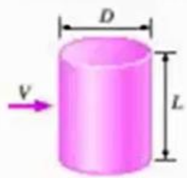


$C_D = 0.4$



$C_D = 1.2$

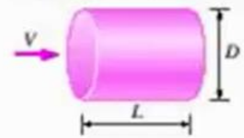
Short cylinder, vertical, $A = LD$



L/D	C_D
1	0.6
2	0.7
5	0.8
10	0.9
40	1.0
∞	1.2

Values are for laminar flow

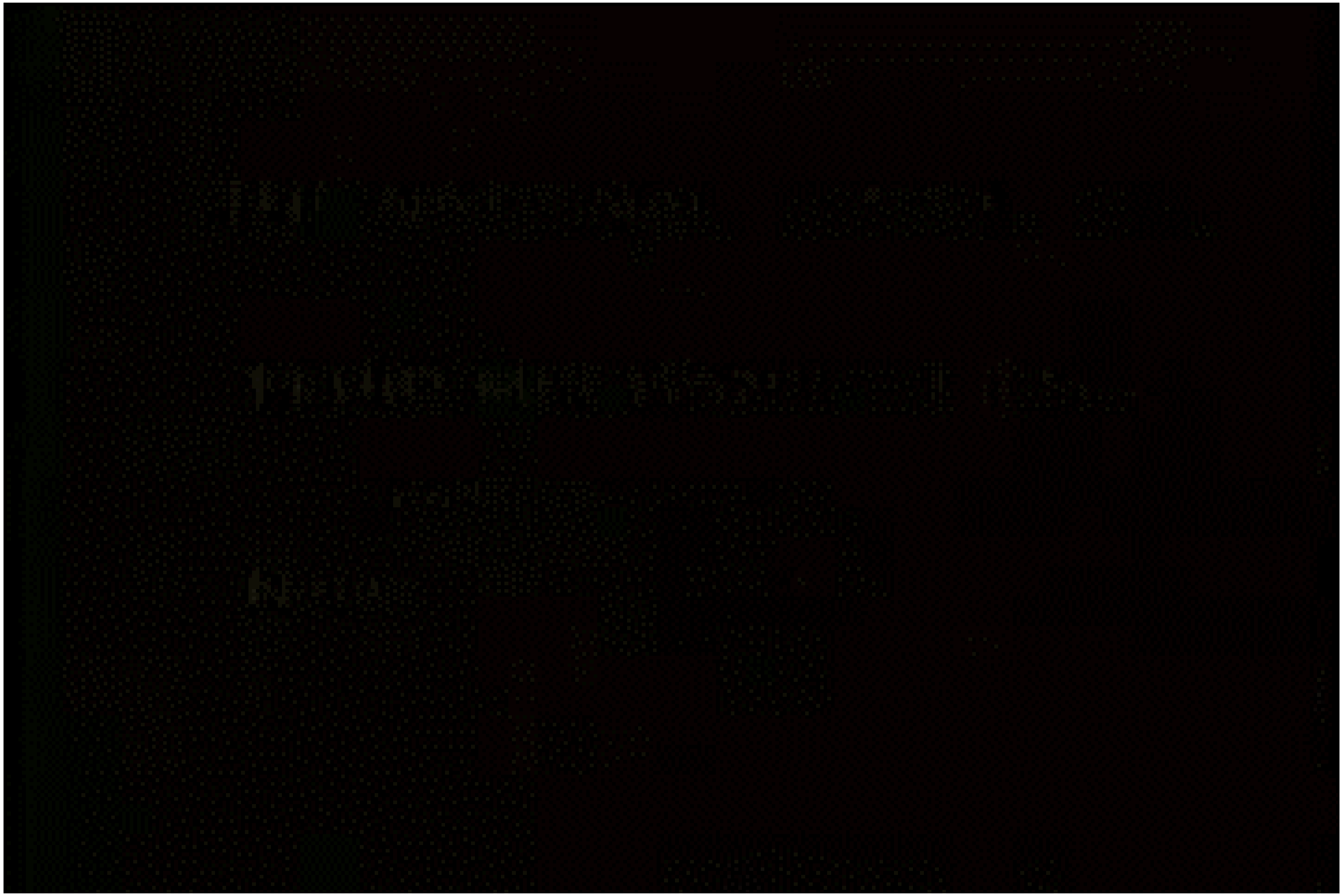
Short cylinder, horizontal, $A = \pi D^2/4$

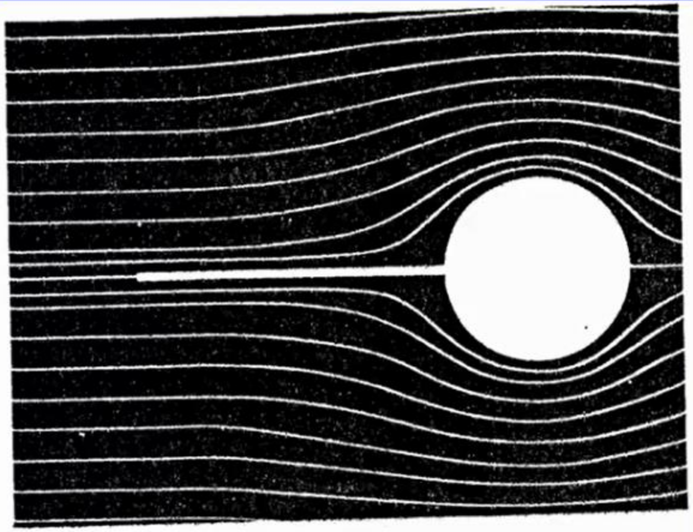


L/D	C_D
0.5	1.1
1	0.9
2	0.9
4	0.9
8	1.0

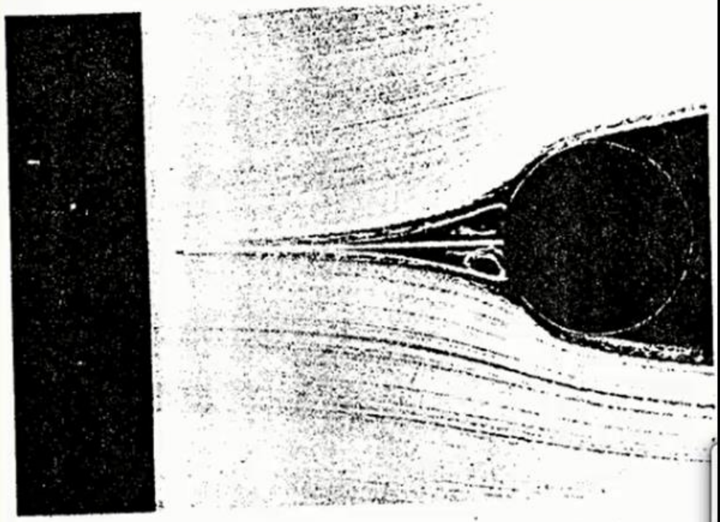
Boundary Layer

Boundary Layer Video

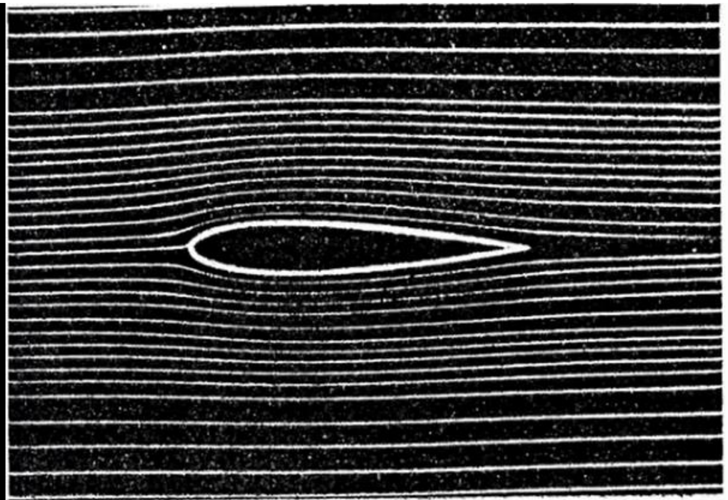




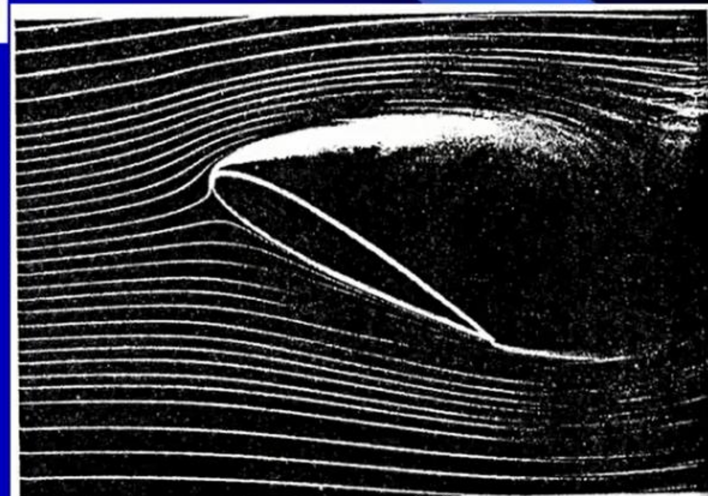
1. Potential flow streamlines about a thin plate attached to a cylinder.



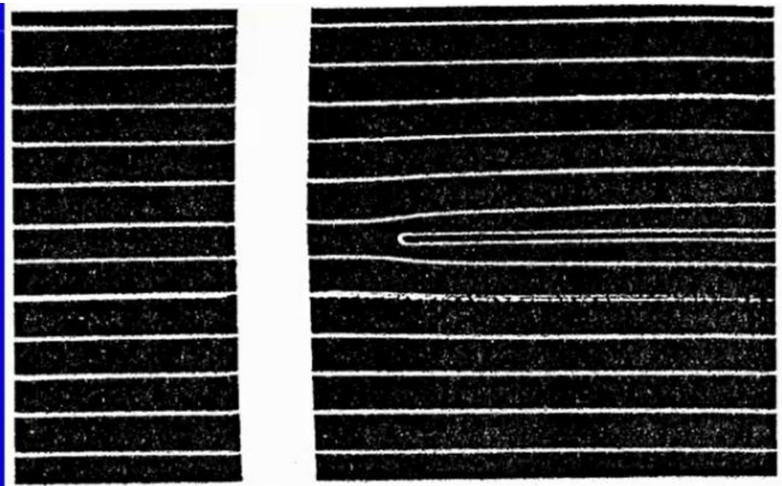
2. Hydrogen-bubble visualization of water flowing pas[t] the object in Fig. 1.



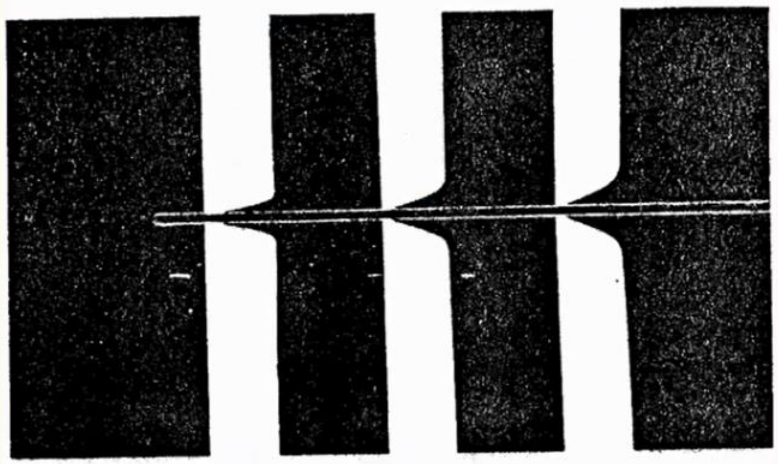
3. Smoke visualization of air flow past an airfoil at a small angle of attack.



4. Same airfoil, at a large angle of attack.



5. Flow approaching a flat plate in a water channel.



6. Timelines produced at wires perpendicular to the plate correspond closely to velocity profiles.

Reynolds Number for a flow over a Flate Plate

$$\boxed{Re = \frac{\rho V x}{\mu}}$$

- Where ρ density, μ viscosity and V velocity.
- Area x is the distance from the leading edge:
  for a flat plate: Plate Length

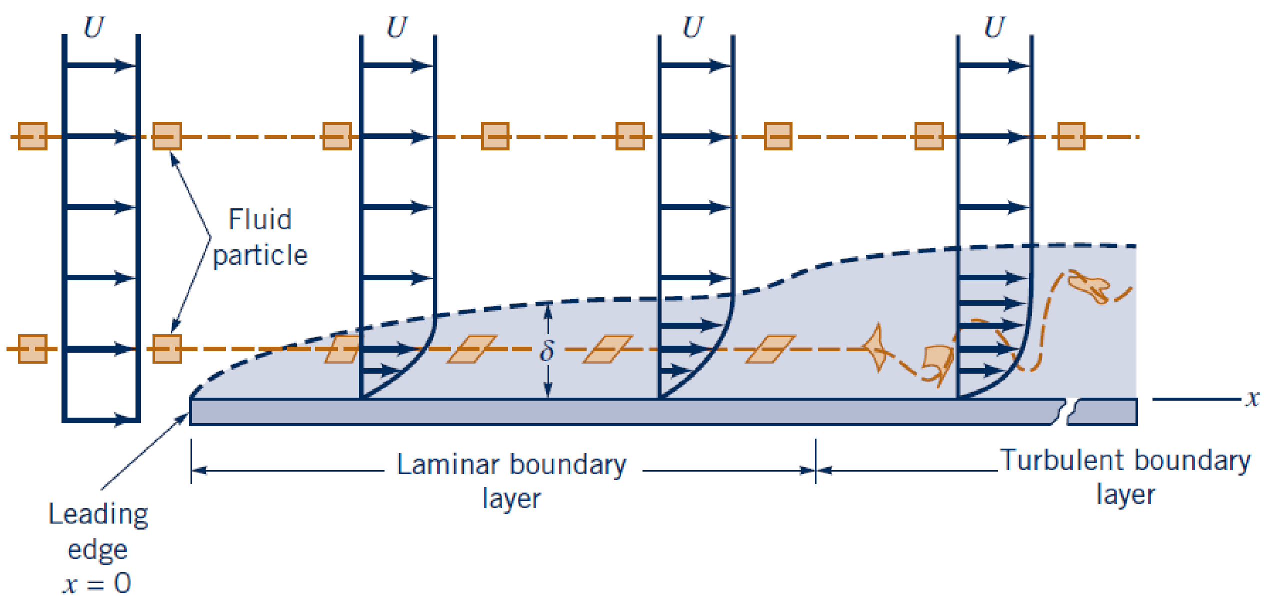
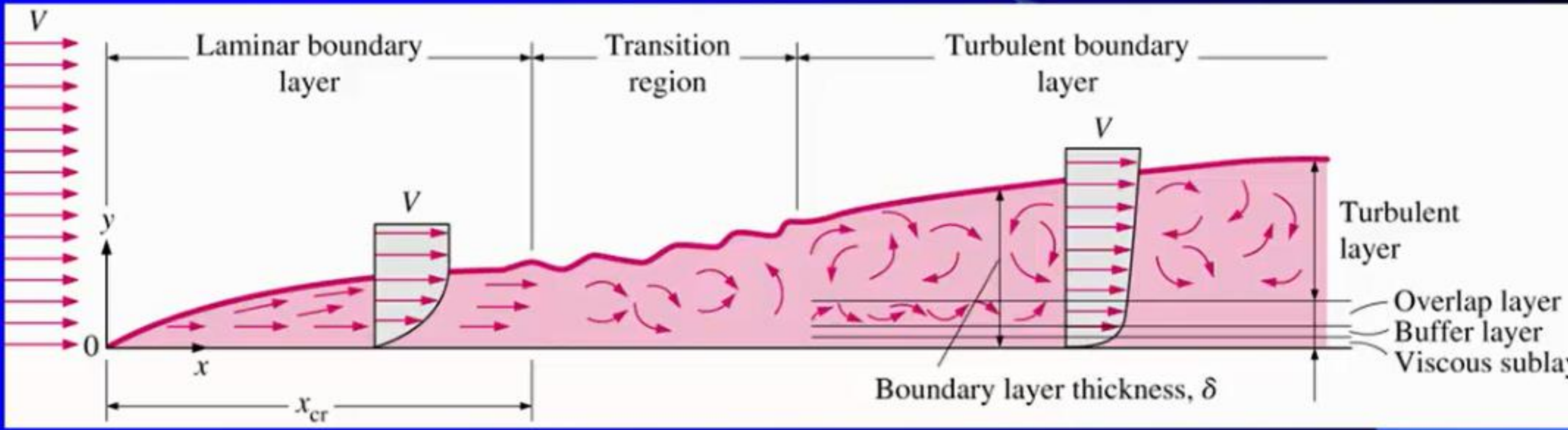


Figure 9.7 Distortion of a fluid particle as it flows within the boundary layer.

Parallel Flow over Plates



- Drag on flat plate is solely due to friction created by laminar, transitional, and turbulent boundary layers.

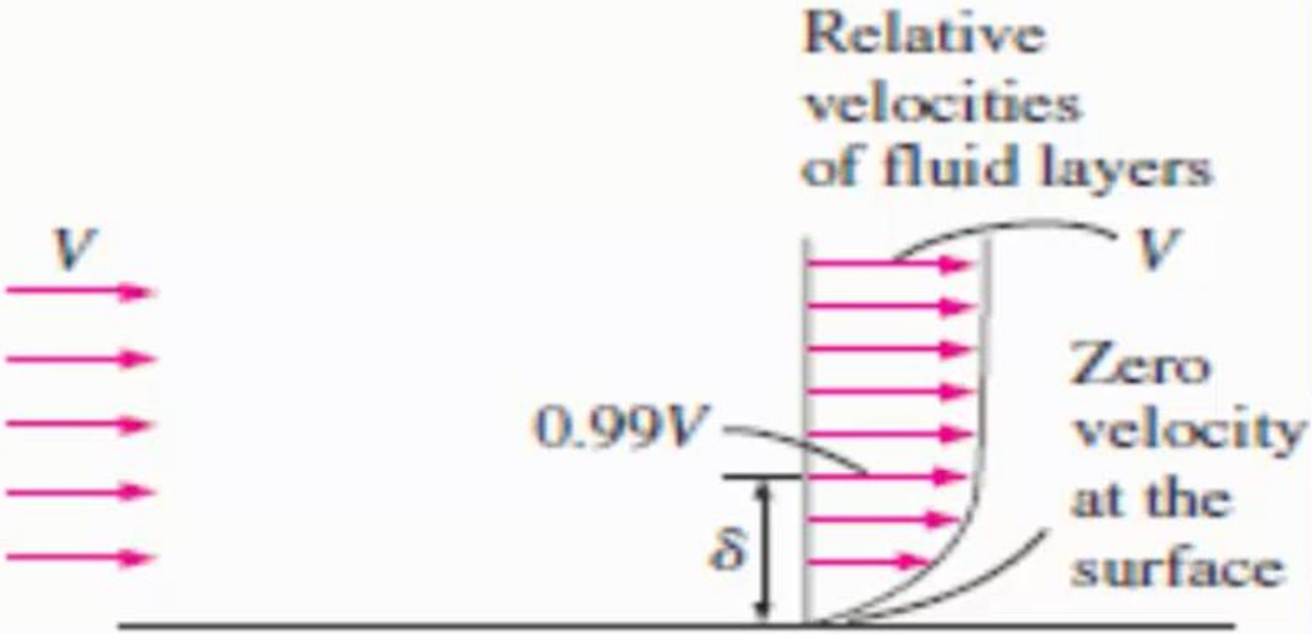
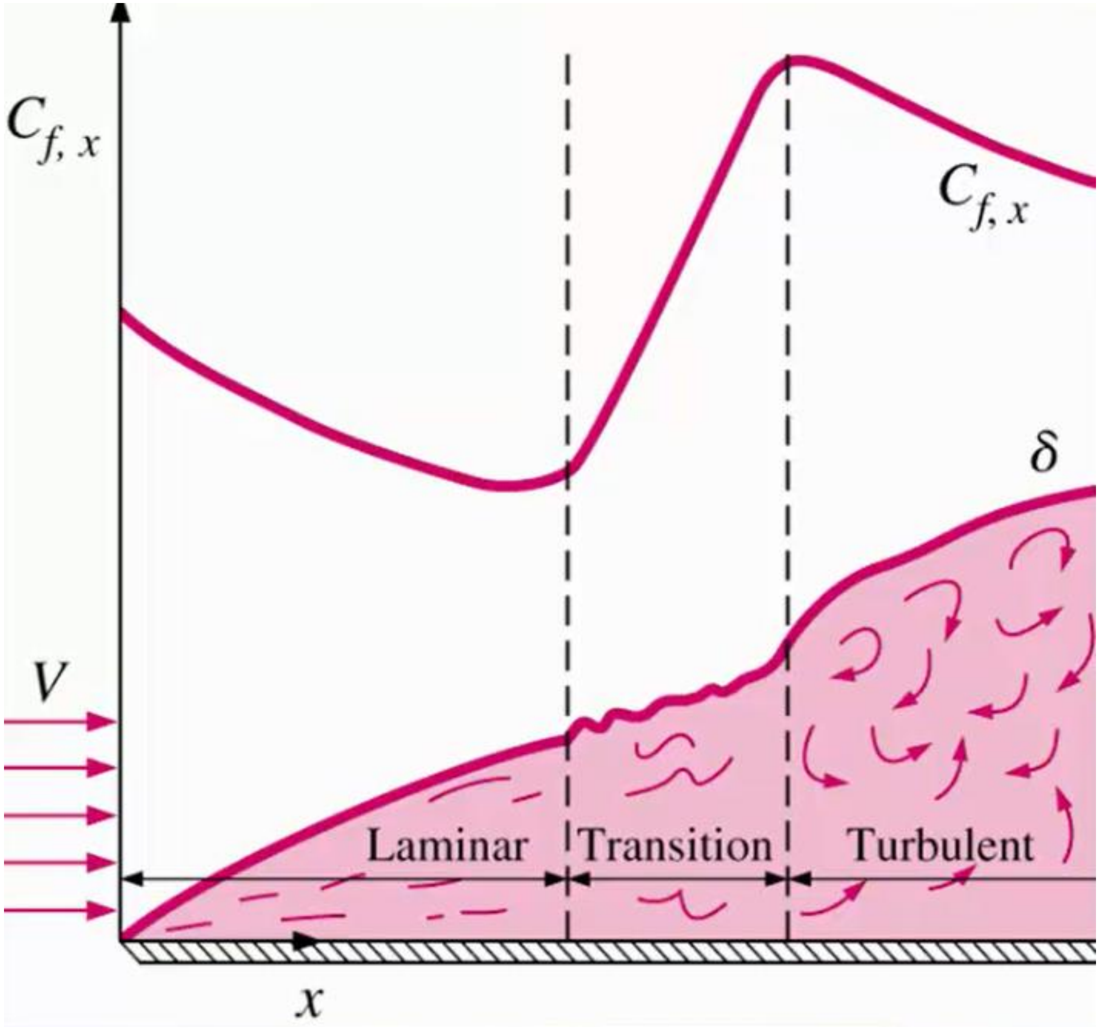
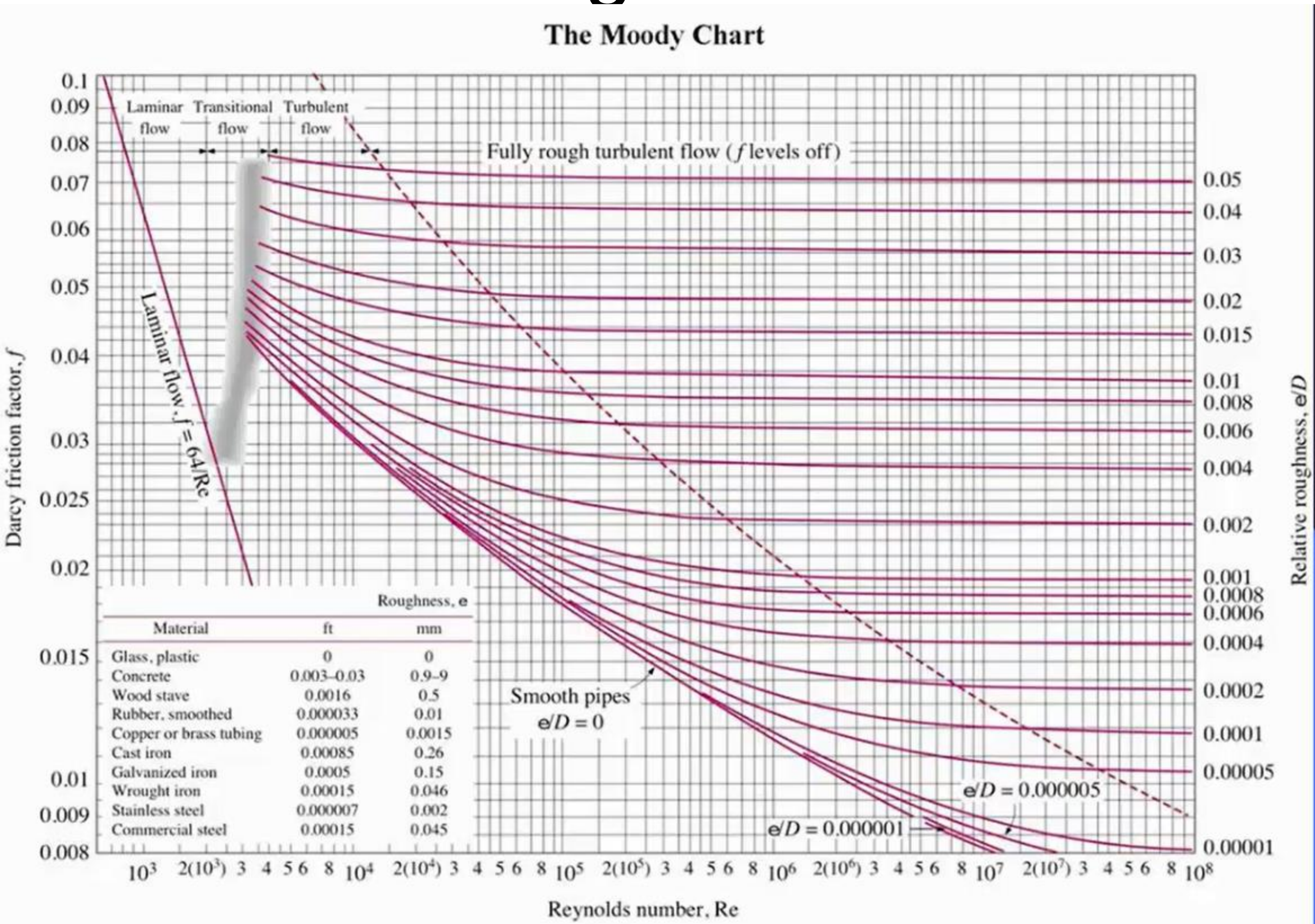


FIGURE 11–26
The development of a boundary layer on a surface is due to the no-slip condition and friction.

Friction Drag on Flat Plate



Friction Drag on Flat Plate



Material	ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Flow over a flat plate

$$C_{D,\,\text{pressure}} = 0$$

$$C_D = C_{D,\,\text{friction}} = C_f$$

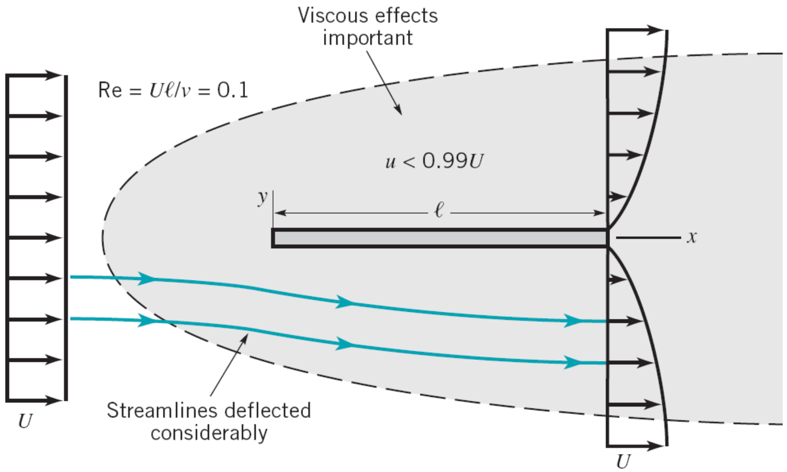
$$F_{D,\,\text{pressure}} = 0$$

$$F_D = F_{D,\,\text{friction}} = F_f = C_f A \frac{\rho V^2}{2}$$

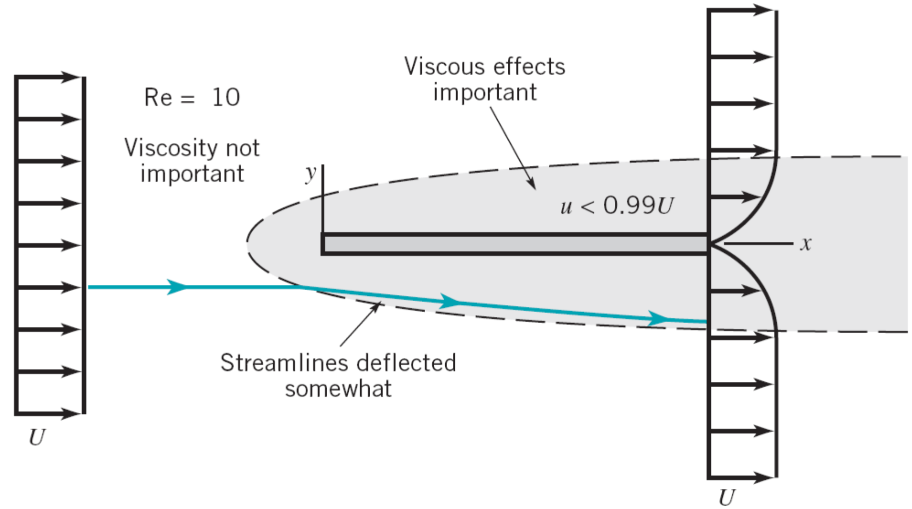
FIGURE 11–27

For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

Character of the steady, viscous flow past a flat plate parallel to the upstream velocity



(a)



(b)

Inertia force = ma =ρL³*V dV/dL= ρV²L²

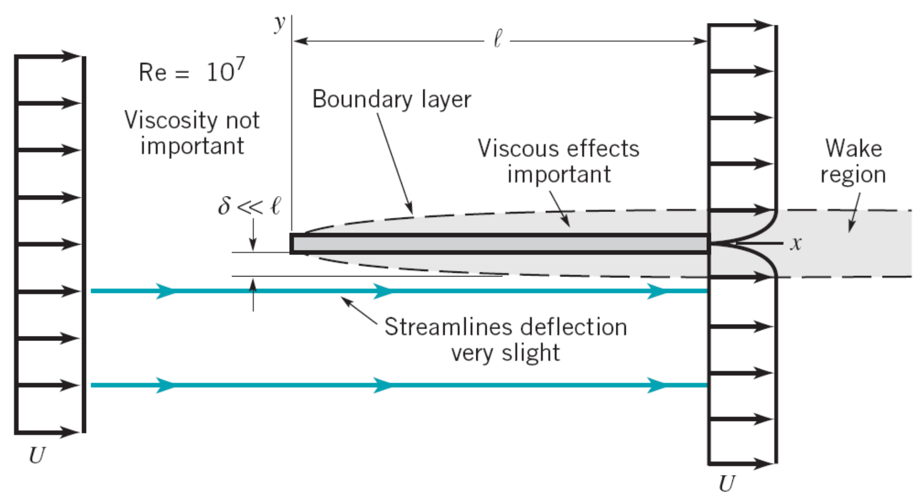
Viscous Force = μ L² dV/dL = μ V L

$$\boxed{Re = \frac{\rho VL}{\mu} = \frac{Inertia\ Force}{Viscous\ Force}}$$

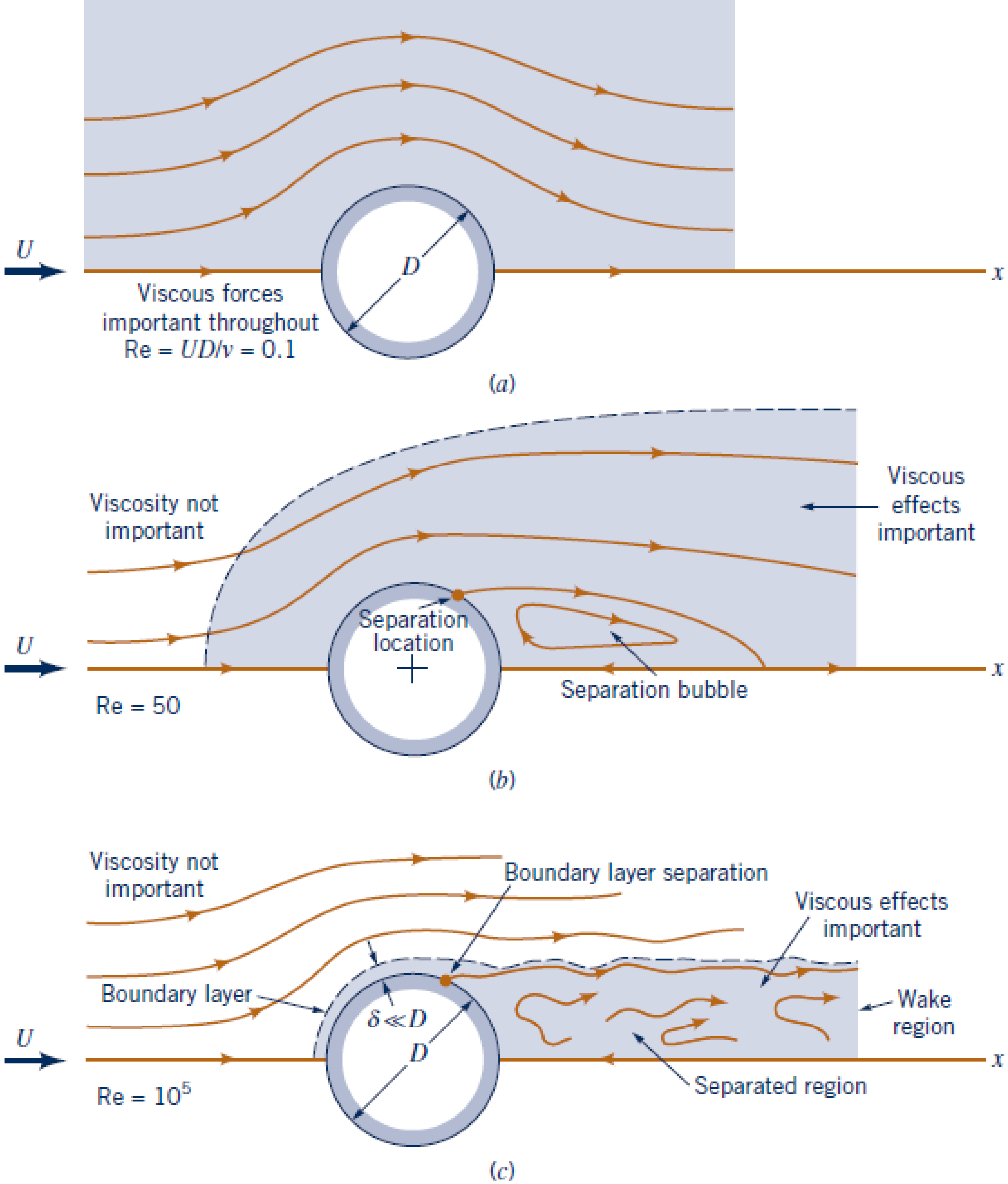
(a) low Reynolds number flow,

(b) moderate Reynolds number flow,

(c) large Reynolds number flow.



(c)



■ **Figure 9.6** **Character of the steady, viscous flow past a circular cylinder: (*a*) low Reynolds number flow, (*b*) moderate Reynolds number flow, (*c*) large Reynolds number flow.**

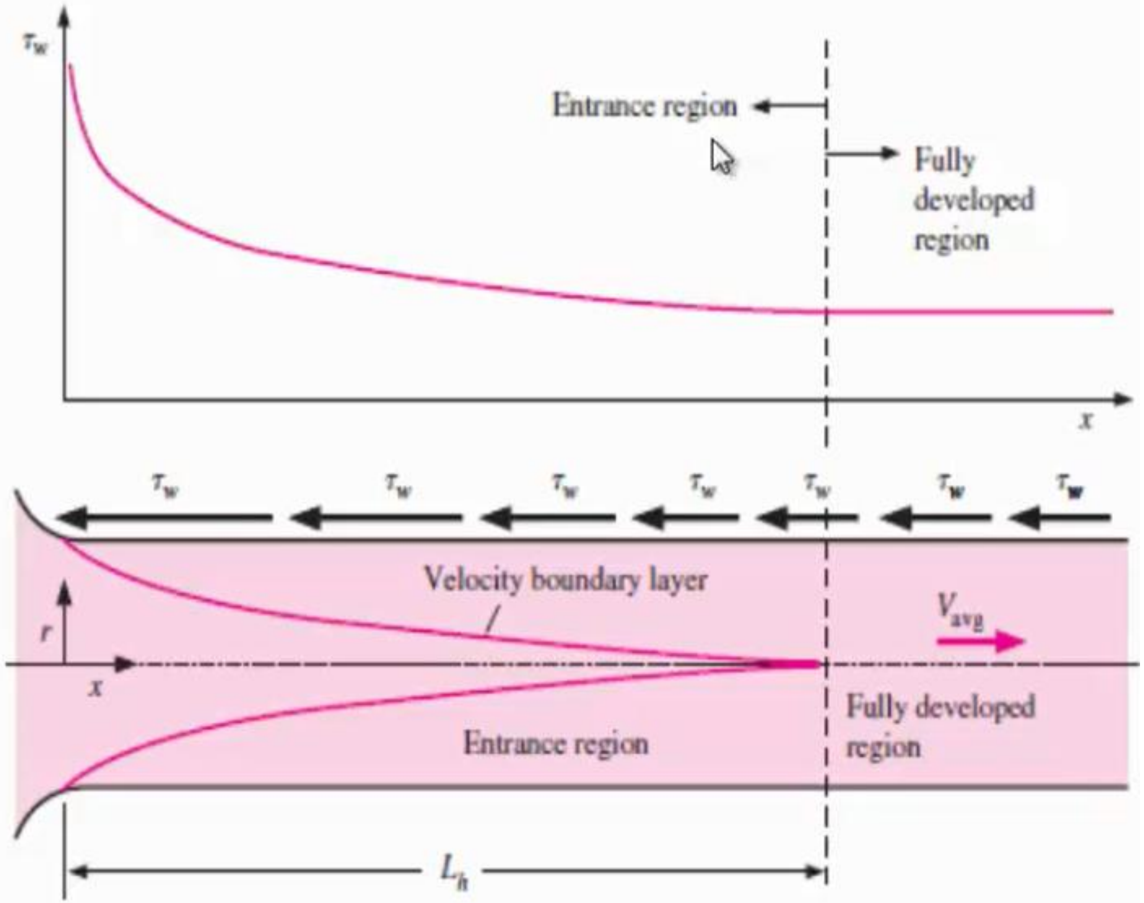
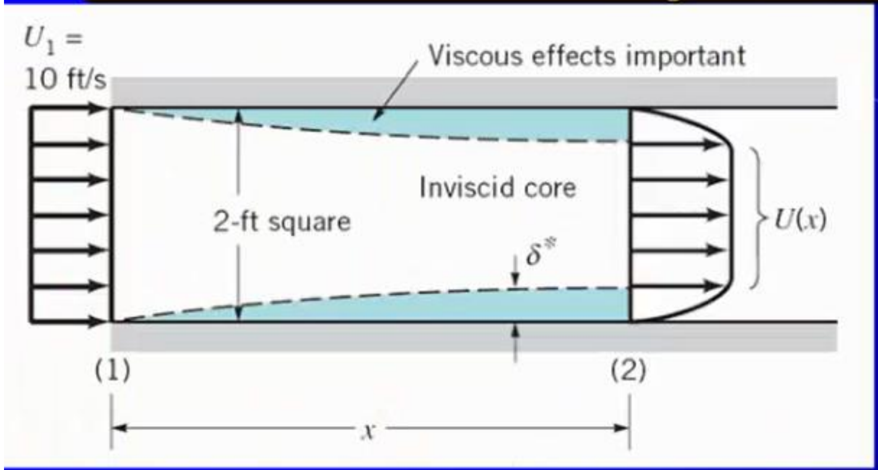
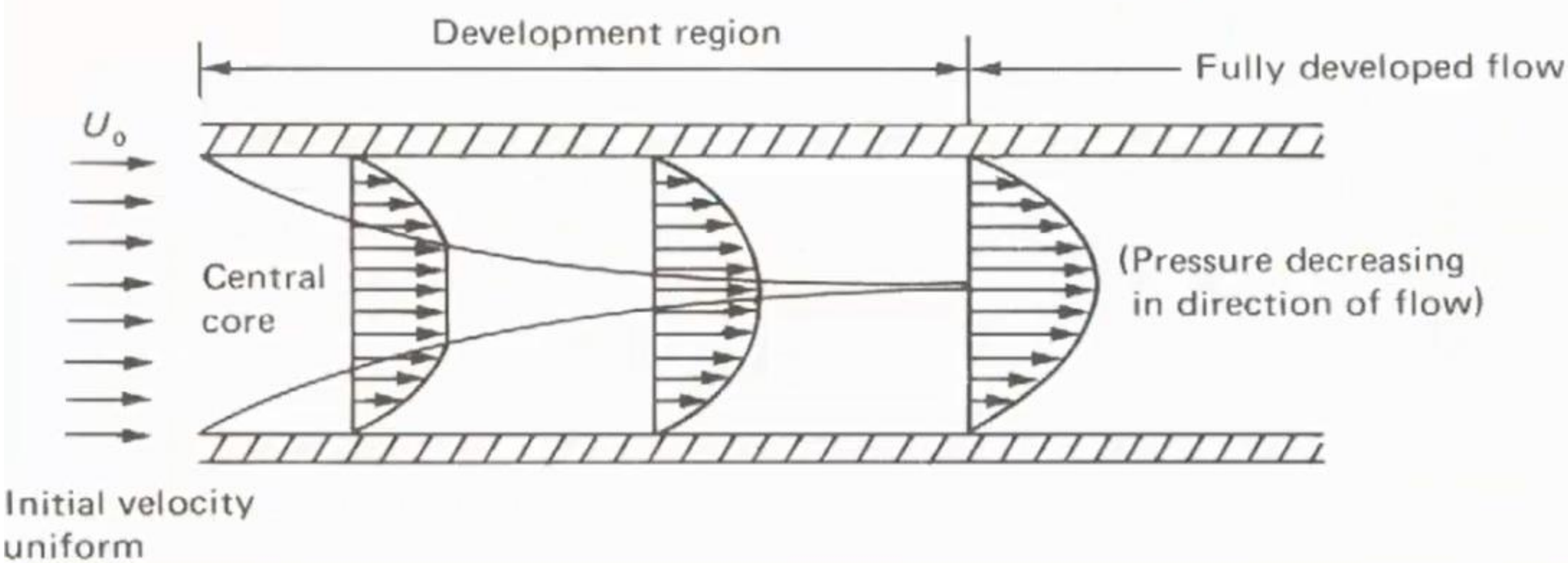
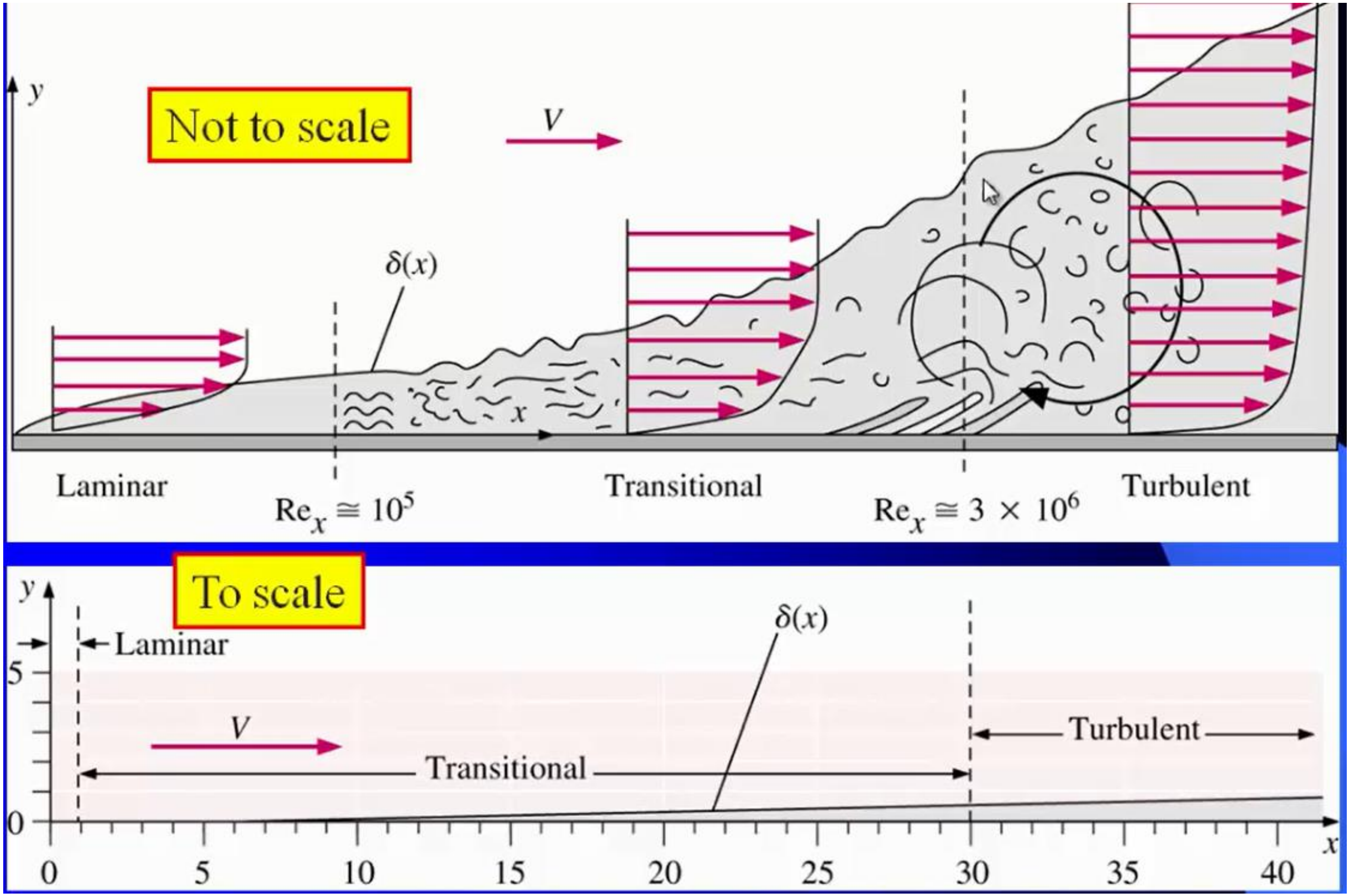


FIGURE 8–10
The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

B. L. Development in pipeline







Not to scale
V
y
δ(x)
x
Laminar
$Re_x \cong 10^5$
Transitional
$Re_x \cong 3 \times 10^6$
Turbulent
To scale
y
δ(x)
Laminar
V
Transitional
Turbulent
0
5
0
5
10
15
20
25
30
35
40
x

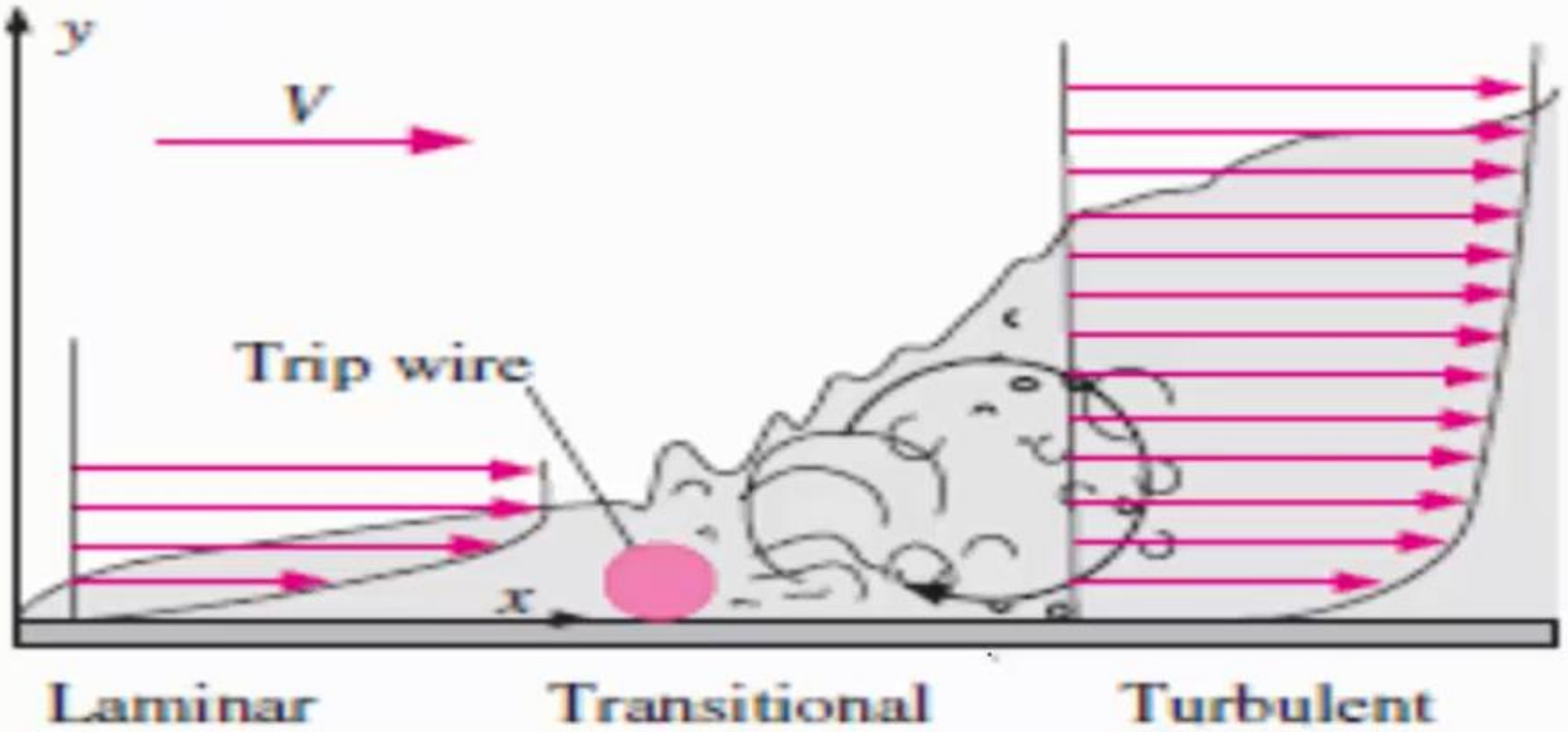
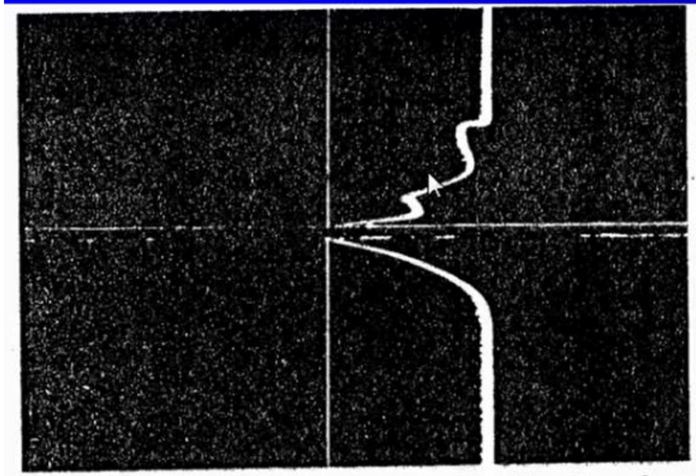


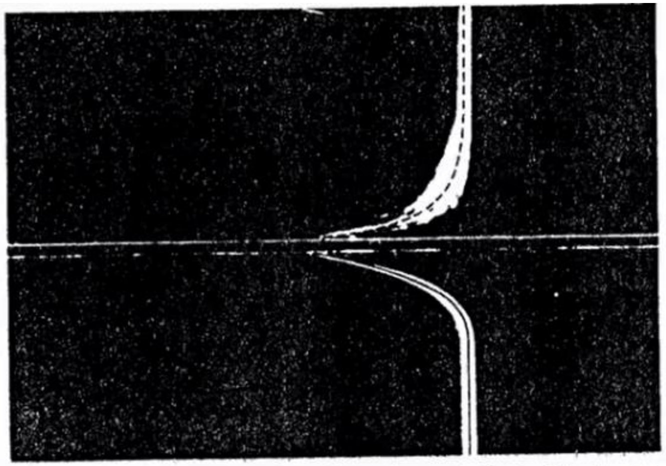
FIGURE 10–83

A trip wire is often used to initiate early transition to turbulence in a boundary layer (not to scale).

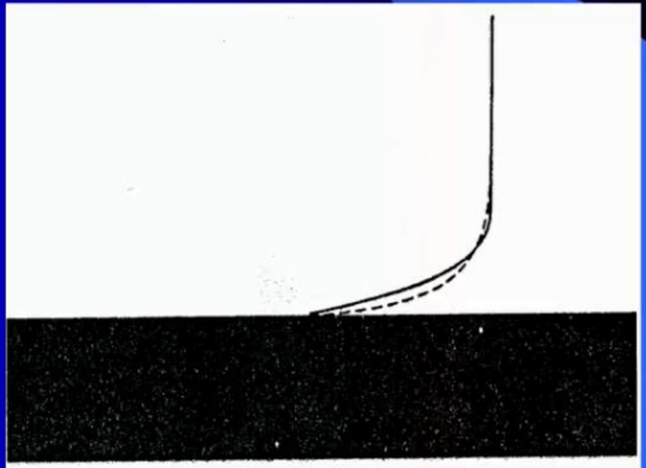
Turbulent Boundary Layer



19. Instantaneous displacement profiles for flow along a thin plate. The boundary layer on the upper surface has been made turbulent, while the flow along the lower surface is laminar.



20a. The upper boundary layer is turbulent; the lower, laminar. Superposition of many instantaneous velocity profiles suggests mean velocity profiles.



20b. The mean laminar (solid) and turbulent (dashed) profiles are compared.

Turbulent Boundary Layer

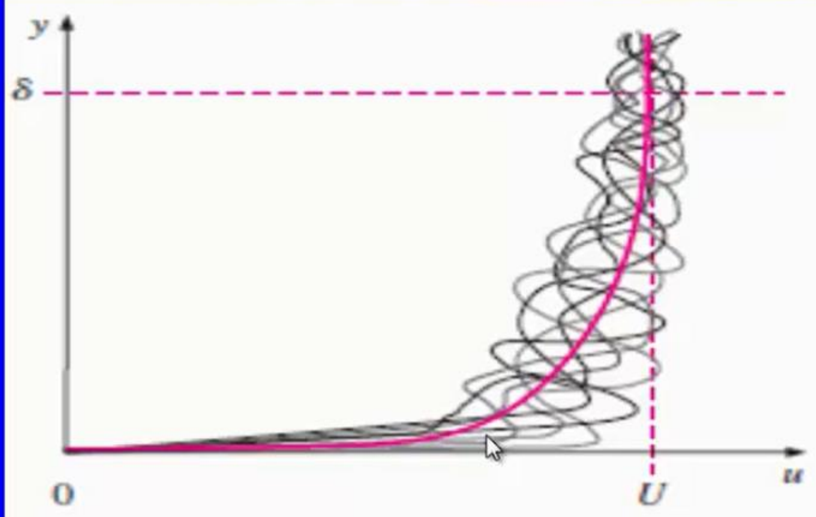


FIGURE 10–112
Illustration of the unsteadiness of a turbulent boundary layer; the thin, wavy black lines are instantaneous profiles, and the thick blue line is a long time-averaged profile.

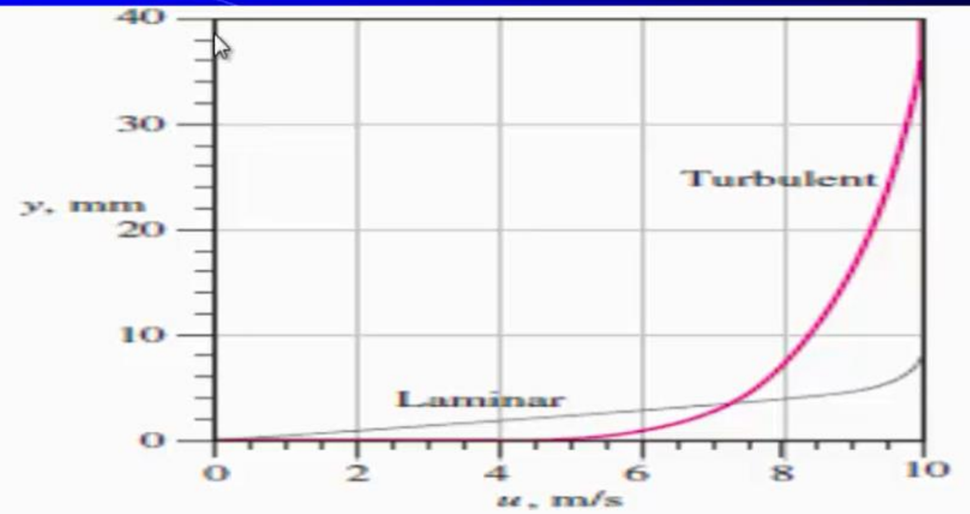


FIGURE 10–115
Comparison of laminar and turbulent flat plate boundary layer profiles in physical variables at the same x-location. The Reynolds number is $Re_x = 1.0 \times 10^6$.

$$\frac{u}{U} \cong \left(\frac{y}{\delta}\right)^{1/7} \quad \text{for } y \le \delta, \quad \rightarrow \quad \frac{u}{U} \cong 1 \quad \text{for } y > \delta \qquad (10\text{–}82)$$

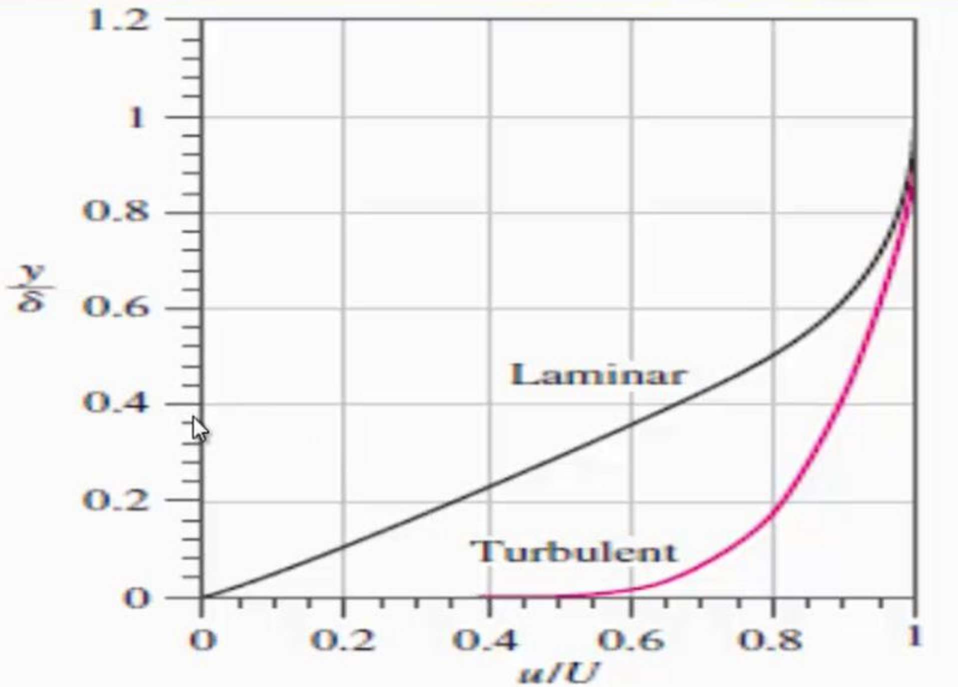


FIGURE 10–113
Comparison of laminar and turbulent flat plate boundary layer profiles, nondimensionalized by boundary layer thickness.

EFFECT OF PRESSURE GRADIENT ON BOUNDARY LAYERS

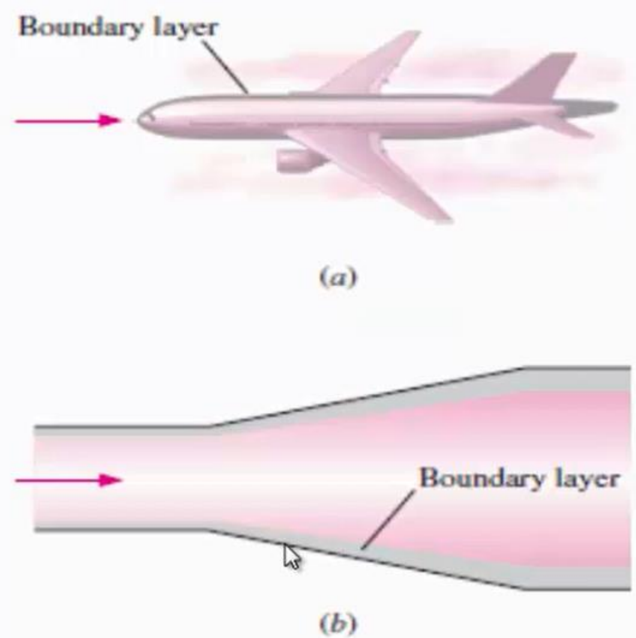
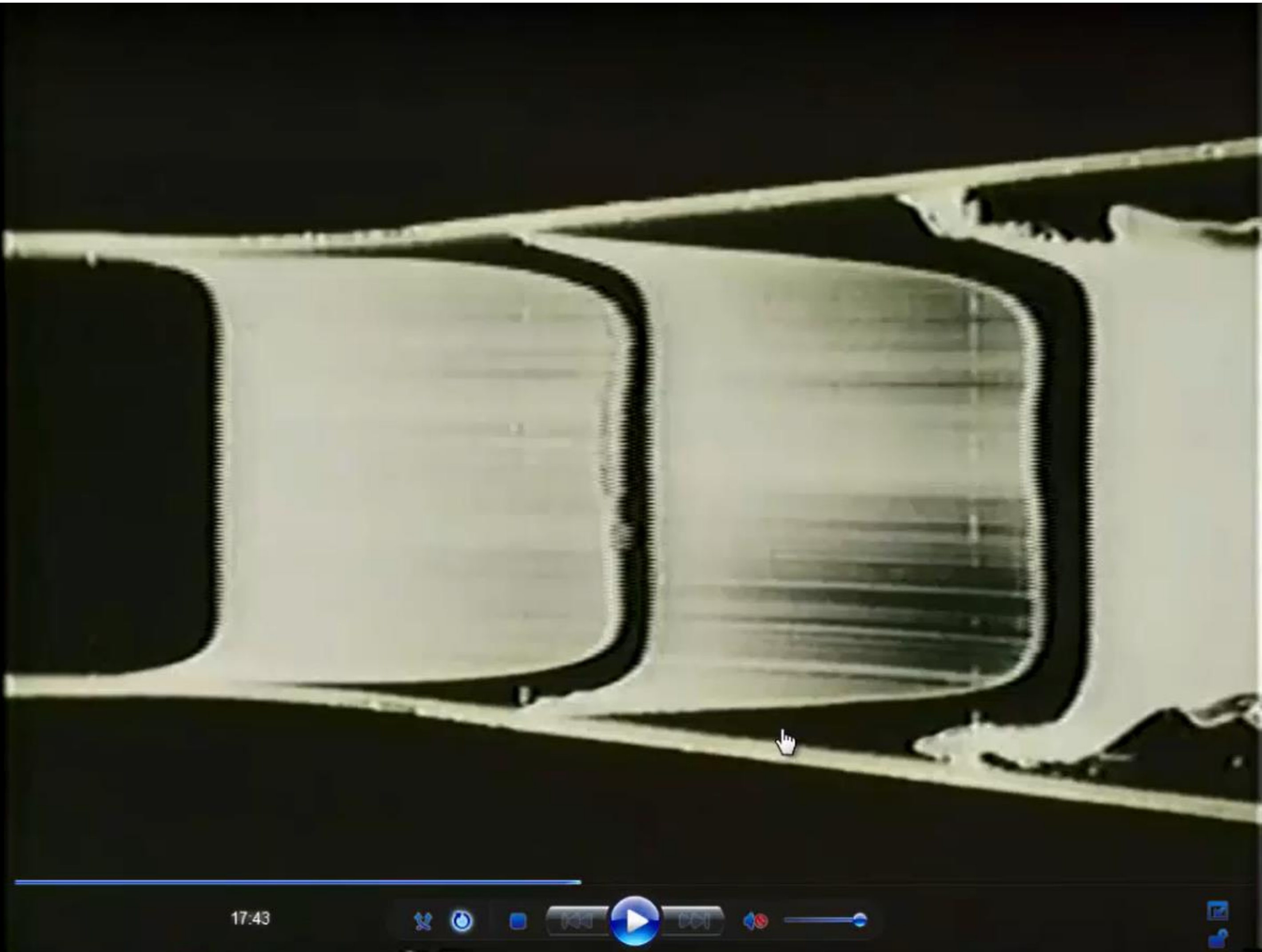


FIGURE 10–120
Boundary layers with nonzero pressure gradients occur in both external flows and internal flows: (*a*) boundary layer developing along the fuselage of an airplane and into the wake, and (*b*) boundary layer growing on the wall of a diffuser (boundary layer thickness exaggerated in both cases).


17:43

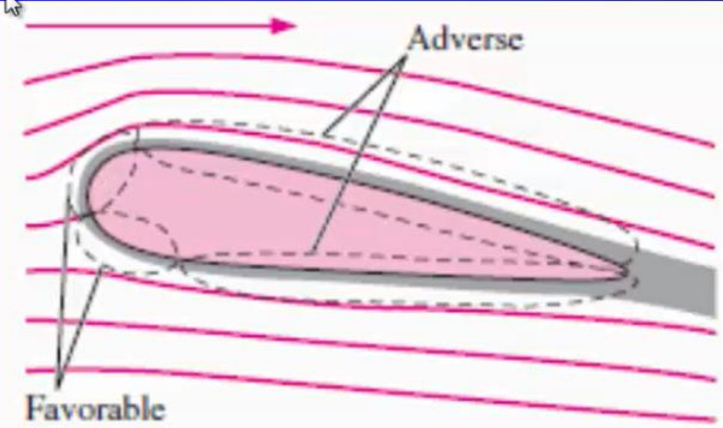


FIGURE 10–121
The boundary layer along a body immersed in a free stream is typically exposed to a favorable pressure gradient in the front portion of the body and an adverse pressure gradient in the rear portion of the body.

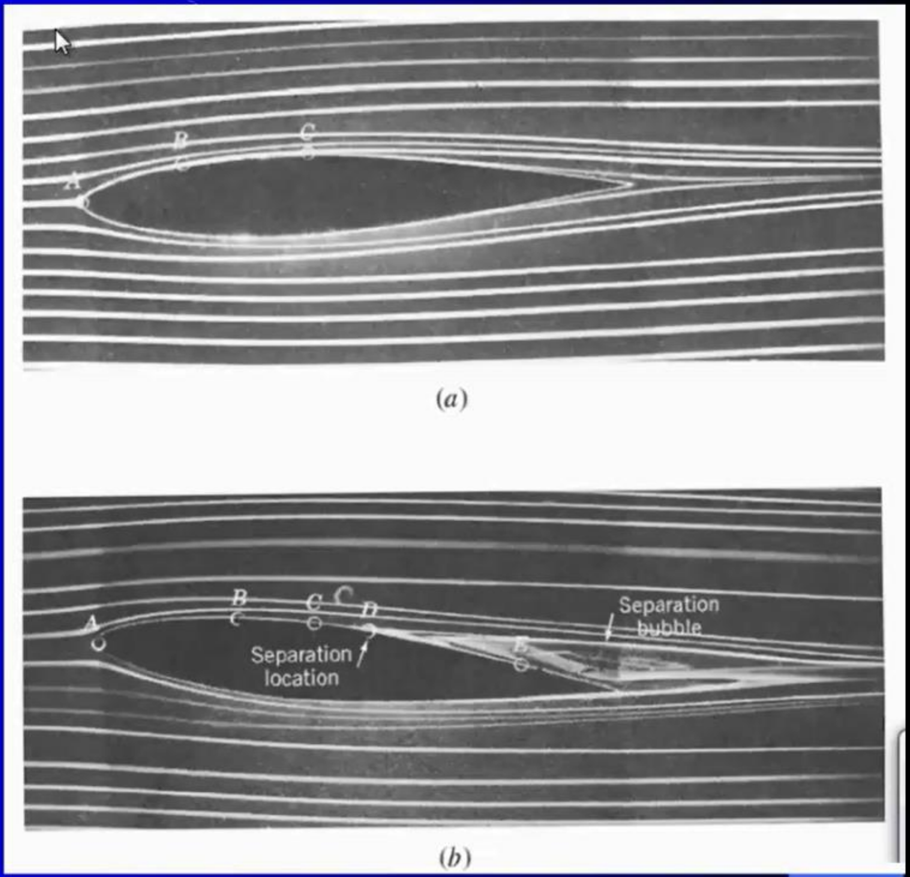
If the adverse pressure gradient is strong enough, the boundary layer is likely to separate off the wall.

BL Separation

Flow visualization photographs of flow past an airfoil (the boundary layer velocity profiles for the points indicated are similar to those indicated in Fig. 9.17*b*):

(*a*) zero angle of attack, no separation,

(*b*) 5° angle of attack, flow separation. Dye in water. (Photograph courtesy of ONERA, France.)



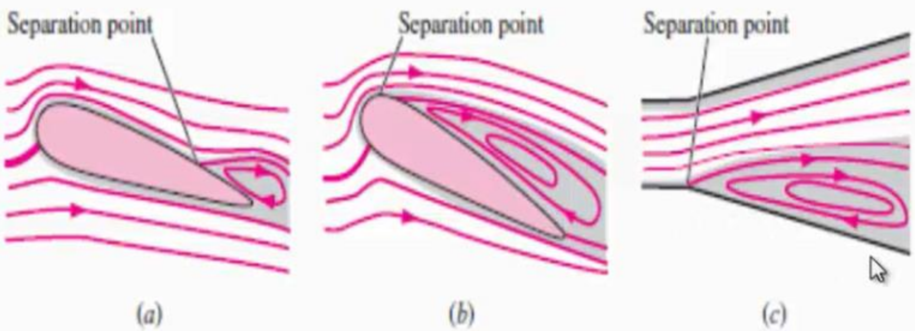


FIGURE 10–122
Examples of boundary layer separation in regions of adverse pressure gradient: (*a*) an airplane wing at a moderate angle of attack, (*b*) the same wing at a high angle of attack (a stalled wing), and (*c*) a wide-angle diffuser in which the boundary layer cannot remain attached and separates on one side.

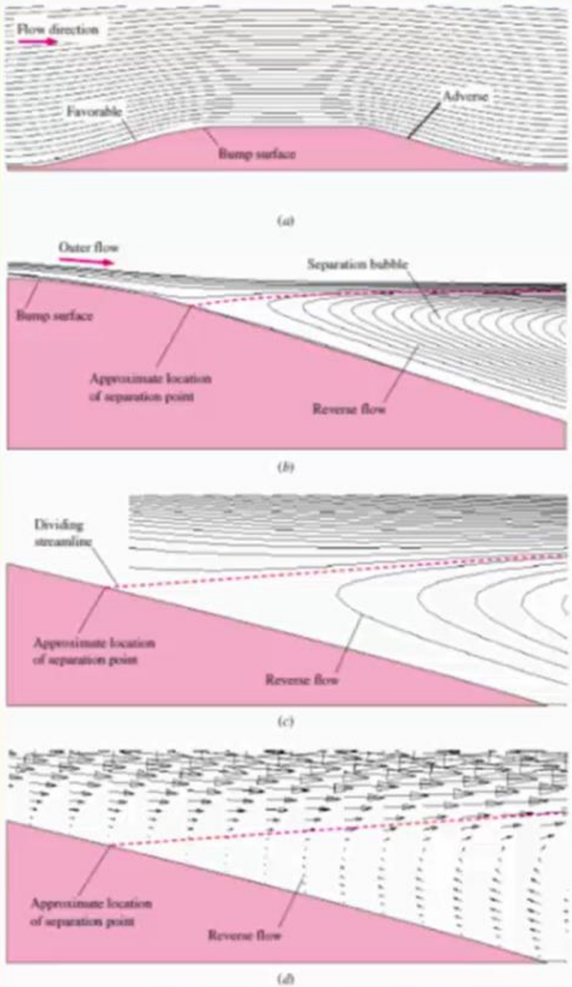
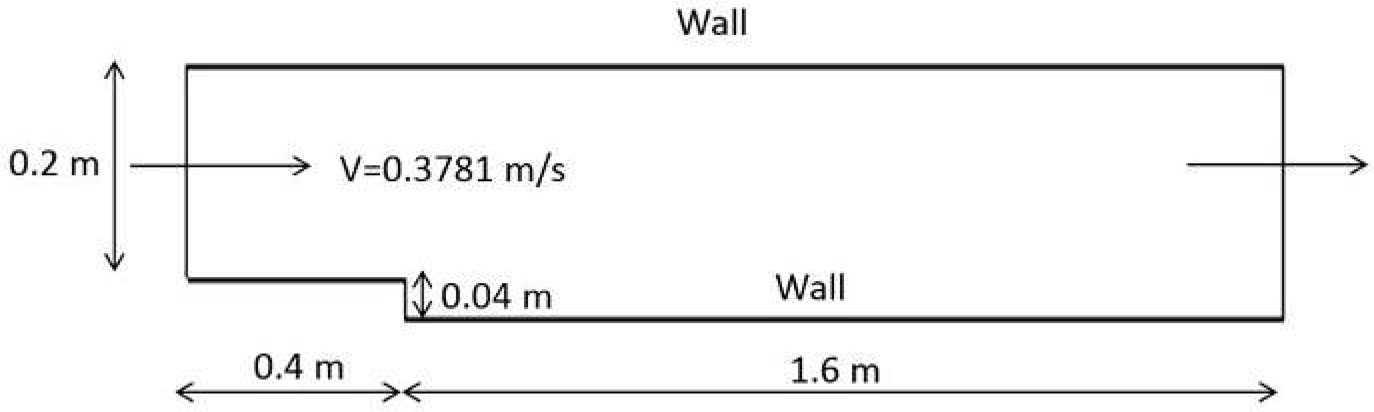
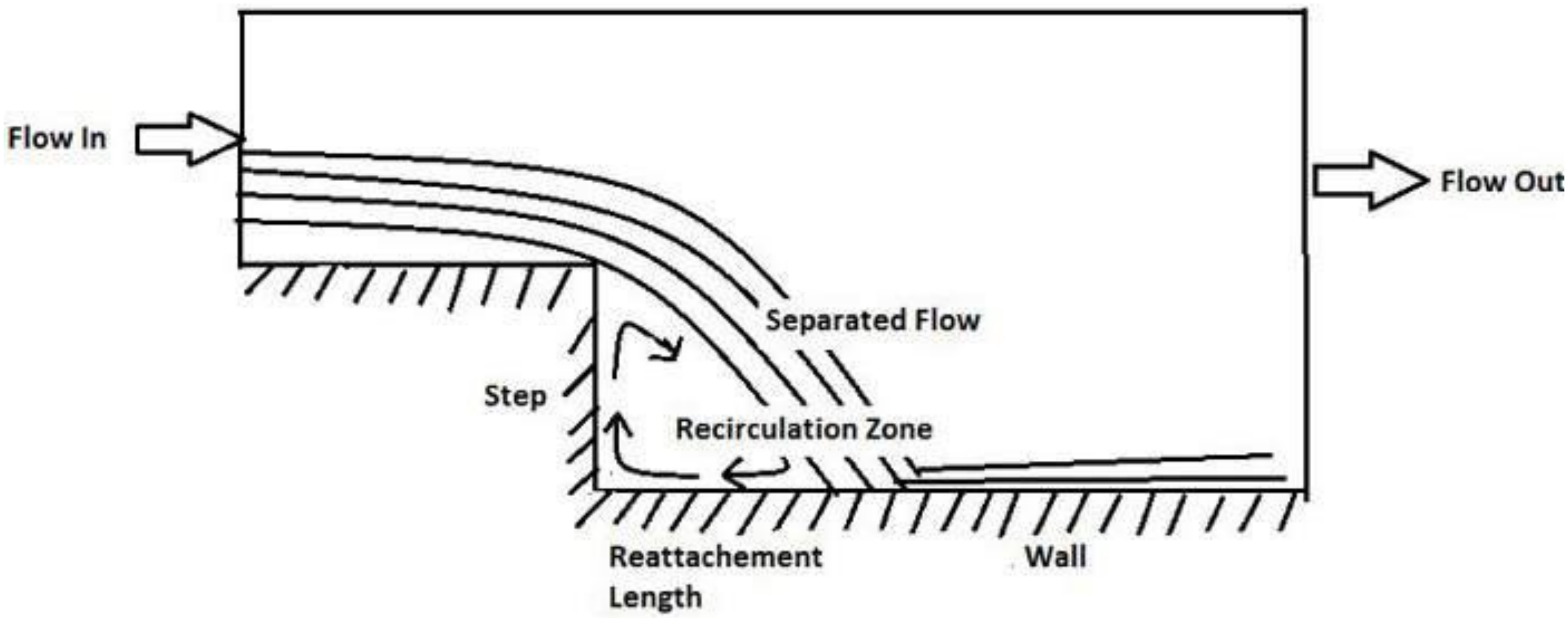


FIGURE 10–124
CFD calculations of flow over a bump: (*a*) solution of the Euler equation with outer flow streamlines plotted (no flow separation), (*b*) laminar flow solution showing flow separation on the downstream side of the bump, (*c*) close-up view of streamlines near the separation point, and (*d*) close-up view of velocity vectors, same view as (*c*).

Back Step Flow Problem

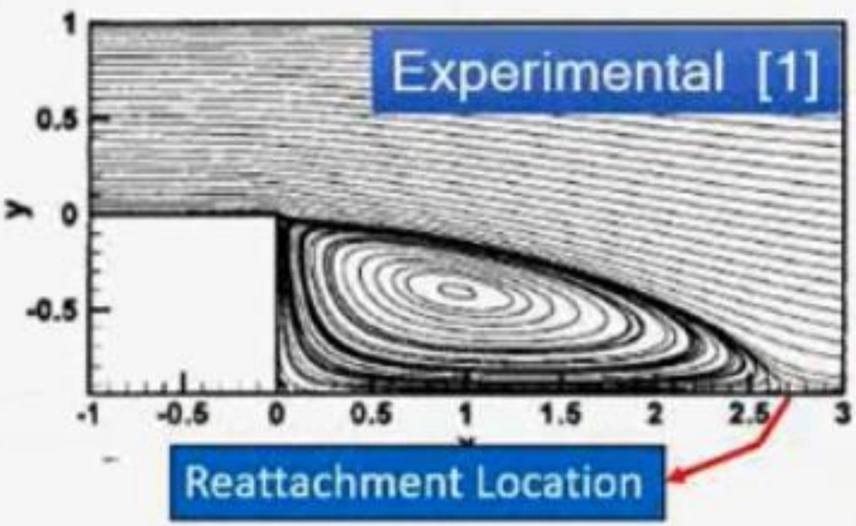


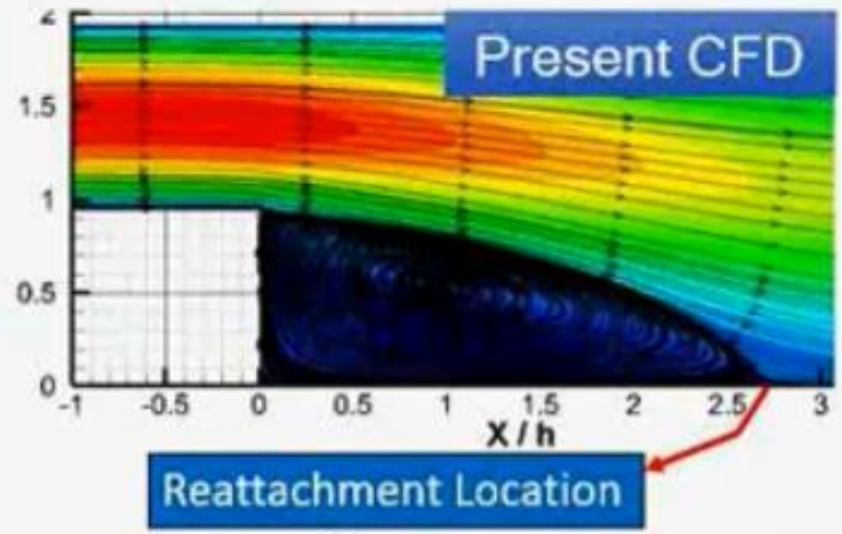
Back Step Flow Problem

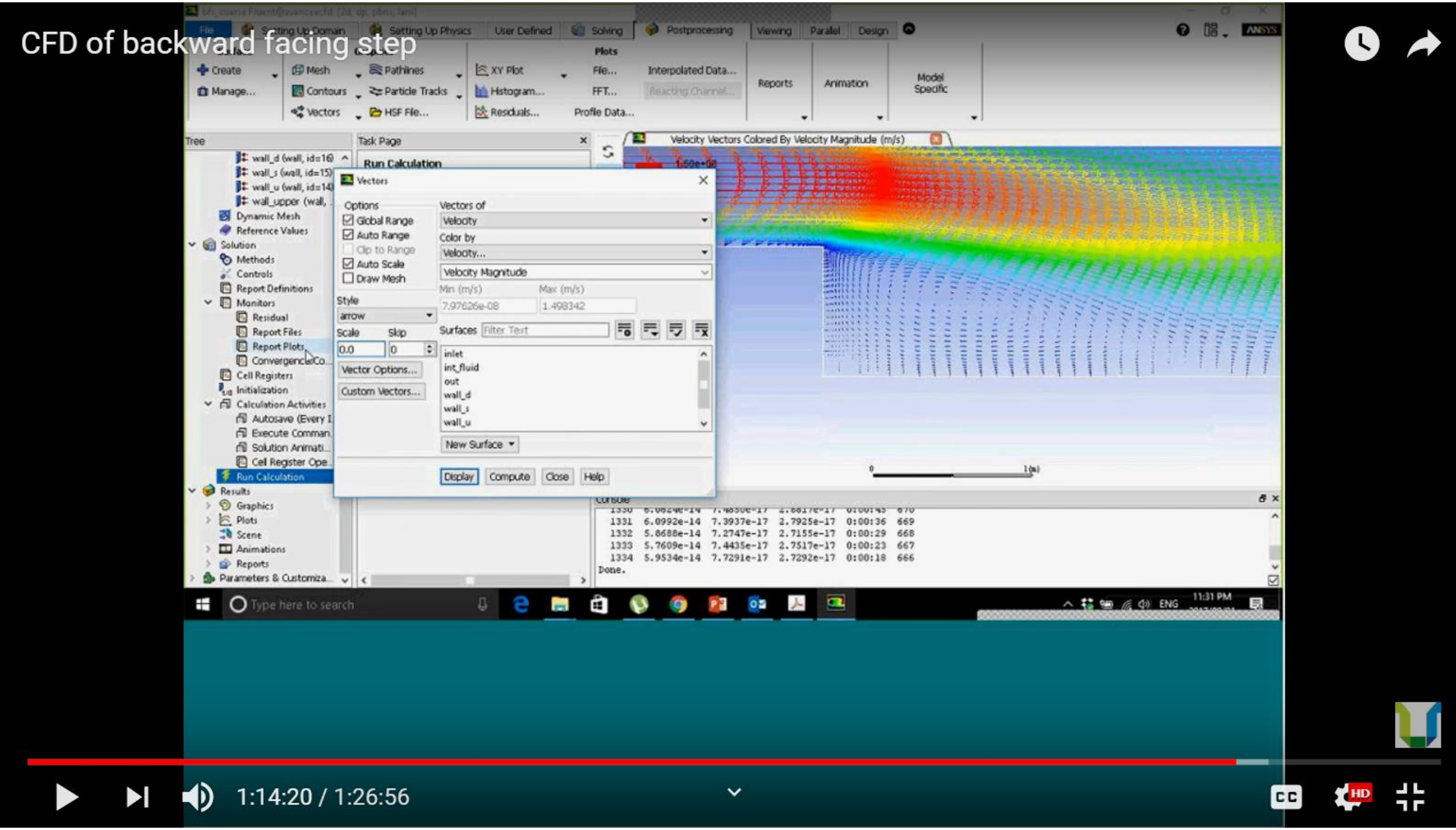


- Reattachment length At H/h = 1.9423 and Re_D = 100

	CFD	Experimental	Error
Reattachment length	2.71 m	2.70584 m	0.153%






CFD of backward facing step
Setting Up Domain
Setting Up Physics
User Defined
Solving
Postprocessing
Viewing
Parallel
Design
Velocity Vectors Colored By Velocity Magnitude (m/s)
Vectors
Options
Global Range
Auto Range
Clip to Range
Auto Scale
Draw Mesh
Vectors of
Velocity
Color by
Velocity...
Velocity Magnitude
Min (m/s)
Max (m/s)
7.97626e-08
1.498342
Style
arrow
Scale
Skip
Surfaces
inlet
int_fluid
out
wall_d
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wall_u
New Surface
Vector Options...
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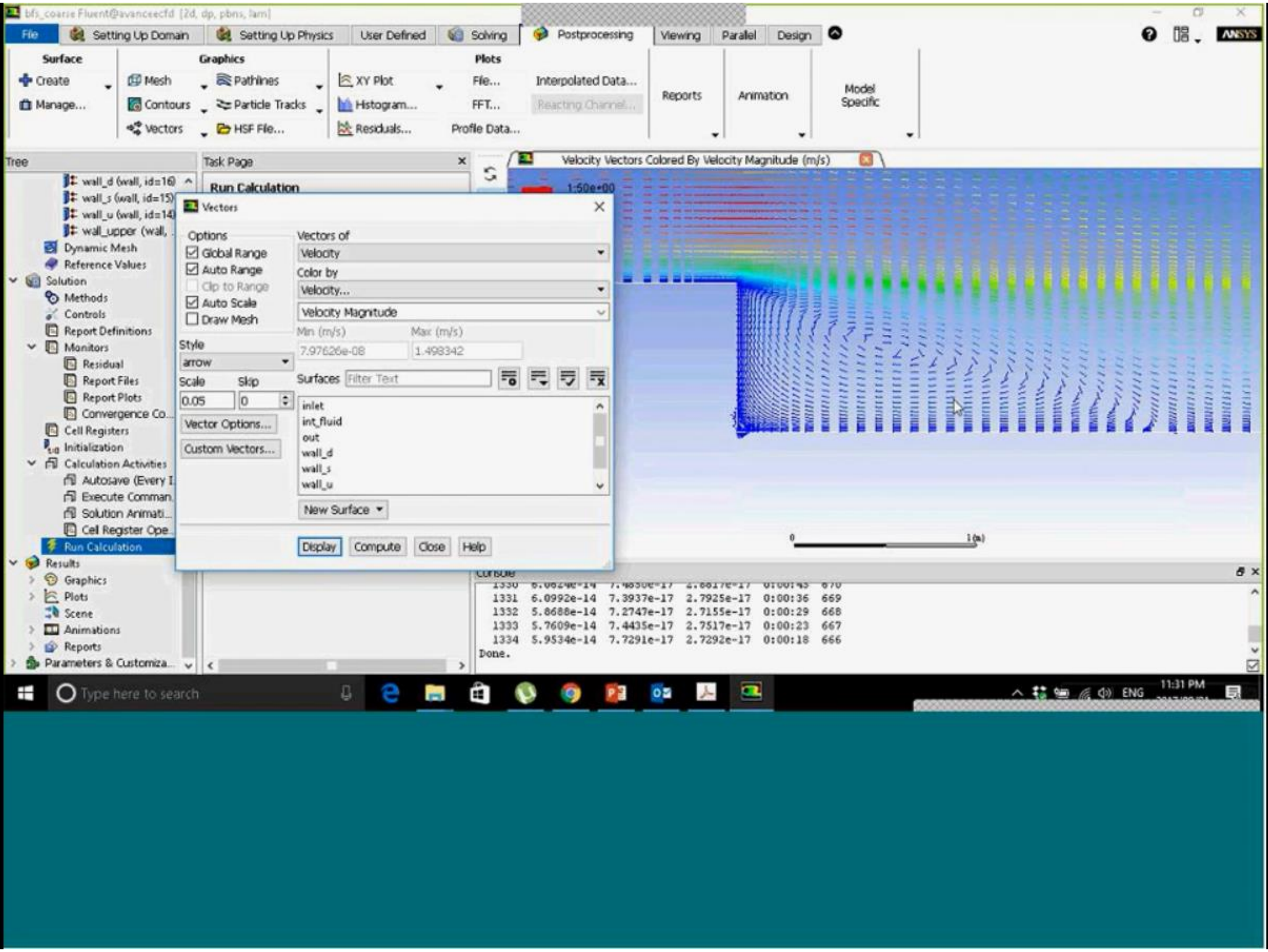
Compute
Close
Help
Run Calculation
Done.
1:14:20 / 1:26:56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HSF File...
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Histogram...
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Custom Vectors...
Vectors of
Velocity
Color by
Velocity...
Velocity Magnitude
Min (m/s)
Max (m/s)
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Reference Values
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Controls
Report Definitions
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Residual
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Cell Registers
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Results
Graphics
Plots
Scene
Animations
Reports
Console
1331 6.0992e-14 7.3937e-17 2.7925e-17 0:00:36 669
1332 5.8688e-14 7.2747e-17 2.7155e-17 0:00:29 668
1333 5.7609e-14 7.4435e-17 2.7517e-17 0:00:23 667
1334 5.9534e-14 7.7291e-17 2.7292e-17 0:00:18 666
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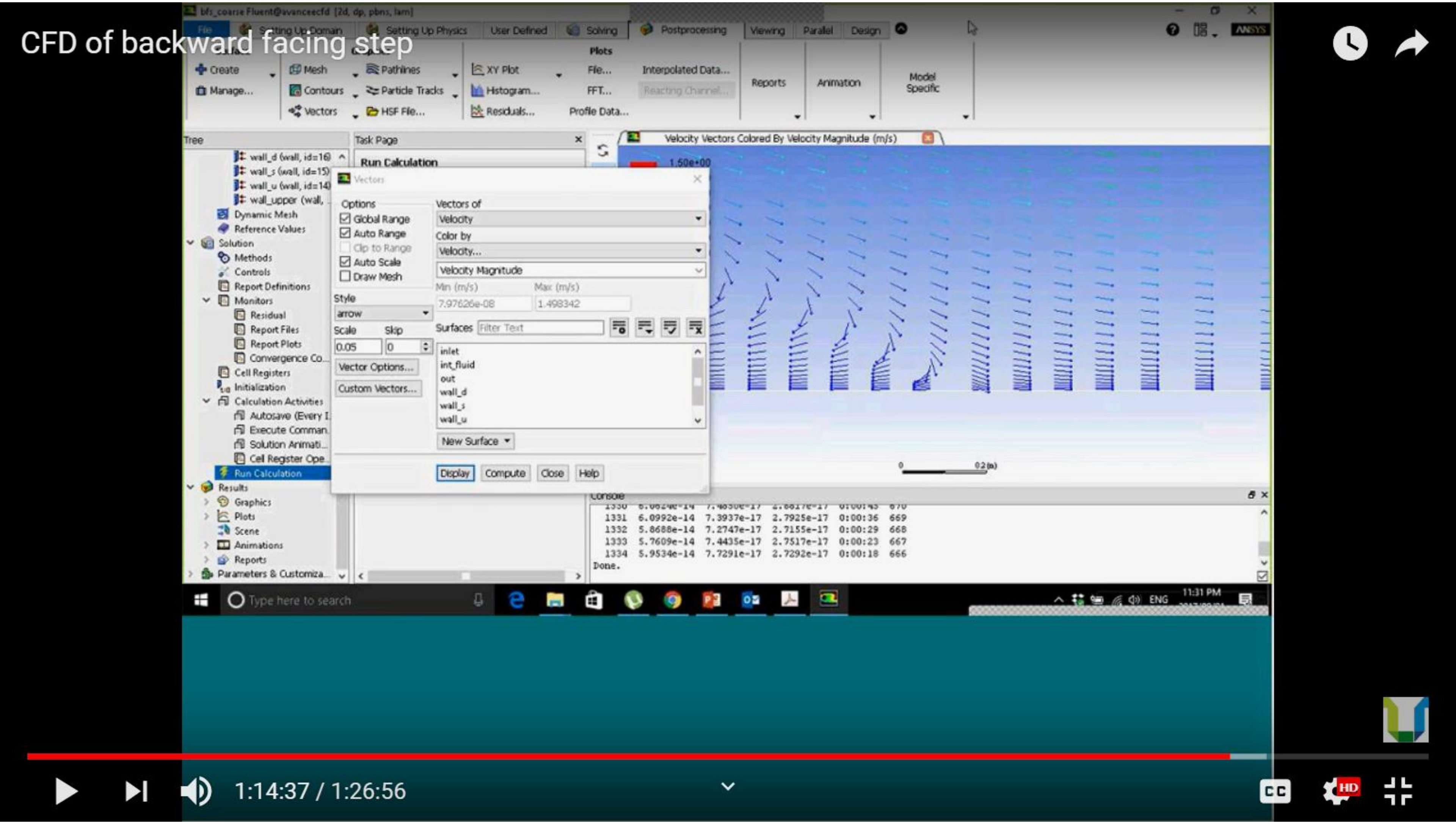
CFD of backward facing step
Setting Up Domain
Setting Up Physics
User Defined
Solving
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Velocity Vectors Colored By Velocity Magnitude (m/s)
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Custom Vectors...
Vectors of
Velocity
Color by
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Min (m/s)
Max (m/s)
7.97626e-08
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Controls
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Report Files
Report Plots
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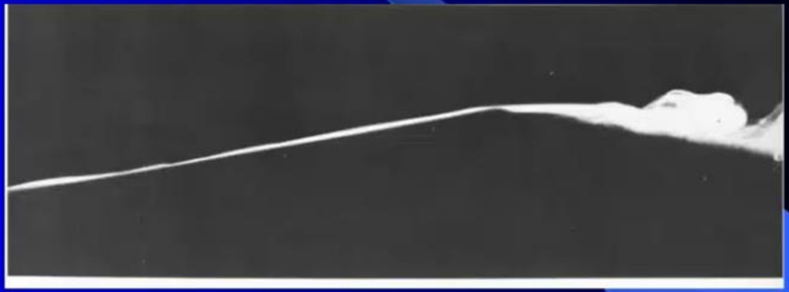
1331 6.0992e-14 7.3937e-17 2.7925e-17 0:00:36 669
1332 5.8688e-14 7.2747e-17 2.7155e-17 0:00:29 668
1333 5.7609e-14 7.4435e-17 2.7517e-17 0:00:23 667
1334 5.9534e-14 7.7291e-17 2.7292e-17 0:00:18 666
Done.
Type here to search
1:14:37 / 1:26:56

Effect of Pressure Gradients on Turbulent B. L.

- **Turbulent BL is more resistant to flow separation than laminar BL exposed to the same adverse pressure gradient**

Laminar flow separates at corner



Turbulent flow does not separate

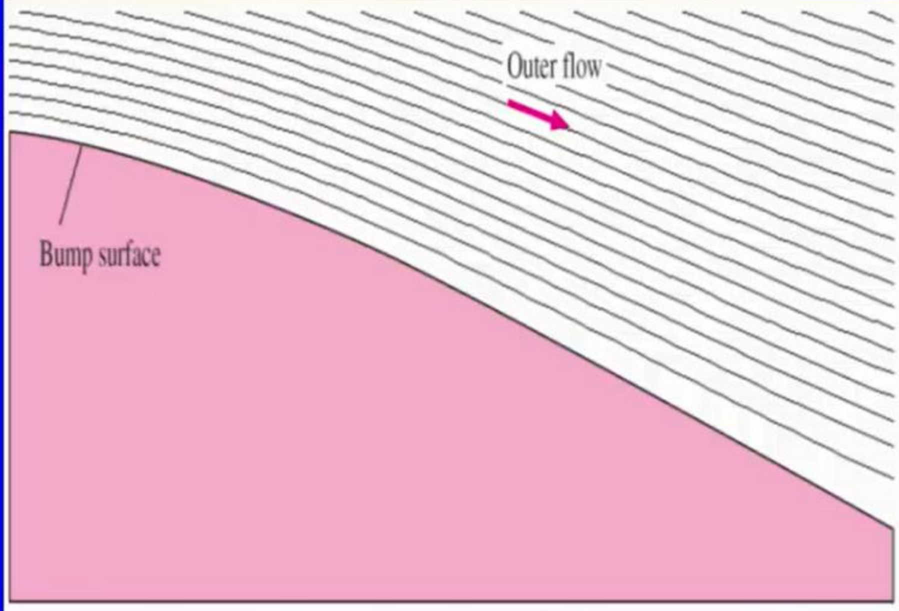
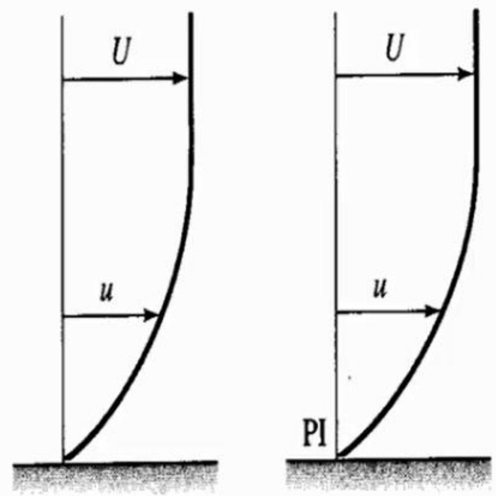


FIGURE 10–126
CFD calculation of turbulent flow over the same bump as that of Fig. 10–124. Compared to the laminar result of Fig. 10–124*b*, the turbulent boundary layer is more resistant to flow separation and does not separate in the adverse pressure gradient region in the rear portion of the bump.



(a) Favorable gradient:

$$\frac{dU}{dx} > 0$$

$$\frac{dp}{dx} < 0$$

No separation, PI inside wall

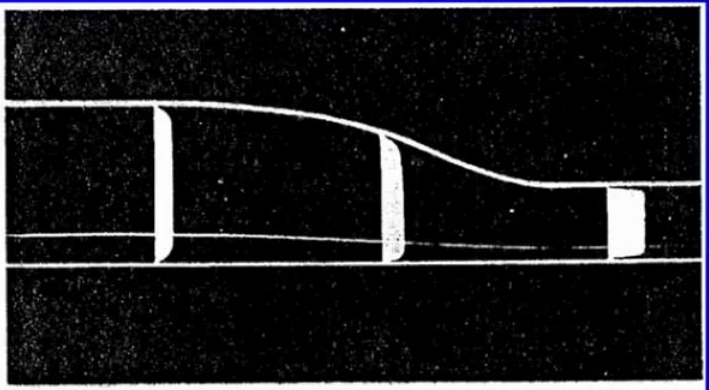
(b) Zero gradient:

$$\frac{dU}{dx} = 0$$

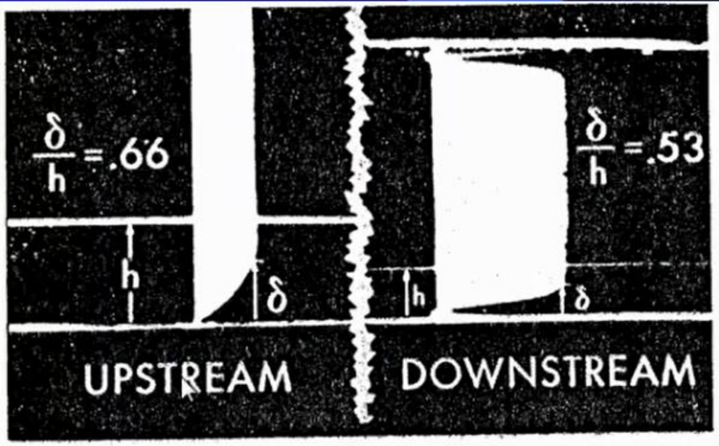
$$\frac{dp}{dx} = 0$$

No separation, PI at wall

Favorable Pressure Gradient

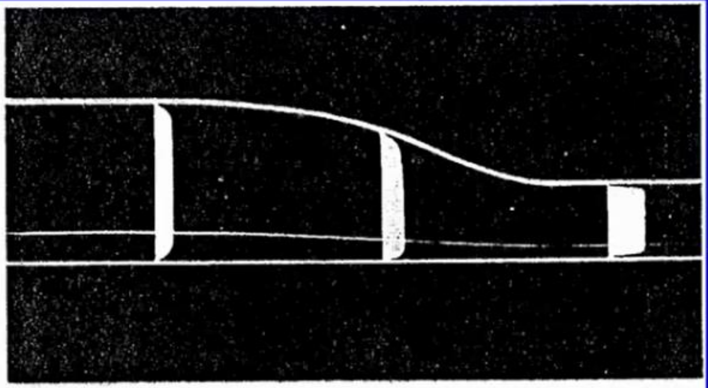


12. Flow in a converging channel (two-to-one contraction ratio).

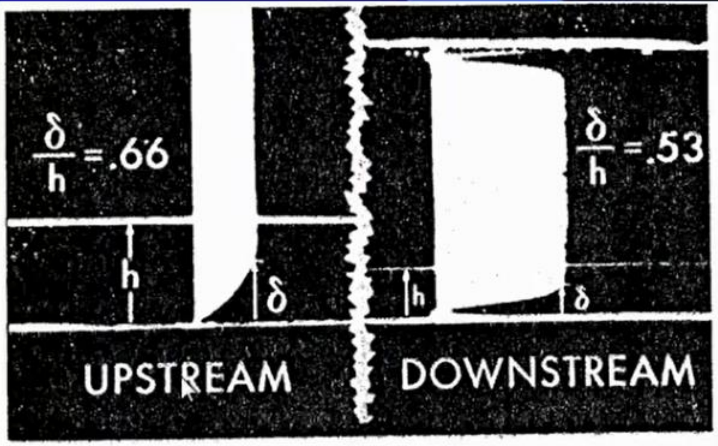


13. Composite blowup of upstream and downstream boundary-layer profiles from Fig. 12.

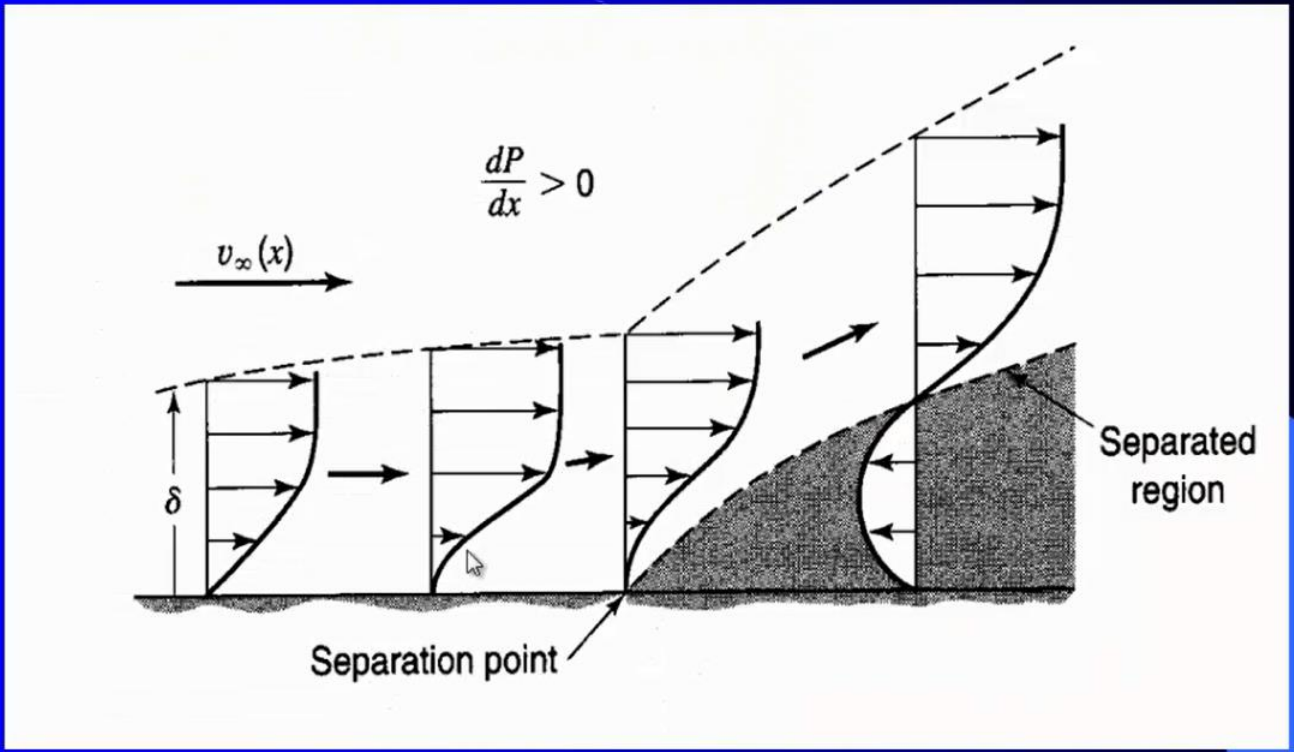
Favorable Pressure Gradient

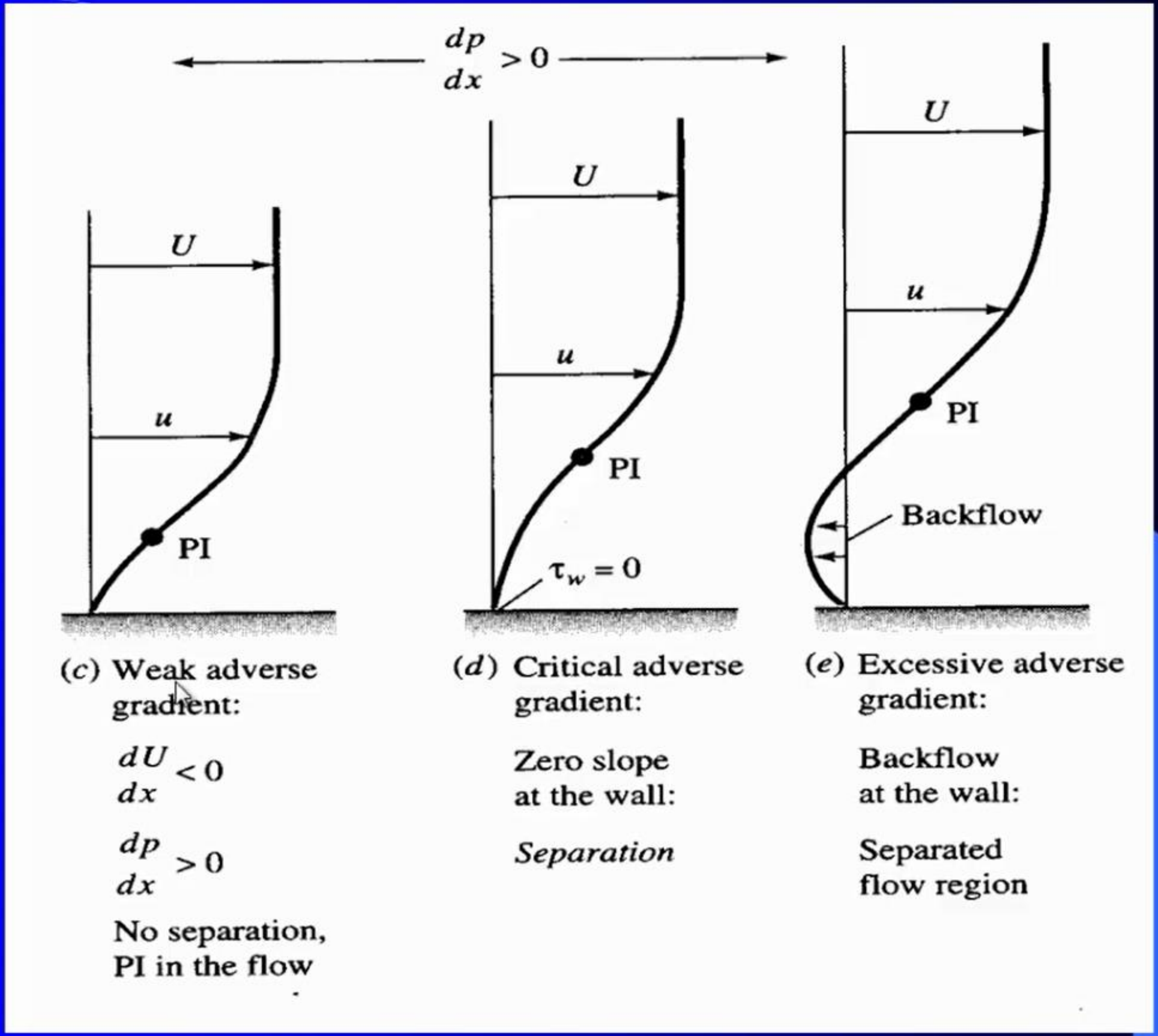


12. Flow in a converging channel (two-to-one contraction ratio).



13. Composite blowup of upstream and downstream boundary-layer profiles from Fig. 12.


$\frac{dP}{dx} > 0$
$v_\infty(x)$
δ
Separated region
Separation point

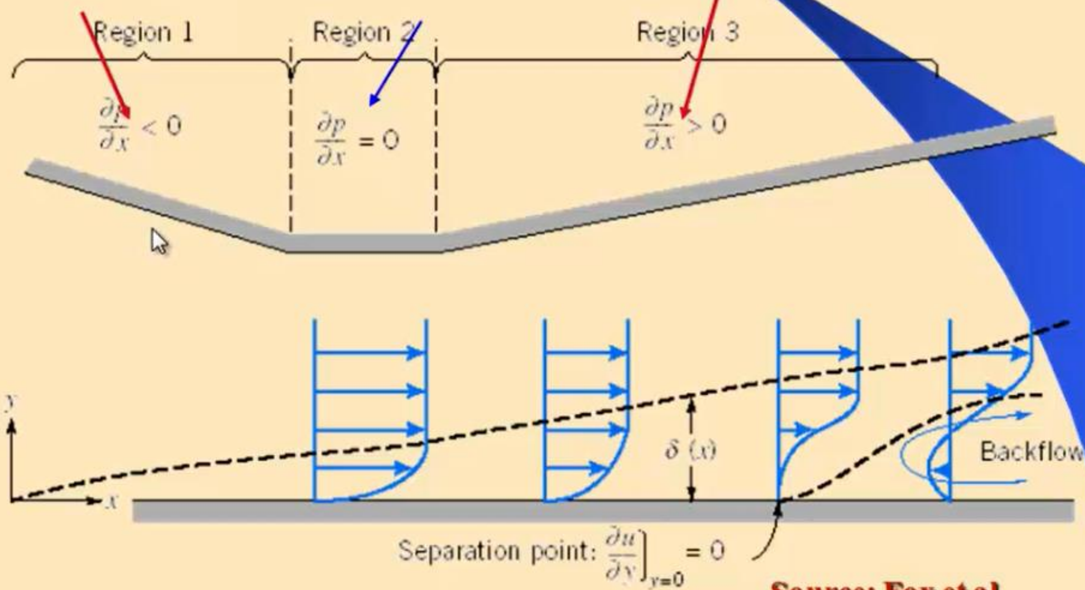

$\frac{dp}{dx} > 0$
U
u
PI
U
u
PI
$\tau_w = 0$
U
u
PI
Backflow
(c) Weak adverse gradient:
$\frac{dU}{dx} < 0$
$\frac{dp}{dx} > 0$
No separation, PI in the flow
(d) Critical adverse gradient:
Zero slope at the wall:
Separation
(e) Excessive adverse gradient:
Backflow at the wall:
Separated flow region

Effect of Pressure Gradients

Outside the boundary layer. The flow accelerates.

Constant velocity

Separation occurs. The flow decelerates.



Source: Fox et al.

Fig. 9.6 Boundary-layer flow with pressure gradient (boundary-layer thickness exaggerated for clarity).

Effect of Pressure Gradients

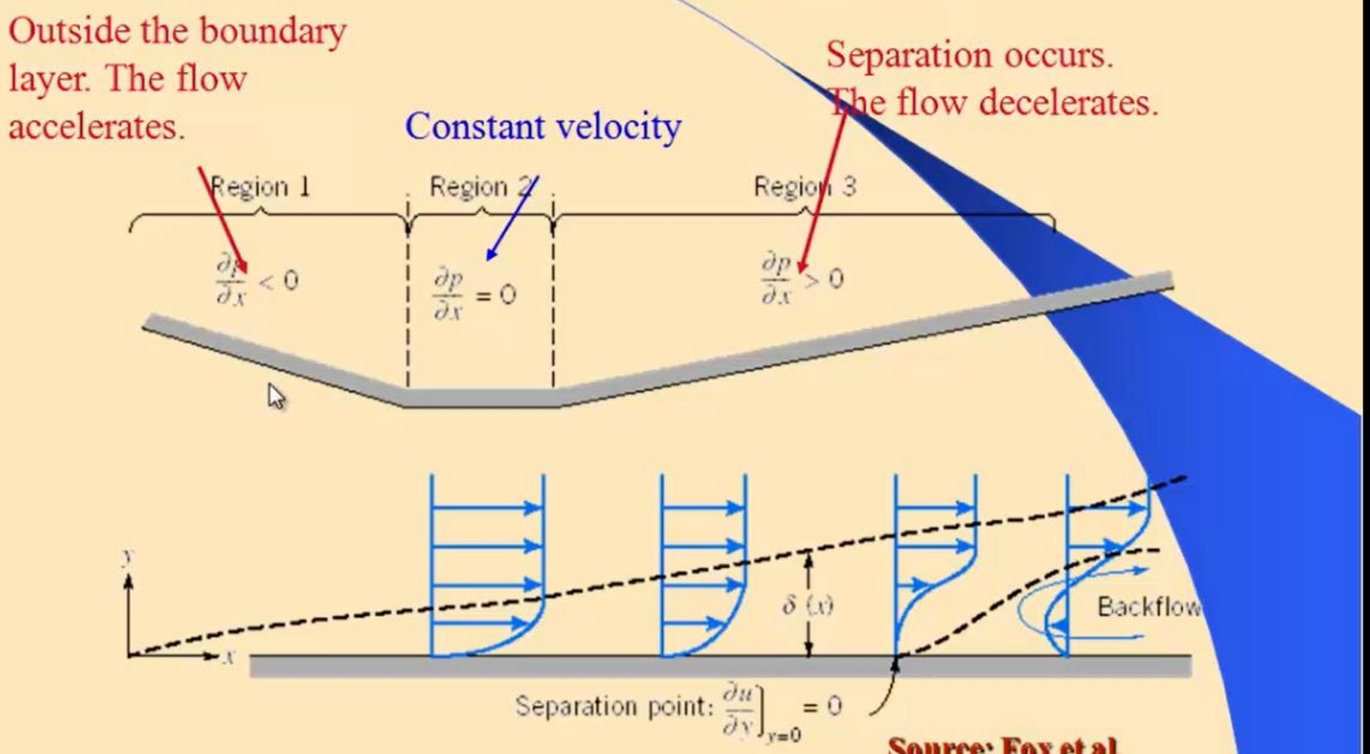
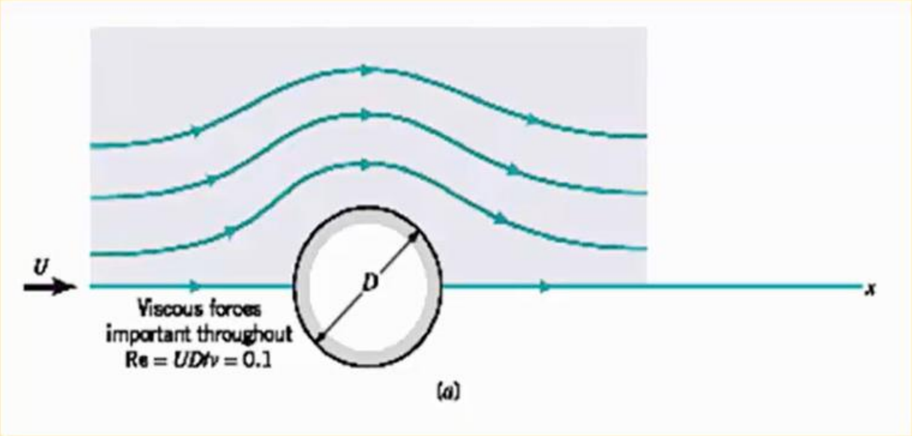


Fig. 9.6 Boundary-layer flow with pressure gradient (boundary-layer thickness exaggerated for clarity).

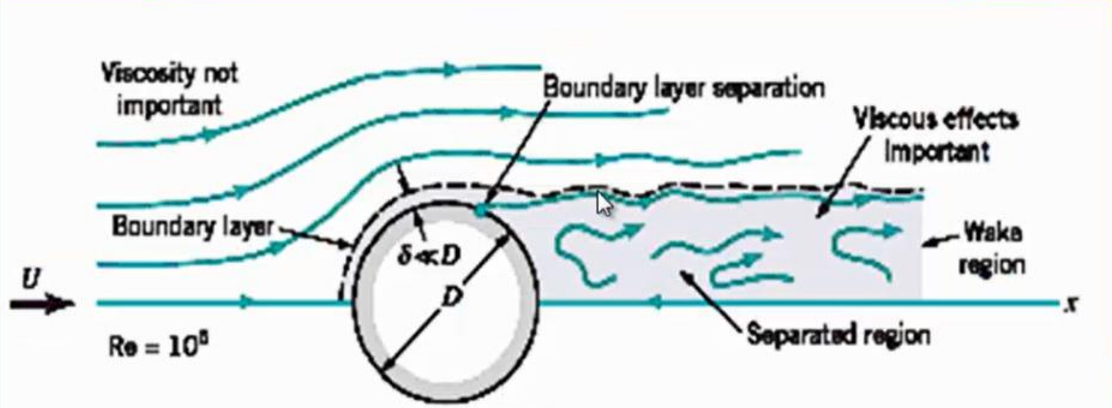
Flow Past an Circular Cylinder 1/4

- When Re= 0.1, the viscous effects are important several diameters in any direction from the cylinder. A somewhat surprising characteristic of this flow is that the streamlines are essentially symmetric about the center of the cylinder-the streamline pattern is the same in front of the cylinder as it is behind the cylinder.



Flow Past an Circular Cylinder 4/4

- With larger Reynolds numbers ($Re > 10^5$), the area affected by the viscous forces is forced farther downstream until it involve only a then ($\delta \ll D$) boundary layer on the front portion of the cylinder and an irregular, unsteady wake region that extends far downstream of the cylinder.
- The velocity gradients within the boundary layer and wake regions are much larger than those in the remainder of the flow field.



(c)

Flow Past an Circular Cylinder 4/4

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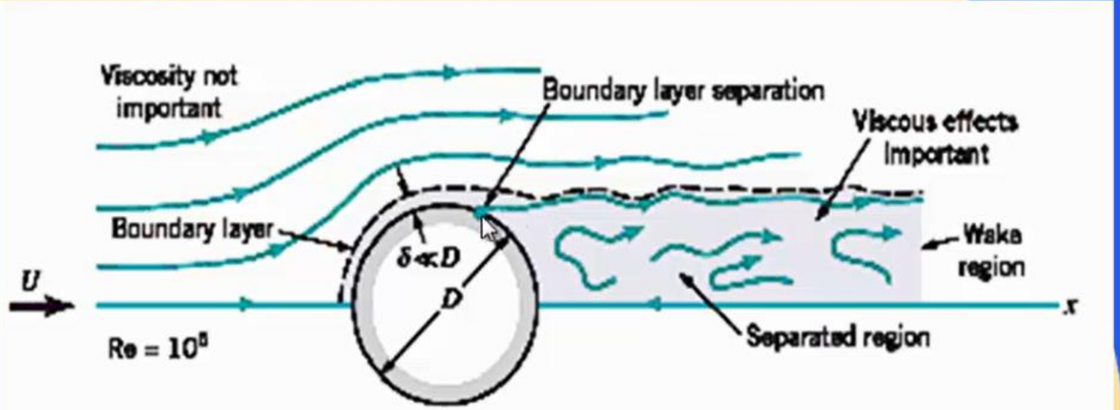
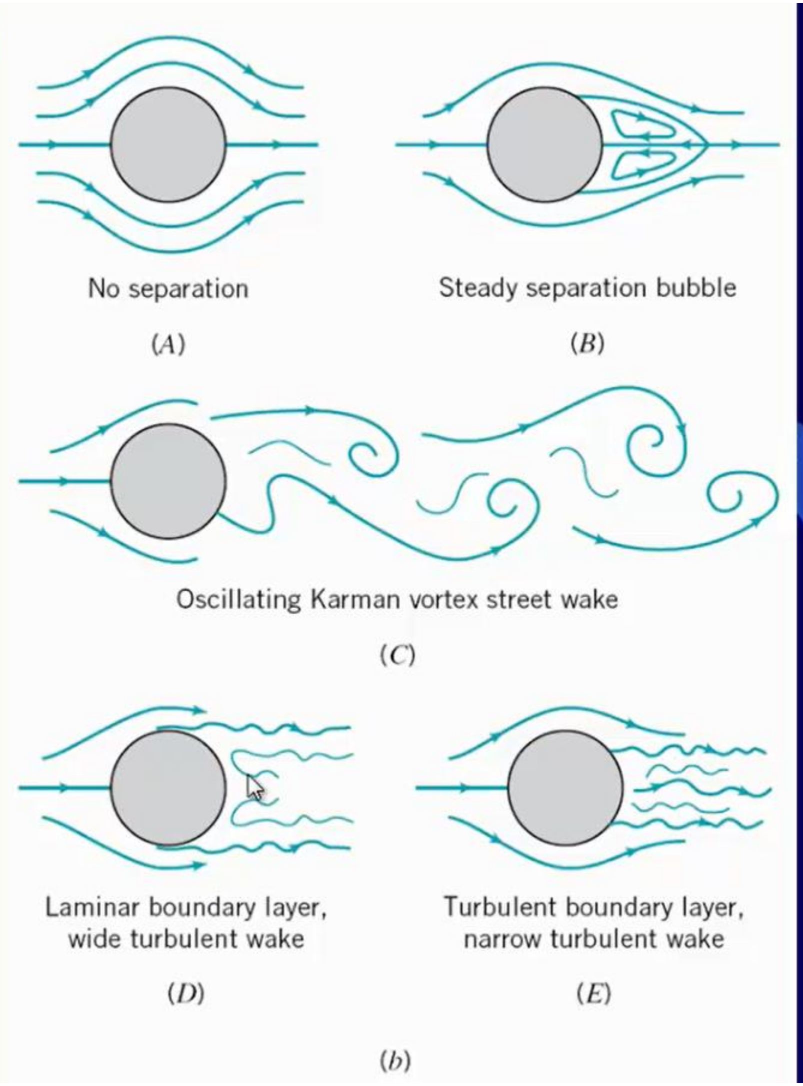
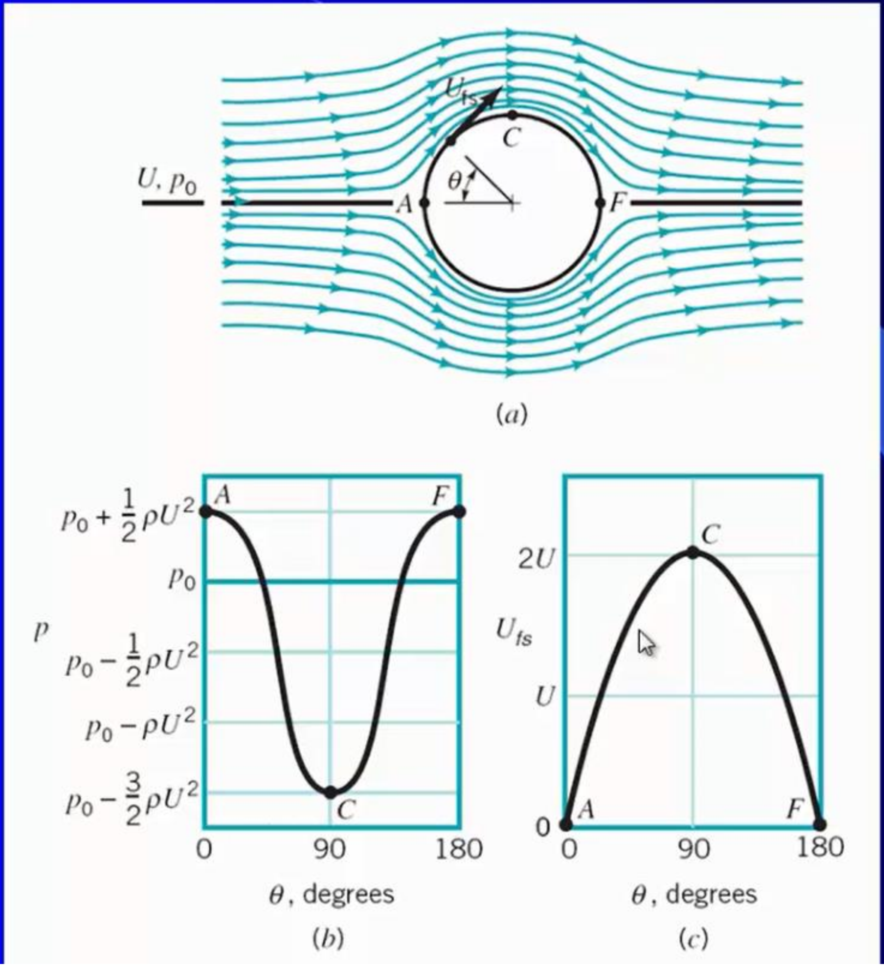


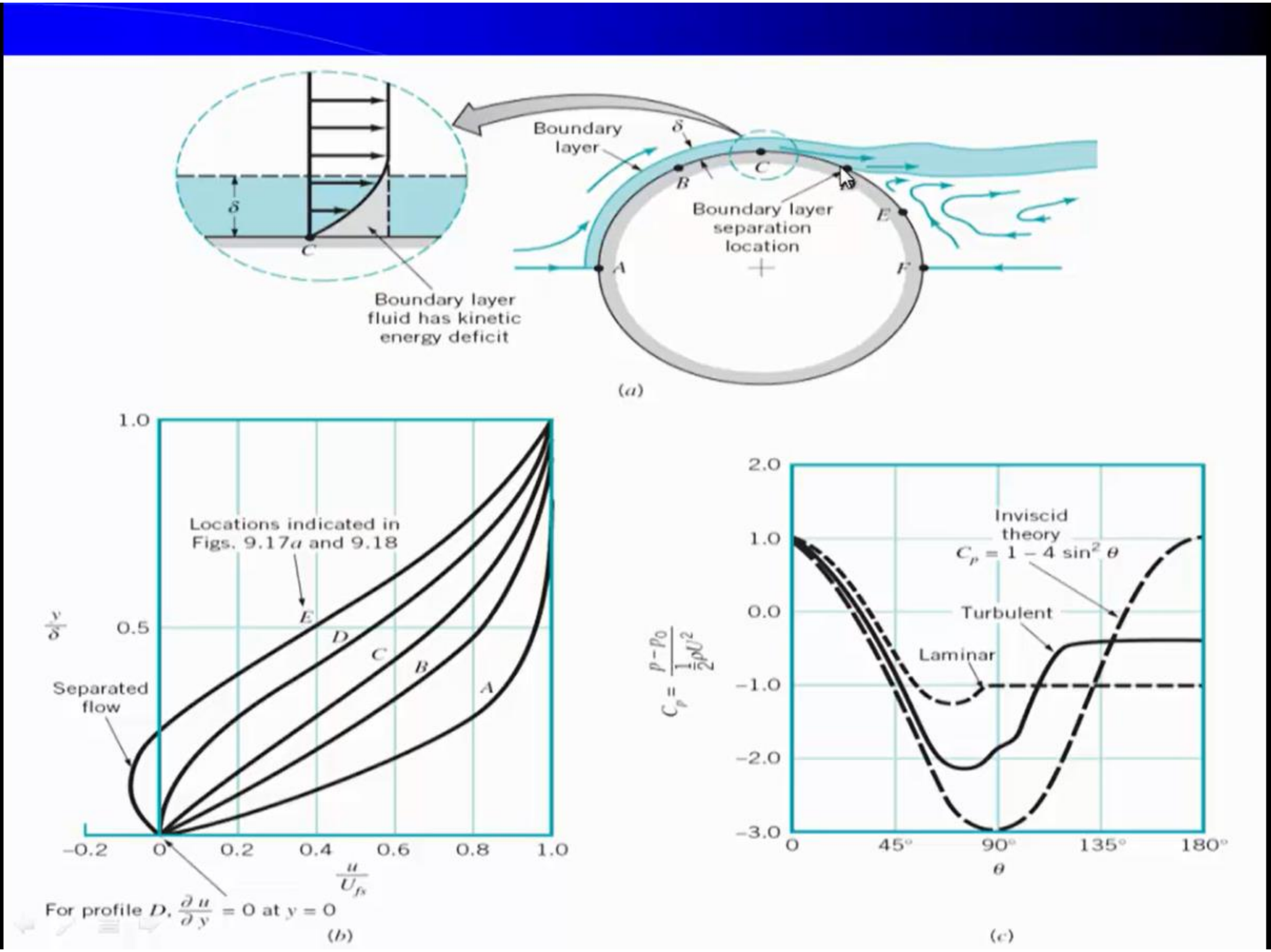
(c)

10:23

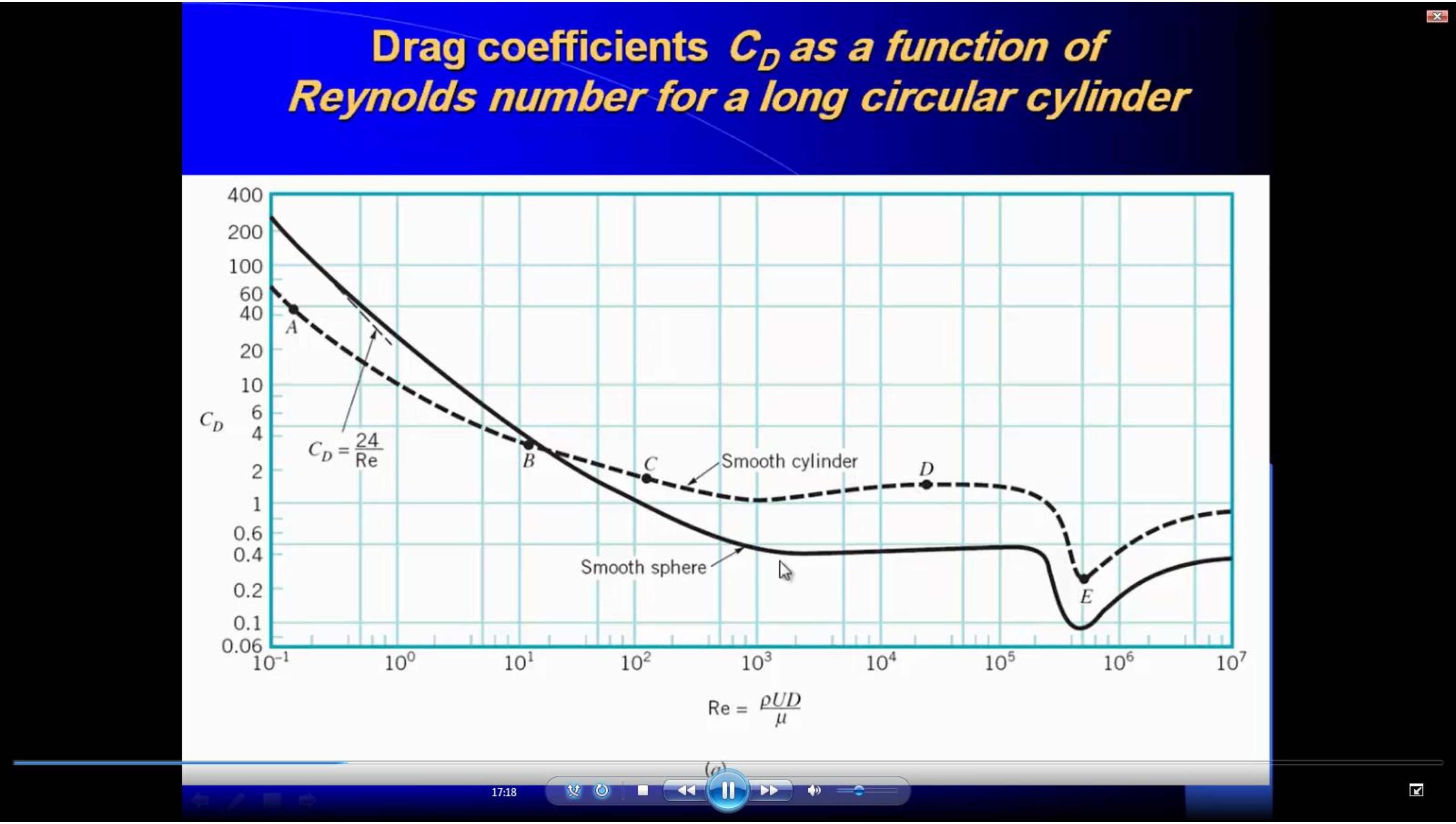

No separation
(A)
Steady separation bubble
(B)
Oscillating Karman vortex street wake
(C)
Laminar boundary layer,
wide turbulent wake
(D)
Turbulent boundary layer,
narrow turbulent wake
(E)
(b)



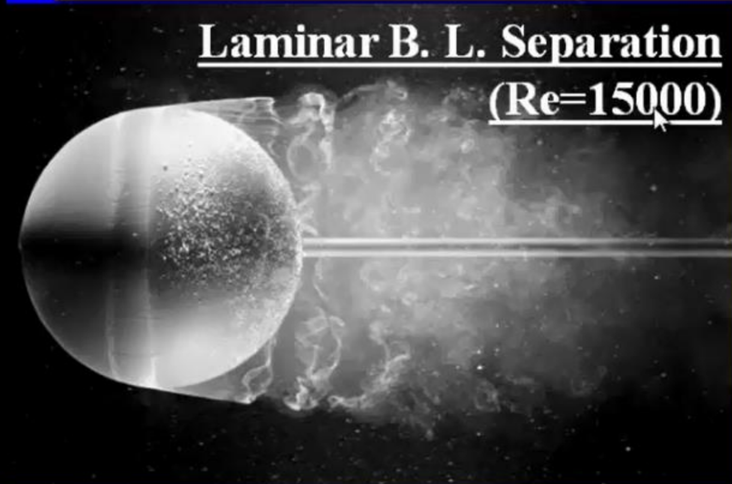
U, p_0
U_{fs}
C
A
F
θ
(a)
$p_0 + \frac{1}{2}\rho U^2$
p_0
p
$p_0 - \frac{1}{2}\rho U^2$
$p_0 - \rho U^2$
$p_0 - \frac{3}{2}\rho U^2$
A
F
C
0
90
180
θ, degrees
(b)
2U
U_{fs}
U
0
C
A
F
0
90
180
θ, degrees
(c)



Boundary layer
δ
B
C
Boundary layer separation location
E
A
F
Boundary layer fluid has kinetic energy deficit
(a)
1.0
0.5
y/δ
Locations indicated in Figs. 9.17a and 9.18
E
D
C
B
A
Separated flow
−0.2
0
0.2
0.4
0.6
0.8
1.0
u/U_fs
For profile D, ∂u/∂y = 0 at y = 0
(b)
2.0
1.0
0.0
−1.0
−2.0
−3.0
Inviscid theory
C_p = 1 − 4 sin² θ
Turbulent
Laminar
C_p = (p − p_0)/(½ρU²)
0
45°
90°
135°
180°
θ
(c)

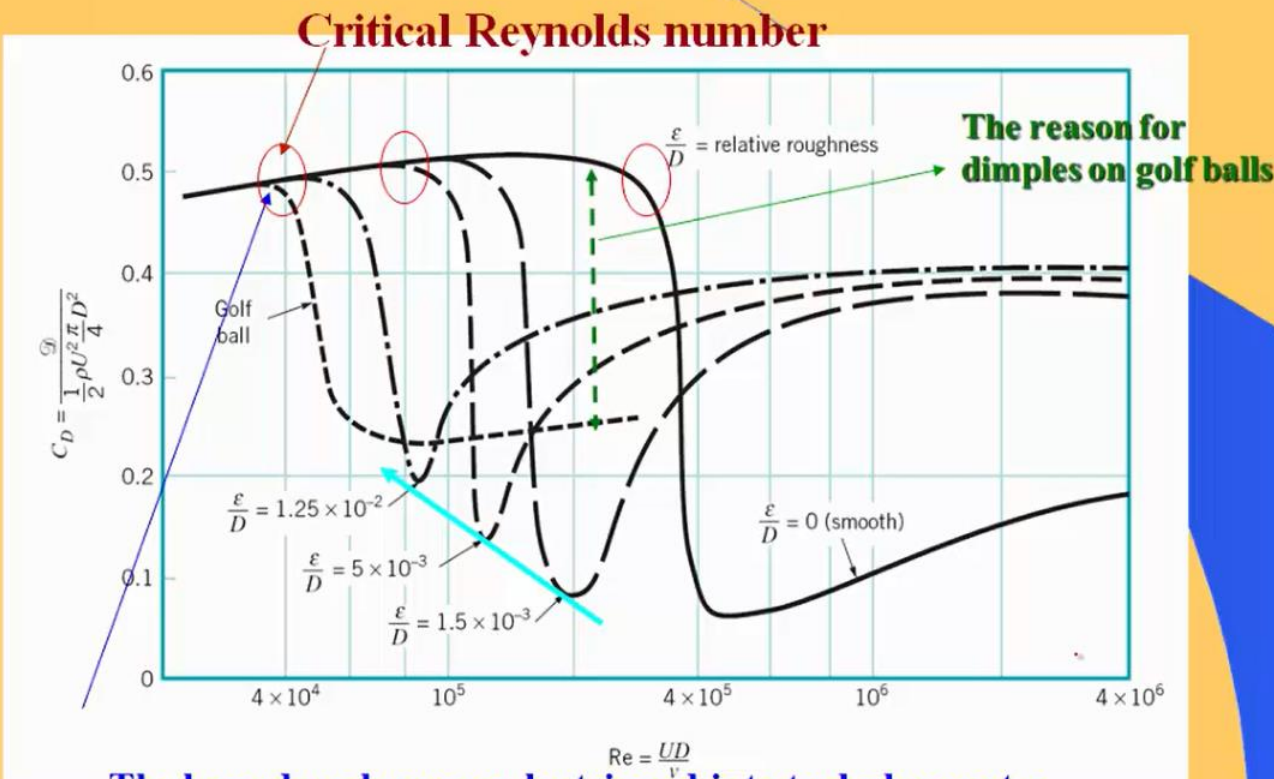

Drag coefficients C_D as a function of Reynolds number for a long circular cylinder
C_D
$C_D = \frac{24}{Re}$
A
B
C
D
E
Smooth cylinder
Smooth sphere
$Re = \frac{\rho U D}{\mu}$
17:18

Turbulent and Laminar B. L. Separation



- Flow is strong function of Re.
- Wake narrows for turbulent flow since TBL (turbulent boundary layer) is more resistant to separation due to adverse pressure gradient.
- $\theta_{sep,\, lam} \approx 80°$
- $\theta_{sep,}$ turb $\approx 140°$

C_D – Surface Roughness 3/3



The boundary layer can be tripped into turbulence at a smaller Reynolds number by using a rough-surfaced sphere.

Boundary Layer

The purpose of the boundary layer is to allow the fluid to change its velocity from the upstream value of U to zero on the surface. Thus, $\mathbf{V} = 0$ at $y = 0$ and $\mathbf{V} \approx U\,\hat{\mathbf{i}}$ at the edge of the boundary layer, with the velocity profile, $u = u(x, y)$ bridging the boundary layer thickness. This boundary layer characteristic occurs in a variety of flow situations, not just on flat plates. For example, boundary layers form on the surfaces of cars, in the water running down the gutter of the street, and in the atmosphere as the wind blows across the surface of the Earth (land or water).

Boundary Layer

If we displace the plate at section *a–a* by an appropriate amount δ^*,the *boundary layer displacement thickness*, the flowrates across each section will be identical.

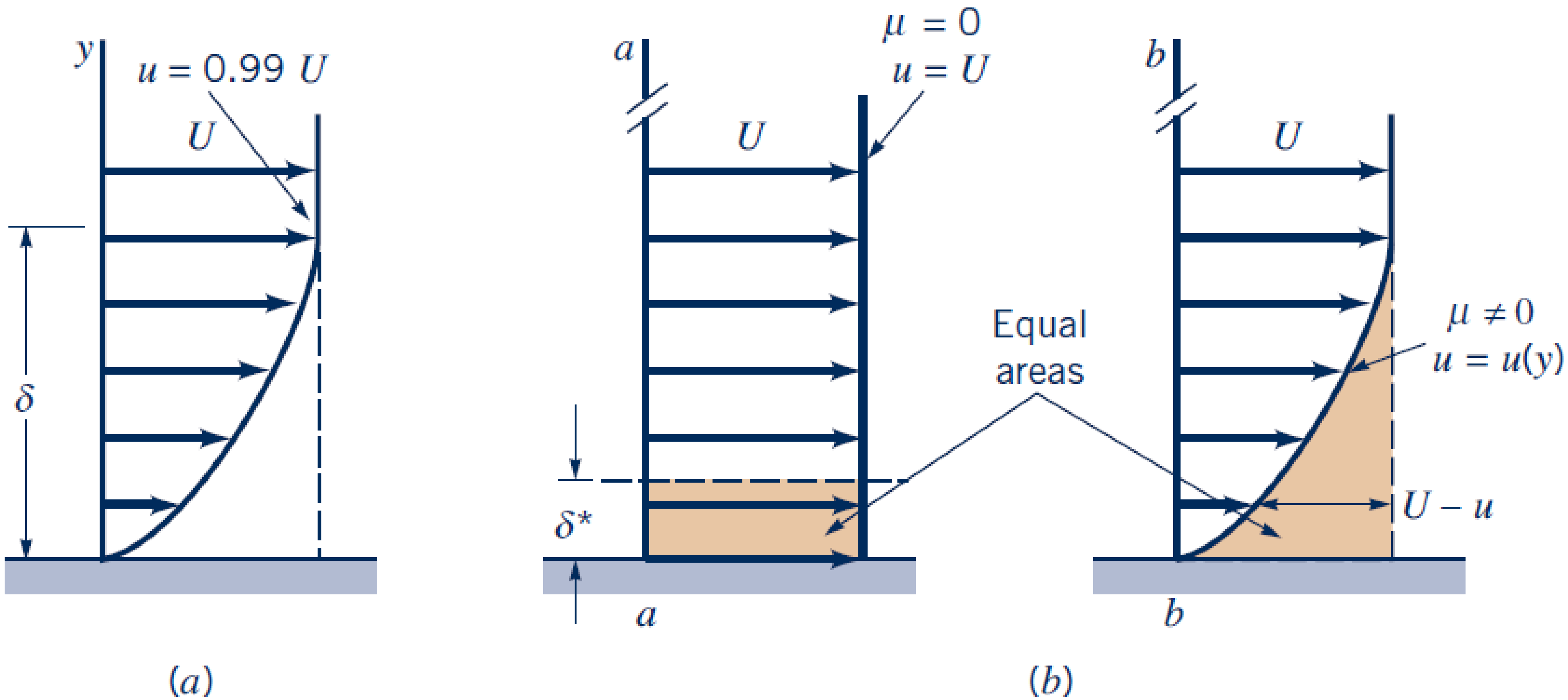
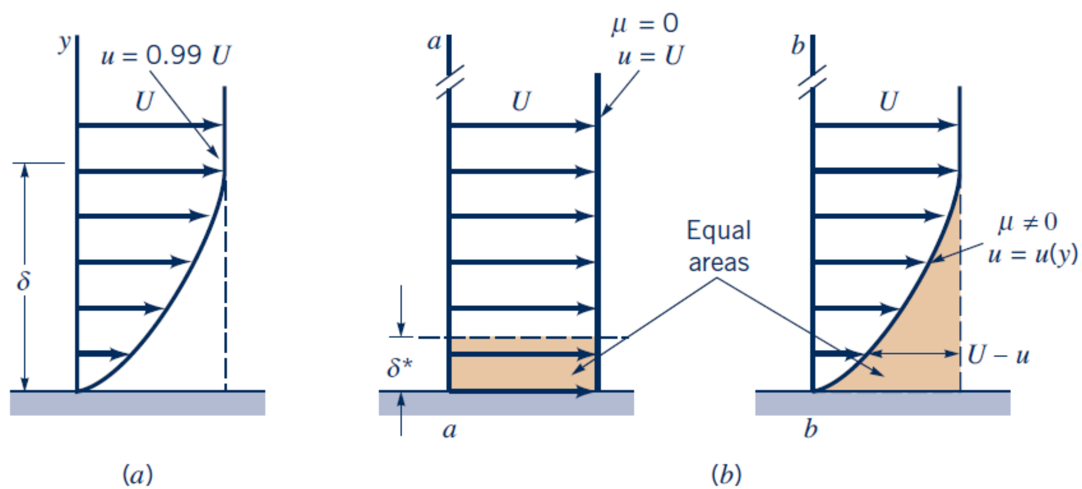


Figure 9.8 **Boundary layer thickness: (*a*) standard boundary layer thickness, (*b*) boundary layer displacement thickness.**

$$\delta = y \qquad \text{where} \qquad u = 0.99U$$

Boundary Layer



■ **Figure 9.8** **Boundary layer thickness: (*a*) standard boundary layer thickness, (*b*) boundary layer displacement thickness.**

$$\delta^* b U = \int_0^\infty (U - u) b \, dy$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

The distance through which the external inviscid flow is displaced by the presence of the boundary layer.

Boundary Layer

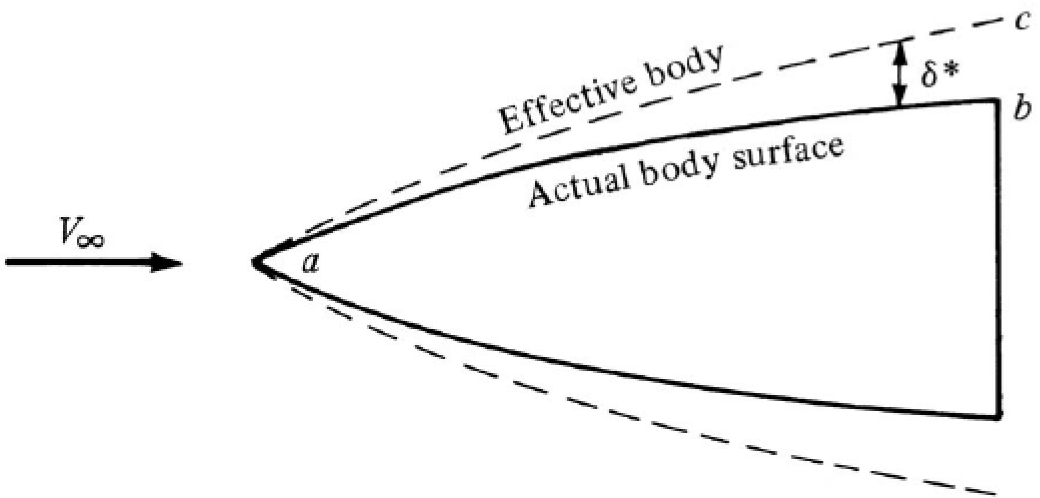
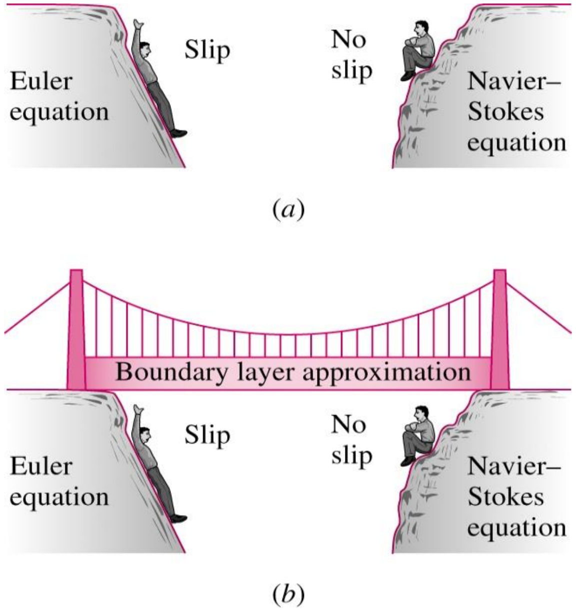
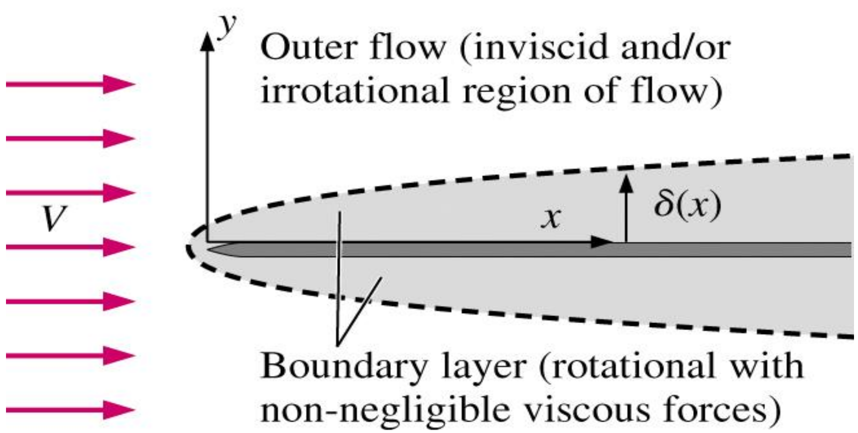


Figure 17.6 The "effective body," equal to the actual body shape plus the displacement thickness distribution.

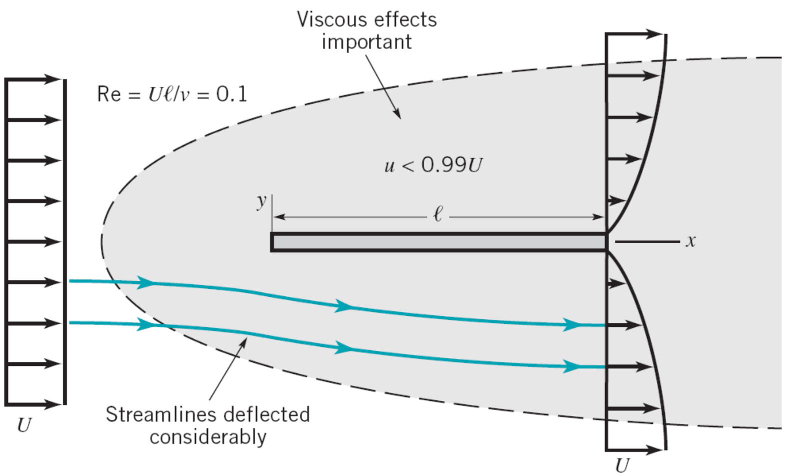
Boundary Layer (BL) Approximation



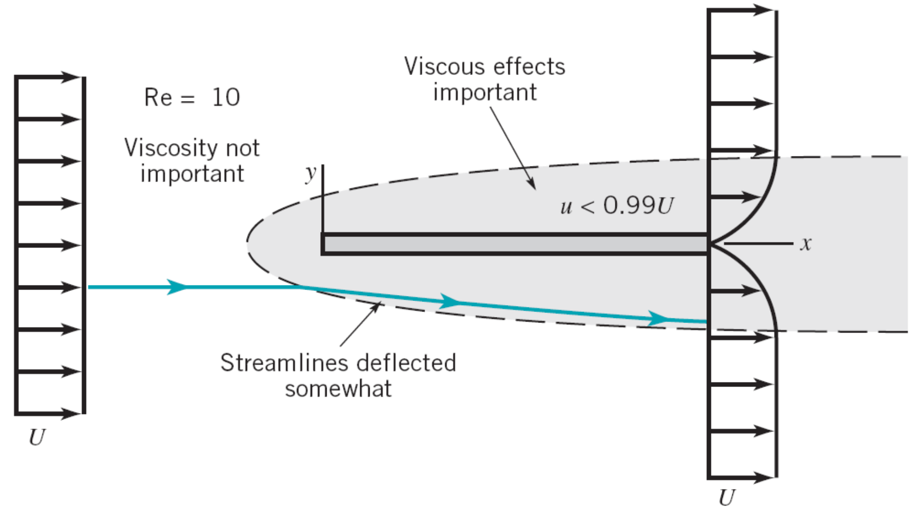


- BL approximation bridges the gap between the Euler (inviscid) and Navier-Stokes (NS) (viscous) equations, and between the slip and no-slip Boundary Conditions (BC) at the wall.
- Prandtl (1904) introduced the BL approximation- BL is **a thin region on the surface of a body in which viscous effects are very important and outside of which the fluid behaves as inviscid.**

Character of the steady, viscous flow past a flat plate parallel to the upstream velocity



(a)



(b)

Inertia force = ma =ρL³*V dV/dL= ρV²L²

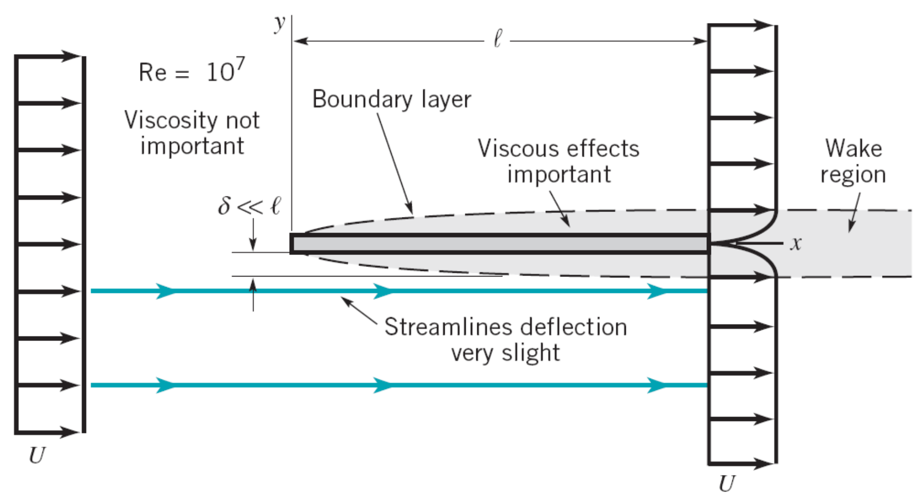
Viscous Force = μ L² dV/dL = μ V L

$$\boxed{\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{Inertia\ Force}{Viscous\ Force}}$$

(a) low Reynolds number flow,

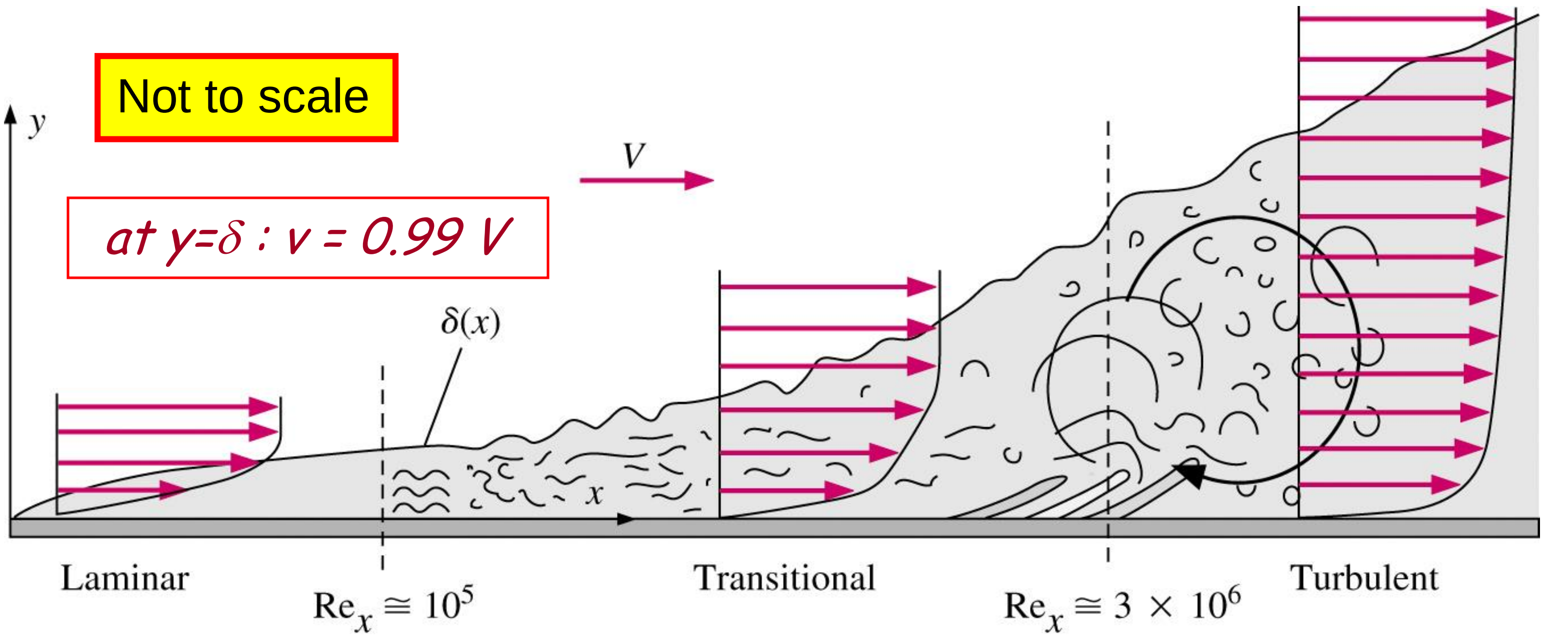
(b) moderate Reynolds number flow,

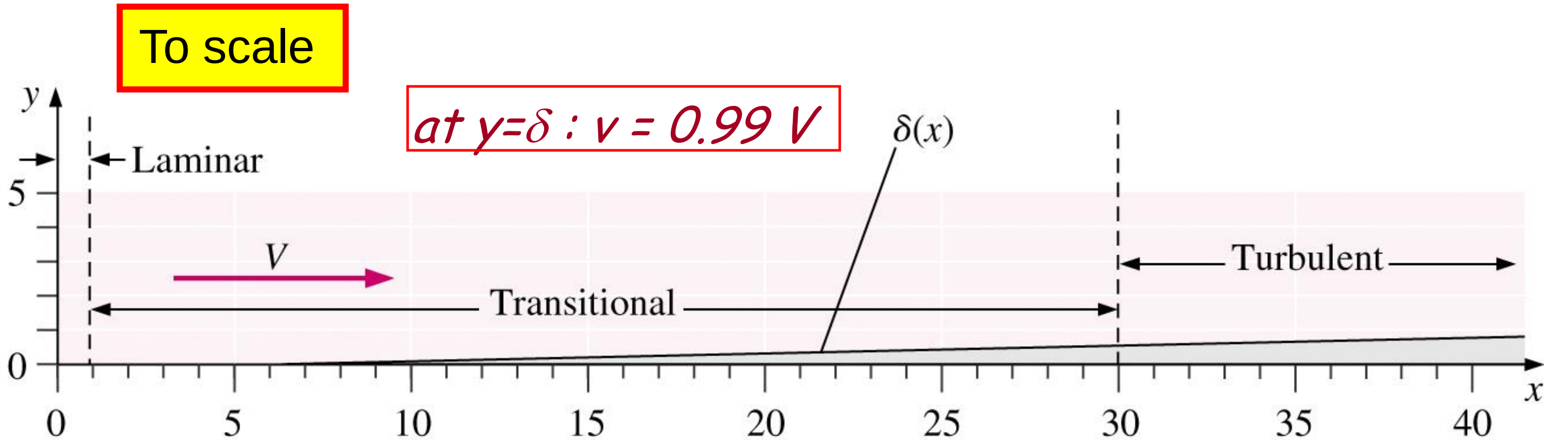
(c) large Reynolds number flow.



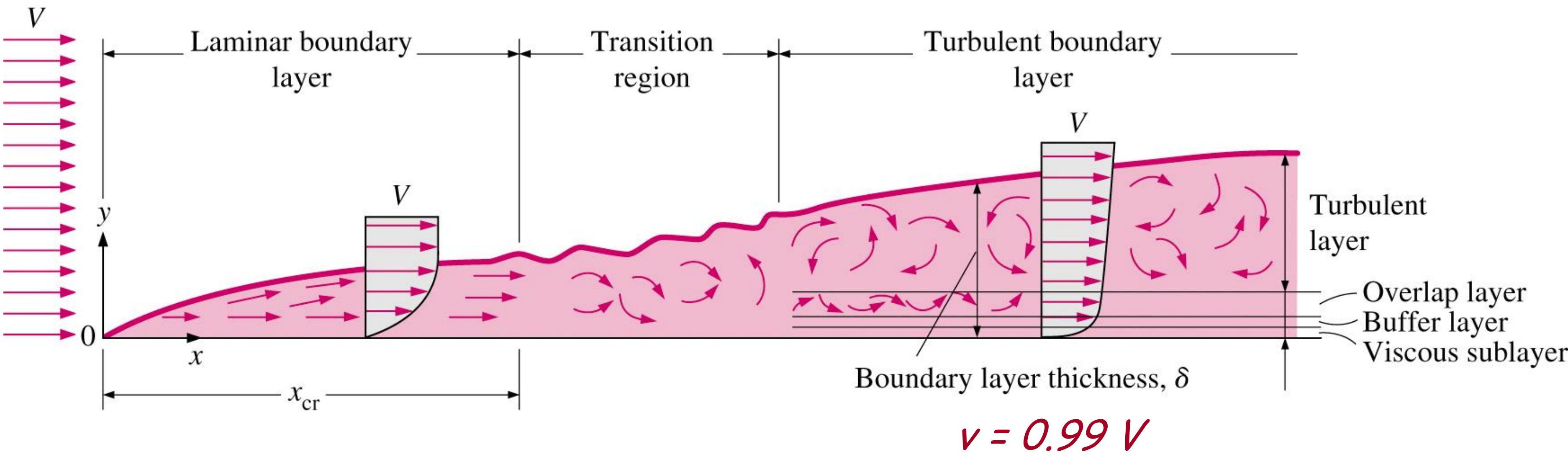
(c)

Boundary Layer on a Flat Plate





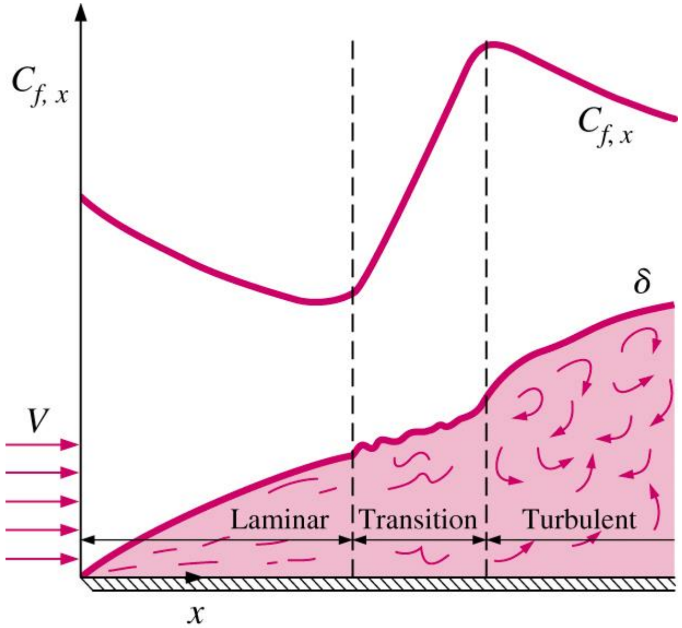
Flat Plate Drag



- Drag on flat plate is solely due to friction ($F_D = F_{Dfriction}$) created by laminar, transitional, and turbulent boundary layers.
- BL thickness, δ, is the distance from the plate at which v = 0.99 V

Flat Plate Drag

$C_{D,\ friction} = C_f$



- Local friction coefficient
  - Laminar: $C_{f,x} = \frac{0.664}{Re_x^{1/2}}$
  - Turbulent: $C_{f,x} = \frac{0.059}{Re_x^{1/5}}$
- Average friction coefficient

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$$

For some cases, plate is long enough for turbulent flow, but not long enough to neglect laminar portion

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,lam}\,dx + \int_{x_{cr}}^{L} C_{f,x,turb}\,dx\right)$$

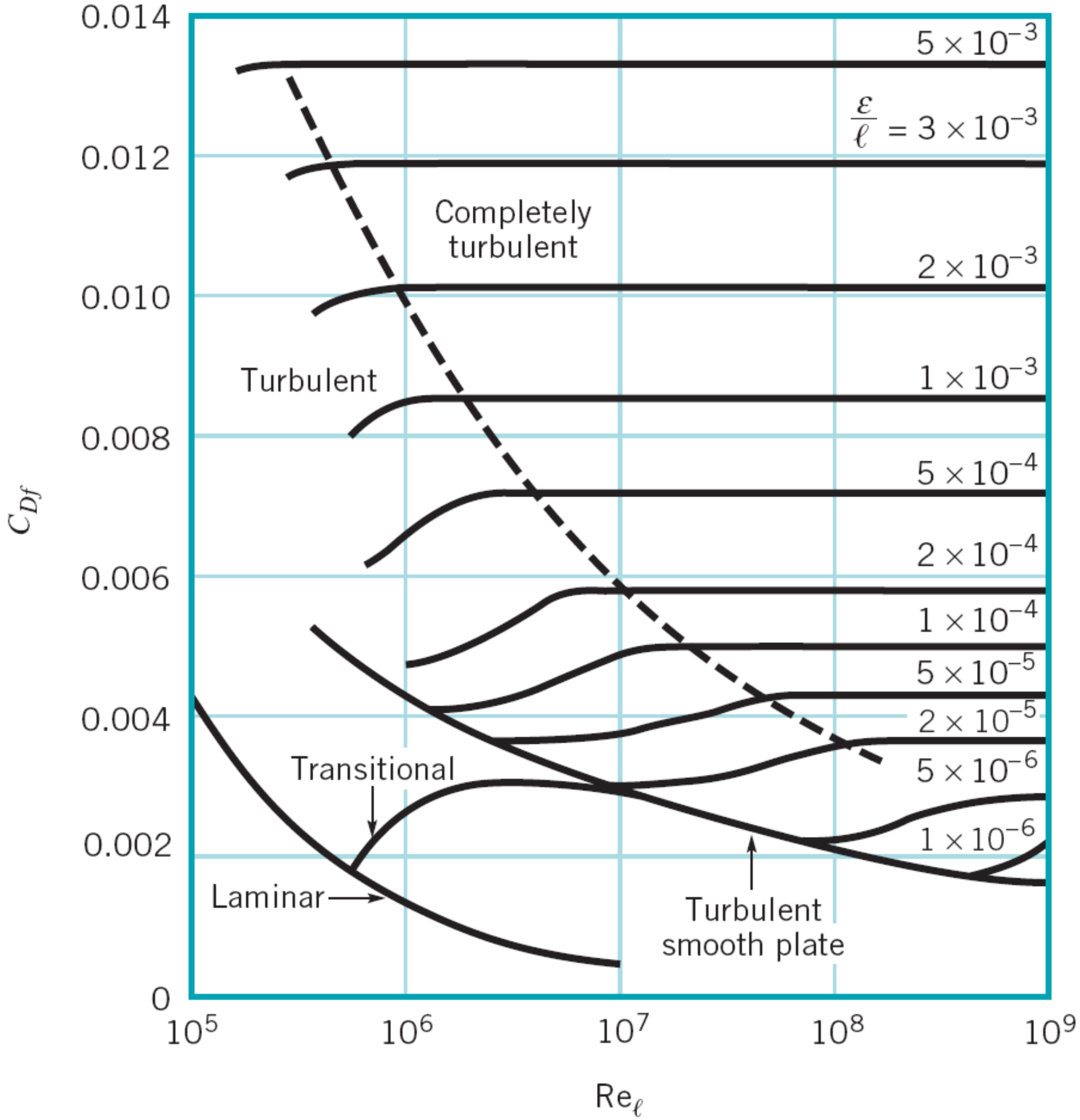
Flat Plate Drag

Empirical Equations for the Flat Plate Drag Coefficient

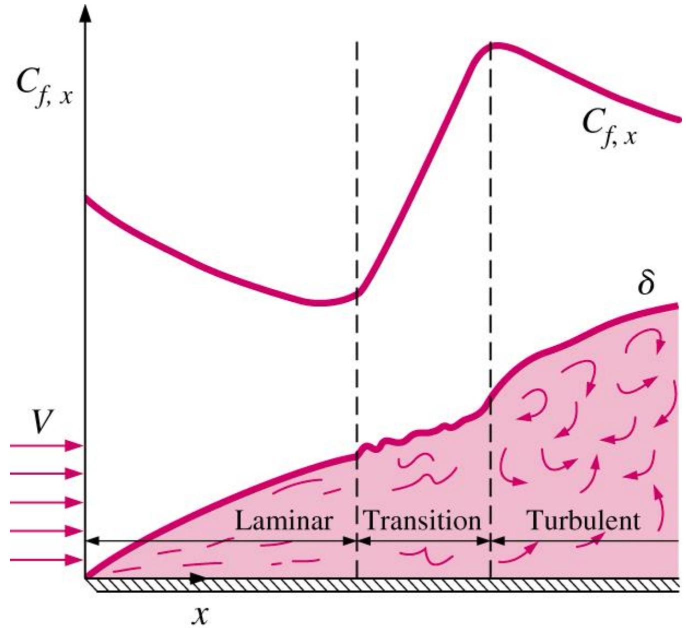
Equation	Flow Conditions
$C_{Df} = 1.328/(\mathrm{Re}_\ell)^{0.5}$	Laminar flow
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58} - 1700/\mathrm{Re}_\ell$	Transitional with $\mathrm{Re}_{xcr} = 5 \times 10^5$
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58}$	Turbulent, smooth plate
$C_{Df} = [1.89 - 1.62 \log(\varepsilon/\ell)]^{-2.5}$	Completely turbulent

Transition takes place at a distance x given by:
Re_{xcr}=2X10^5 to 3X10^6 – $\boxed{\text{We will use } Re_{xcr}= 5X10^5}$

Drag coefficient may also be obtained from charts such as those on the next slides



Friction drag coefficient for a flat plate parallel to the upstream flow.

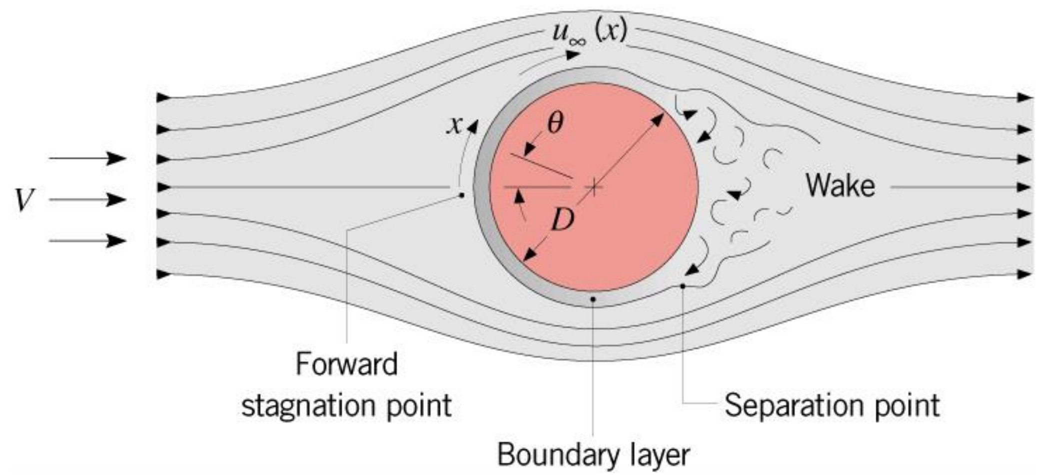


Laminar: $C_{Df} = f\,(Re)$

Turbulent: $C_{Df} = f\,(Re, \varepsilon/L)$

Cylinders in Cross Flow

Conditions depend on special features of boundary layer development, including onset at a stagnation point and separation, as well as transition to turbulence.



- Stagnation point: Location of zero velocity and maximum pressure. Followed by boundary layer development under a favorable pressure gradient and hence acceleration of the free stream flow
- As the rear of the cylinder is approached, the pressure must begin to increase.
- Hence, there is a minimum in the pressure distribution, *p(x)*, after which boundary layer development occurs under the influence of an adverse pressure gradient

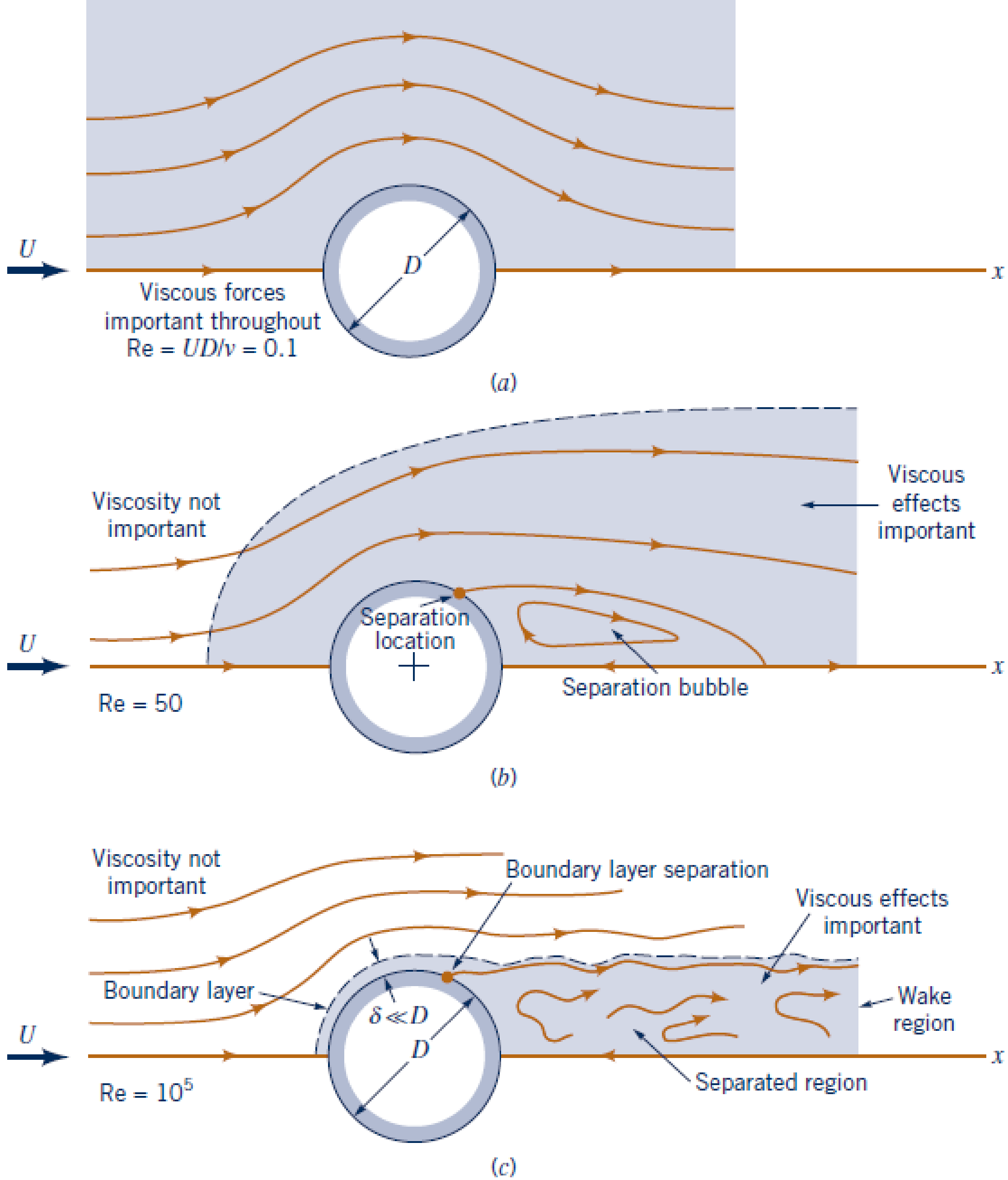


Figure 9.6 Character of the steady, viscous flow past a circular cylinder: (*a*) low Reynolds number flow, (*b*) moderate Reynolds number flow, (*c*) large Reynolds number flow.

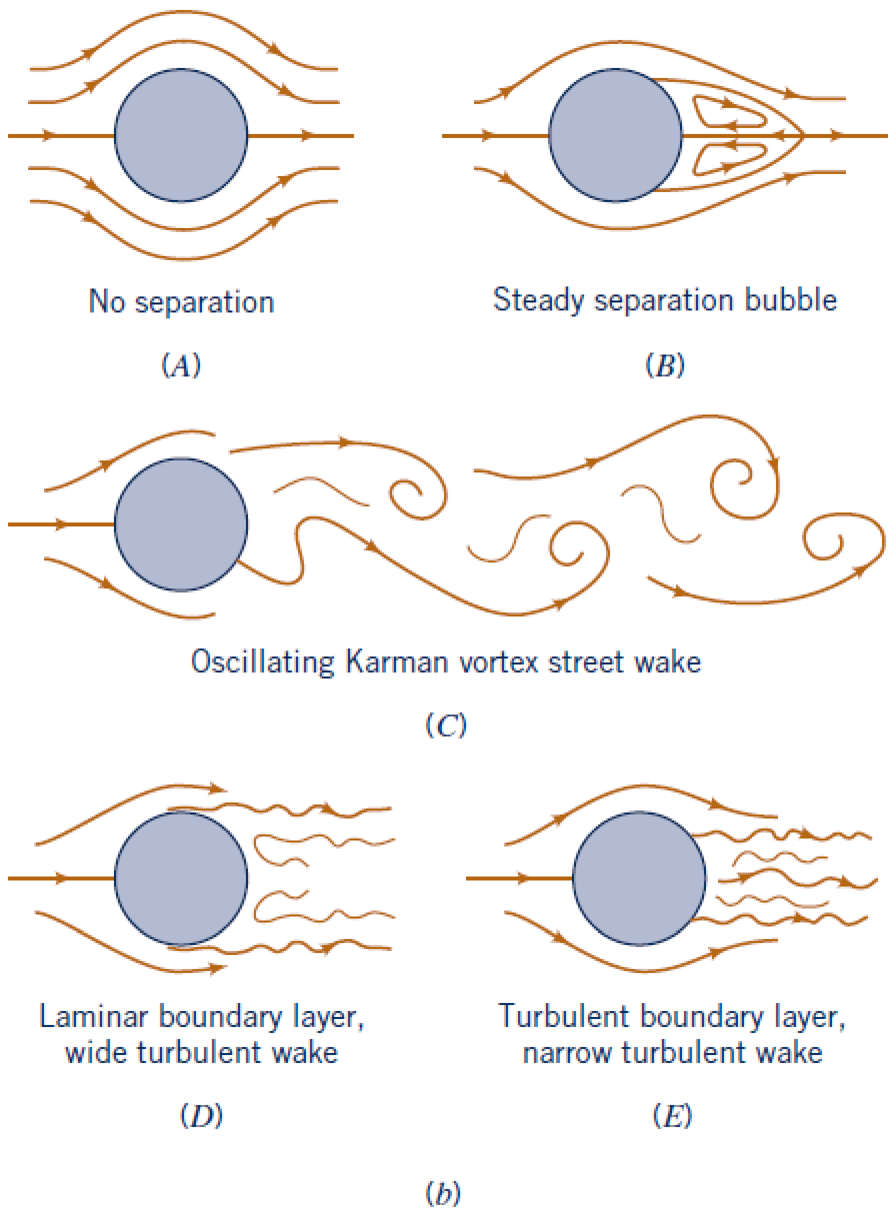
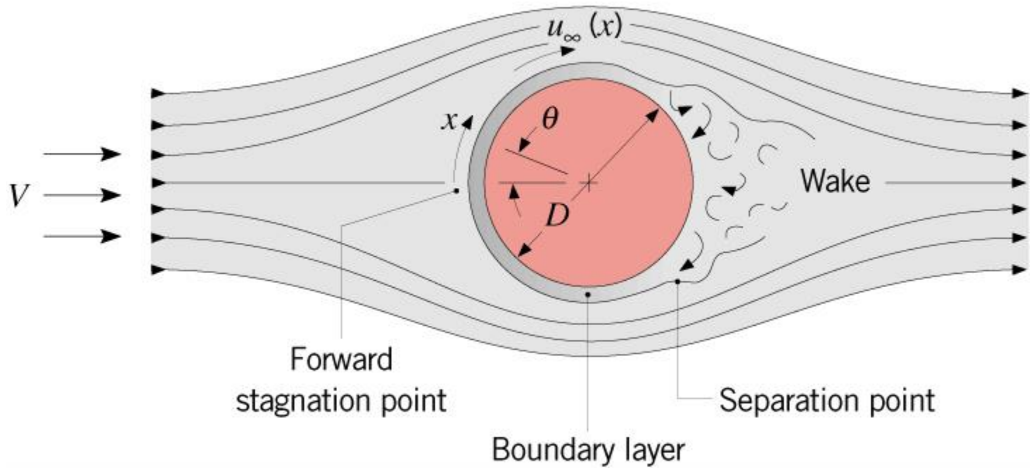
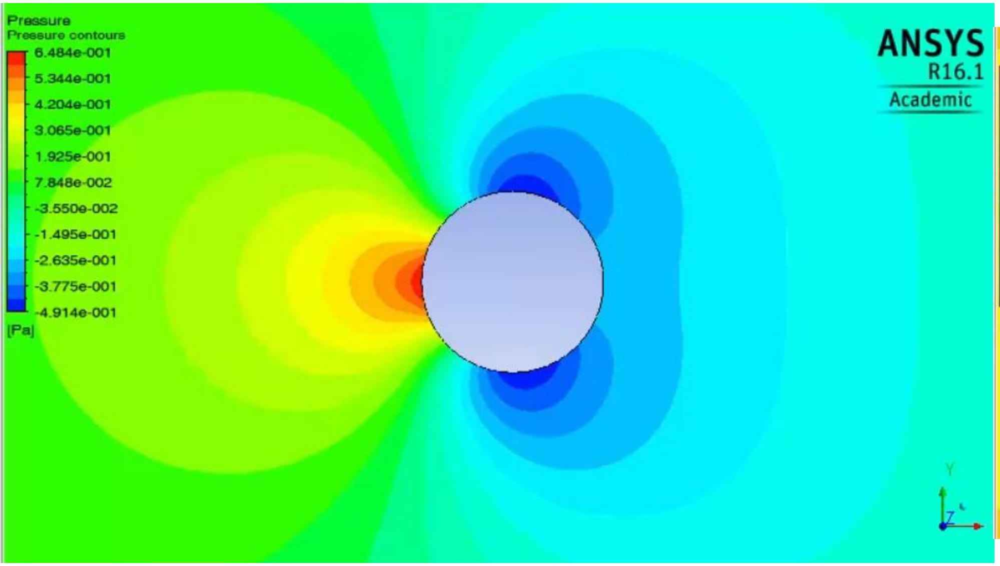
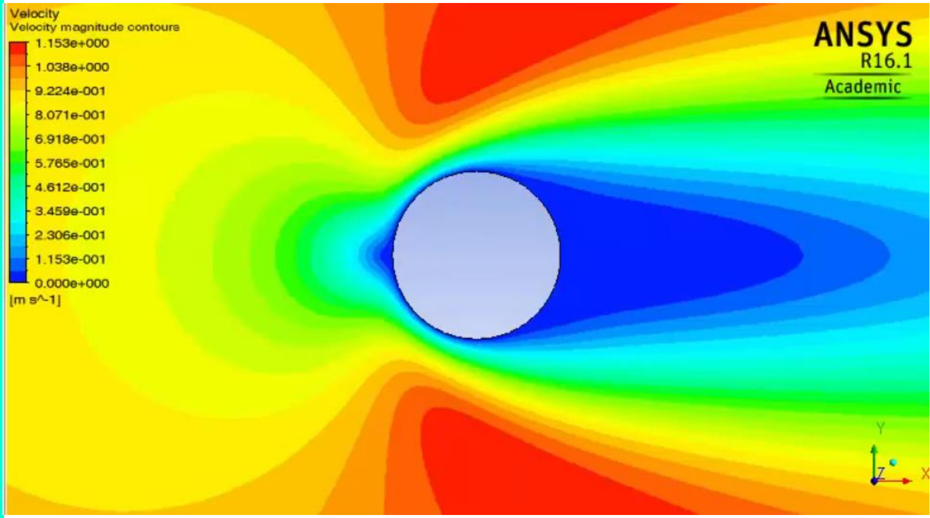


Figure 9.21 (*a*) Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere. (*b*) Typical flow patterns for flow past a circular cylinder at various Reynolds numbers as indicated in (*a*).

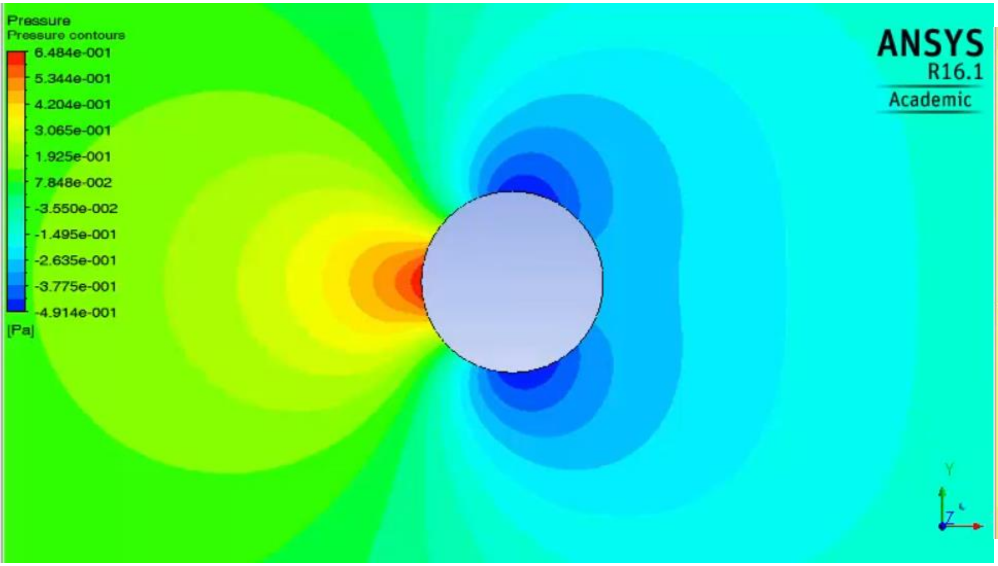
Cylinders in Cross Flow

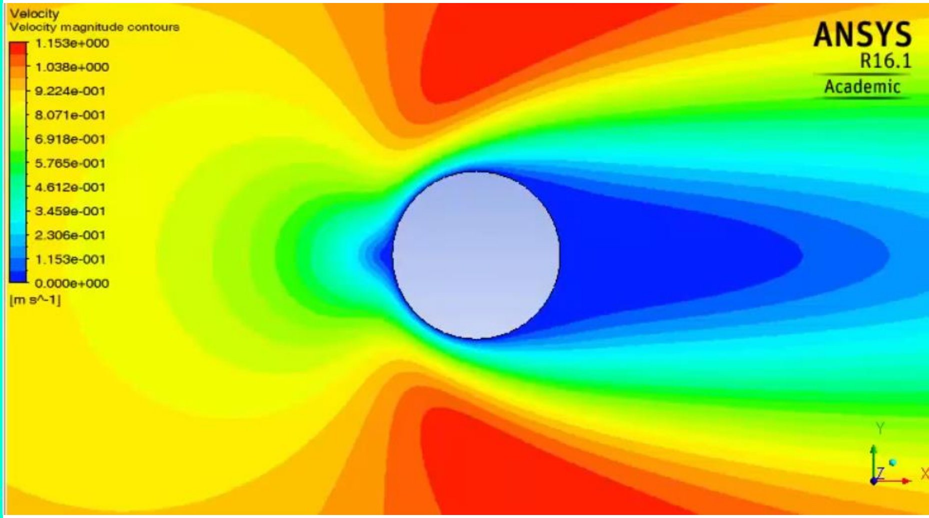


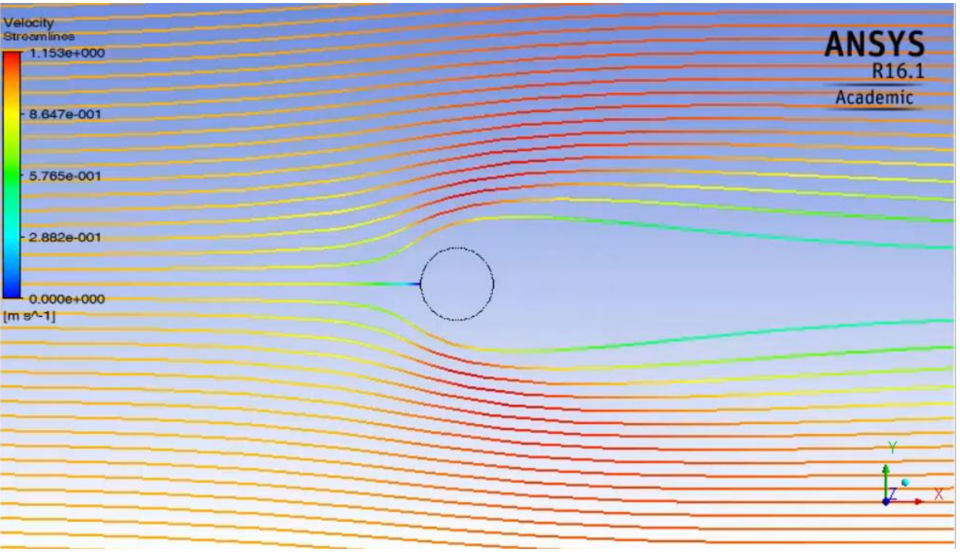


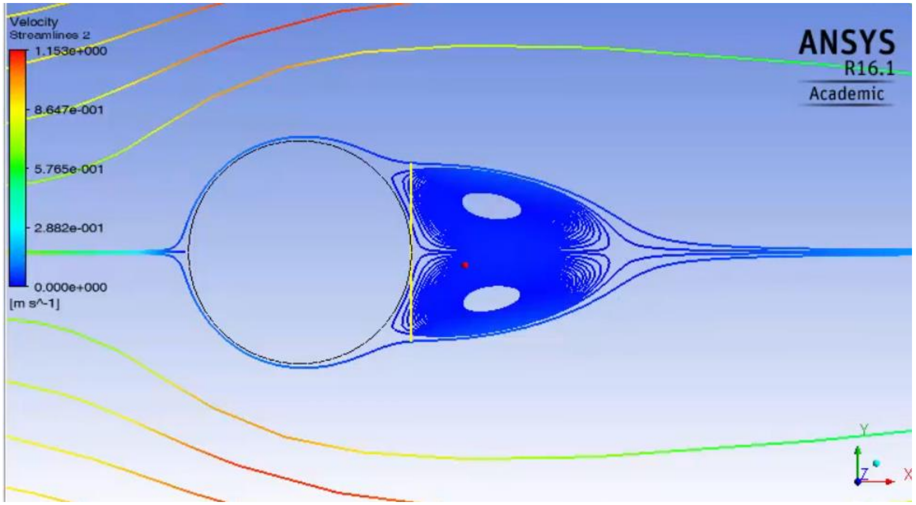


Cylinders in Cross Flow

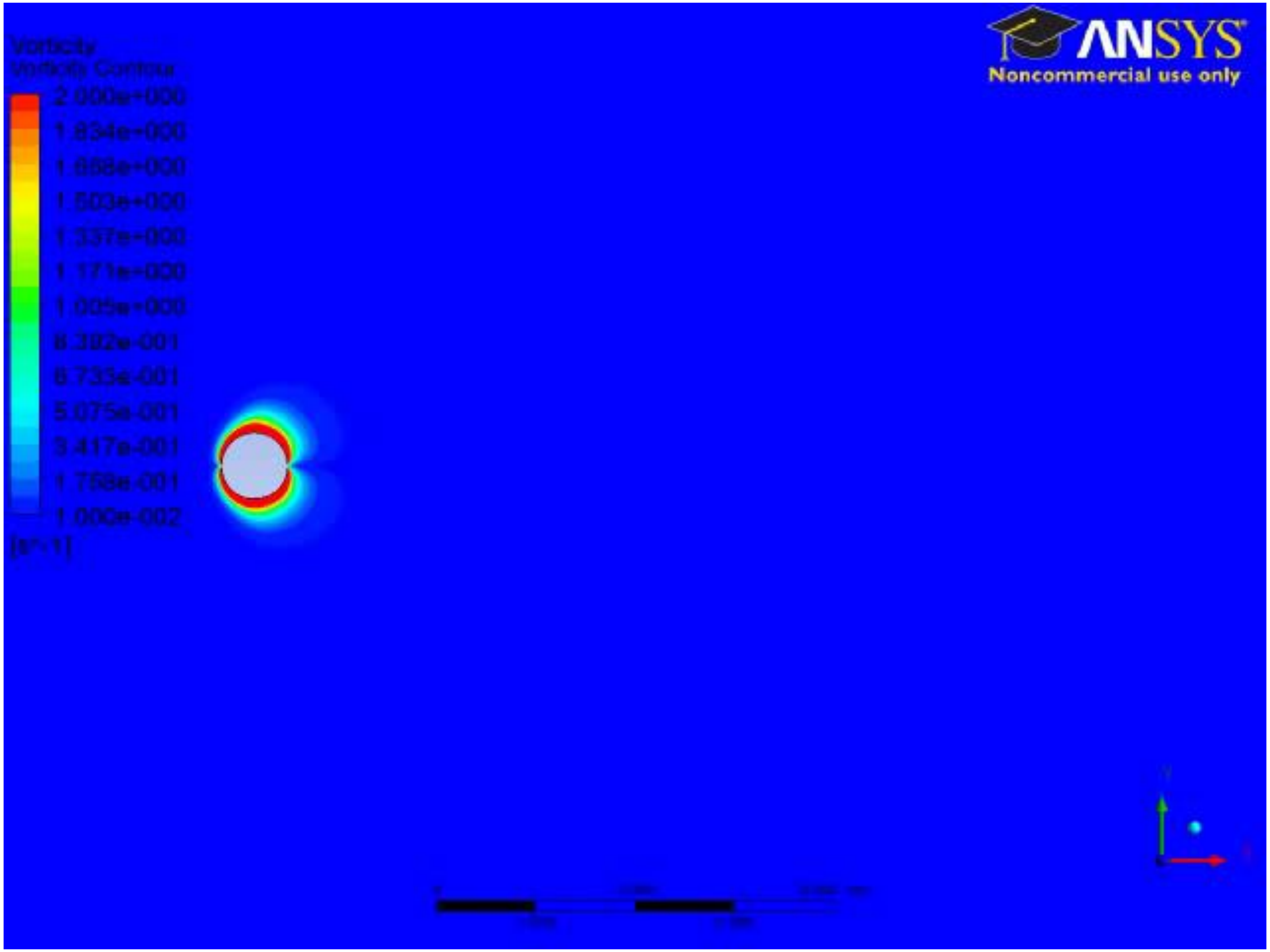




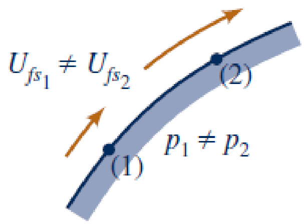




Cylinders in Cross Flow



Cylinders in Cross Flow



The free-stream velocity on a curved surface is not constant.

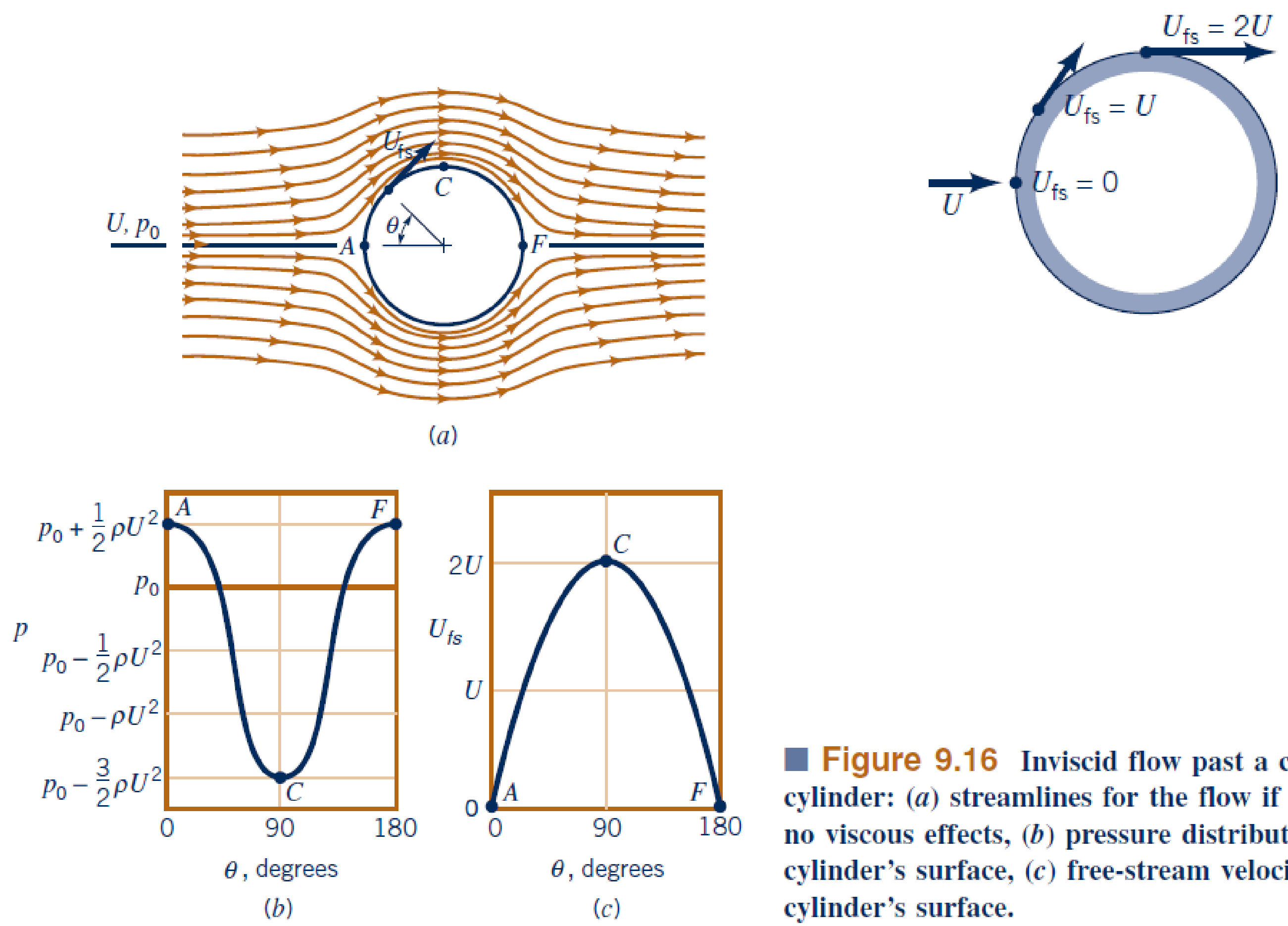


Figure 9.16 Inviscid flow past a circular cylinder: (*a*) streamlines for the flow if there were no viscous effects, (*b*) pressure distribution on the cylinder's surface, (*c*) free-stream velocity on the cylinder's surface.

Cylinders in Cross Flow

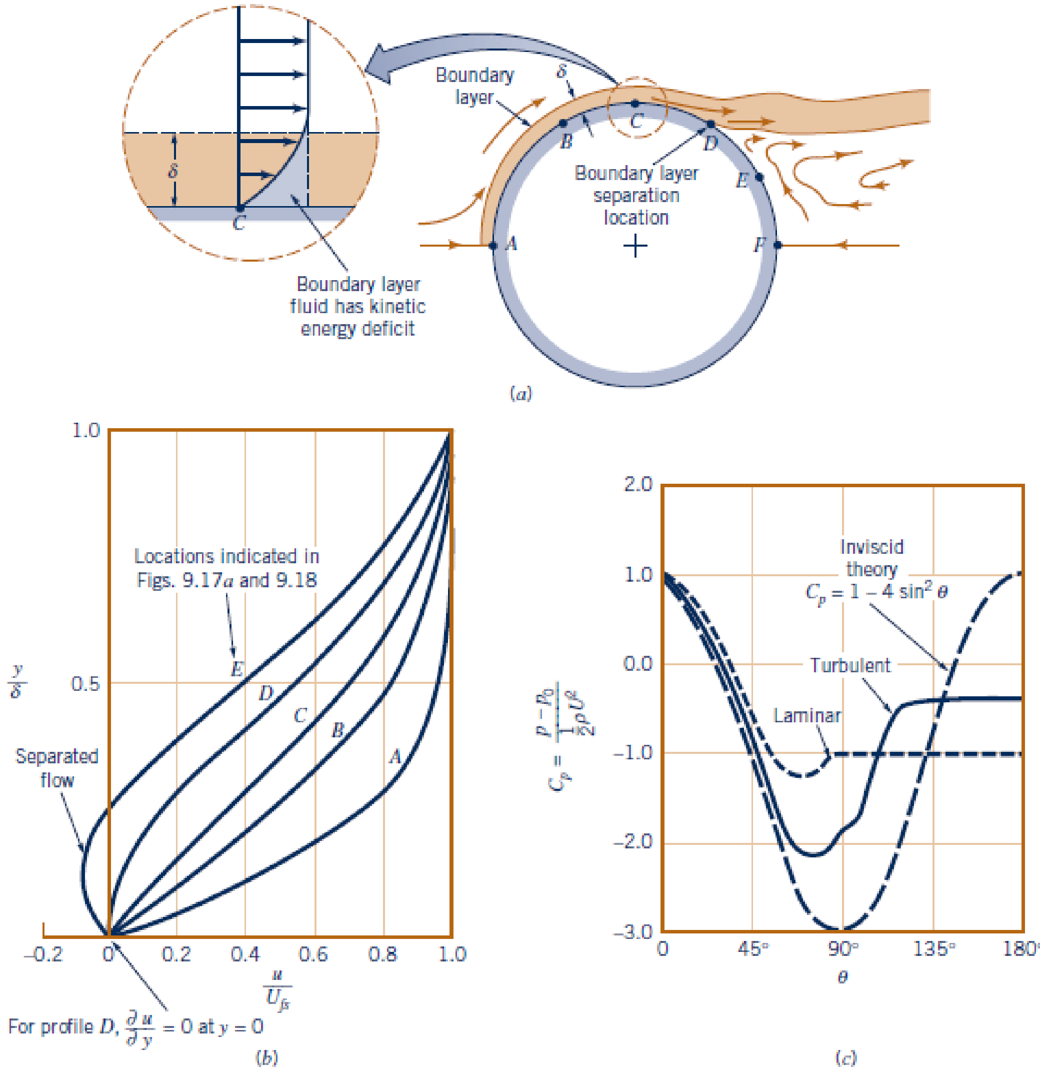
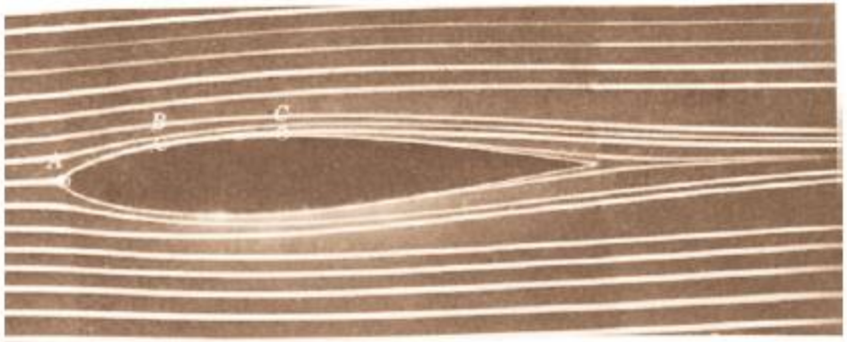
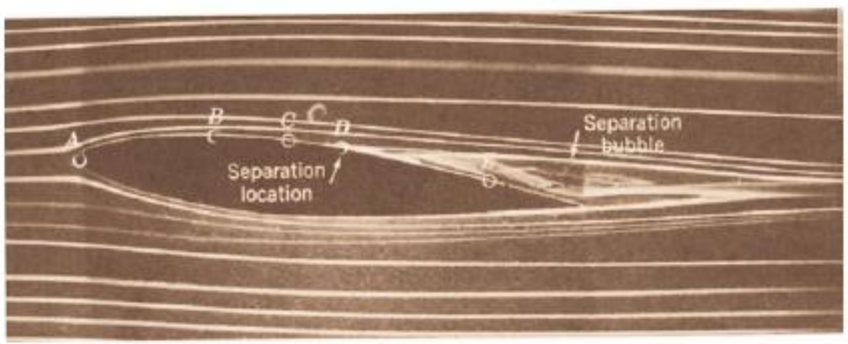


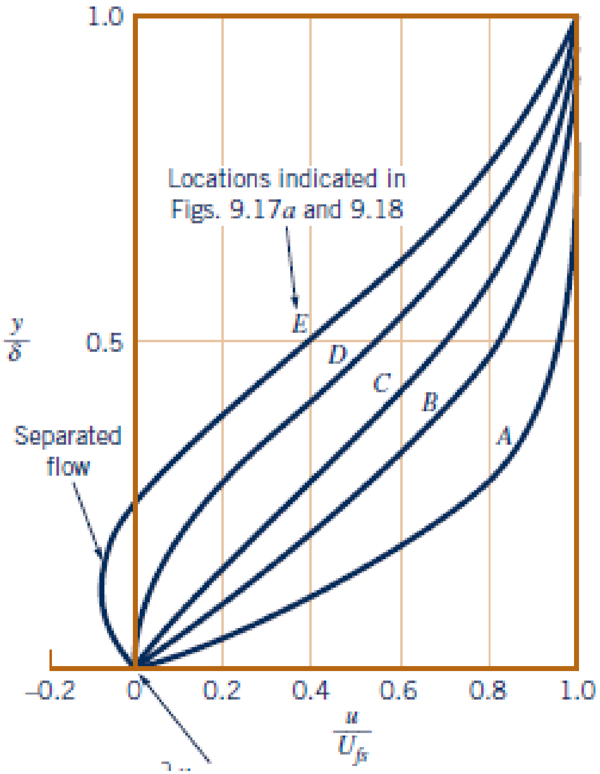
Figure 9.17 Boundary layer characteristics on a circular cylinder: (*a*) boundary layer separation location, (*b*) typical boundary layer velocity profiles at various locations on the cylinder, (*c*) surface pressure distributions for inviscid flow and boundary layer flow.



(a)



(b)

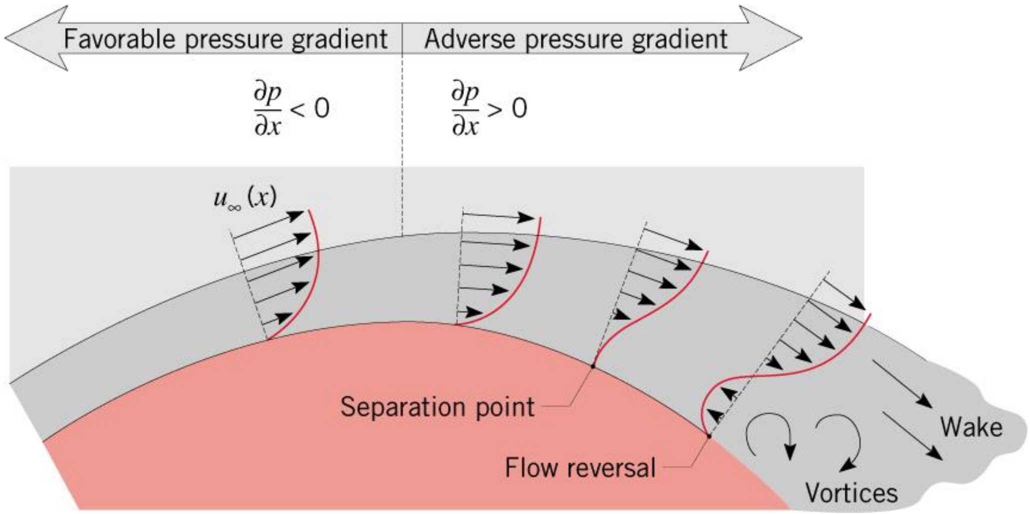


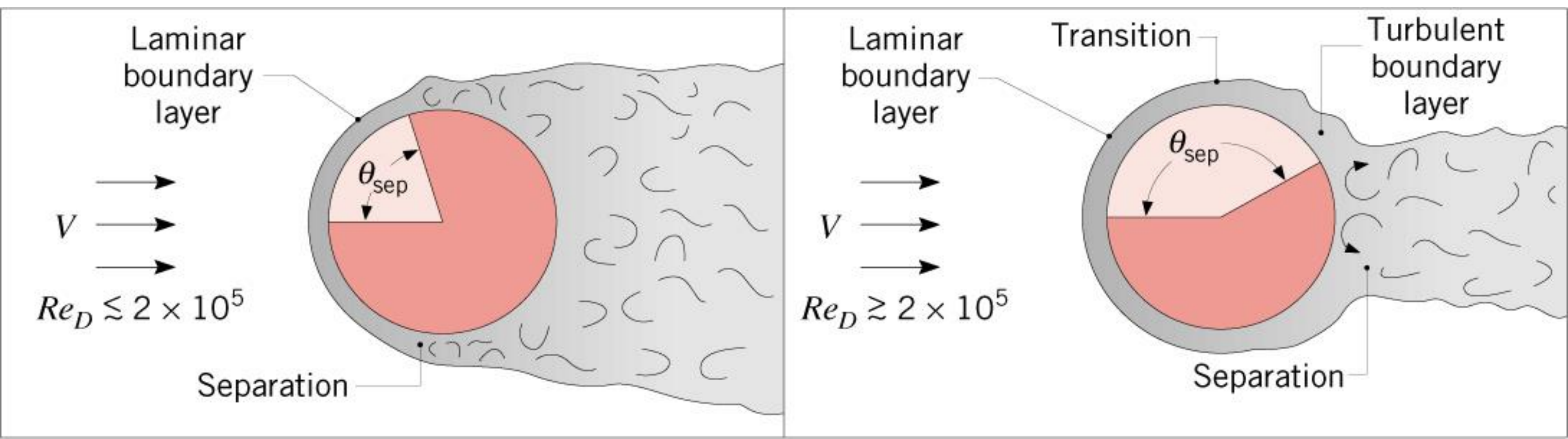
■ **Figure 9.18** Flow visualization photographs of flow past an airfoil (the boundary layer velocity profiles for the points indicated are similar to those indicated in Fig. 9.17*b*): (*a*) zero angle of attack, no separation, (*b*) 5° angle of attack, flow separation. Dye in water. (Photographs courtesy of ONERA, The French Aerospace Lab.)

– What features differentiate boundary development for the flat plate in parallel flow from that for flow over a cylinder?

–Separation occurs when the velocity gradient reduces to zero.

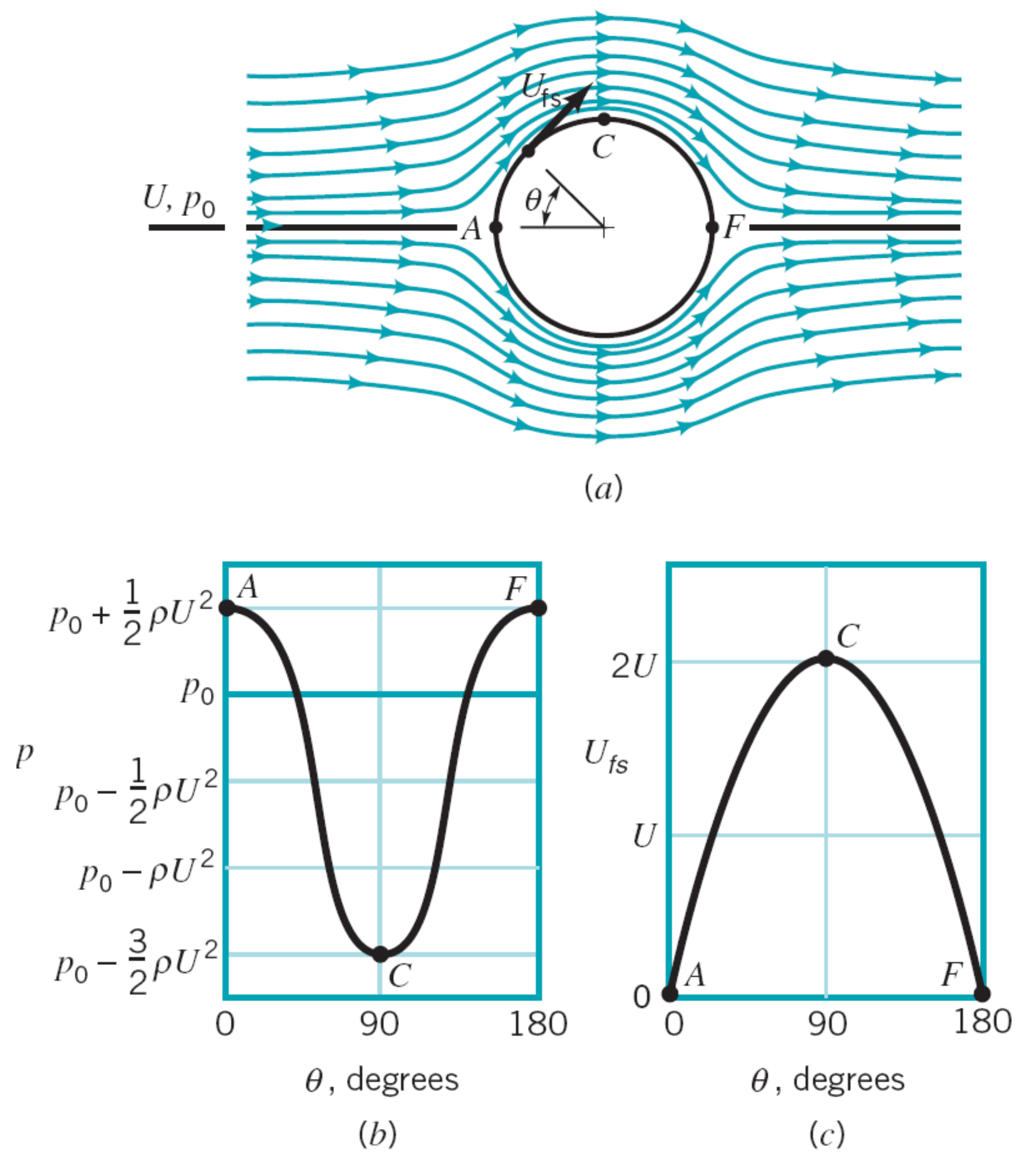
and is accompanied by flow reversal and a downstream wake.
Location of separation depends on boundary layer transition.





Flow around a cylinder

- Inviscid flow past a circular cylinder: (*a*) streamlines for the flow if there were no viscous effects. (*b*) pressure distribution on the cylinder's surface,
  (*c*) free-stream velocity on the cylinder's surface.

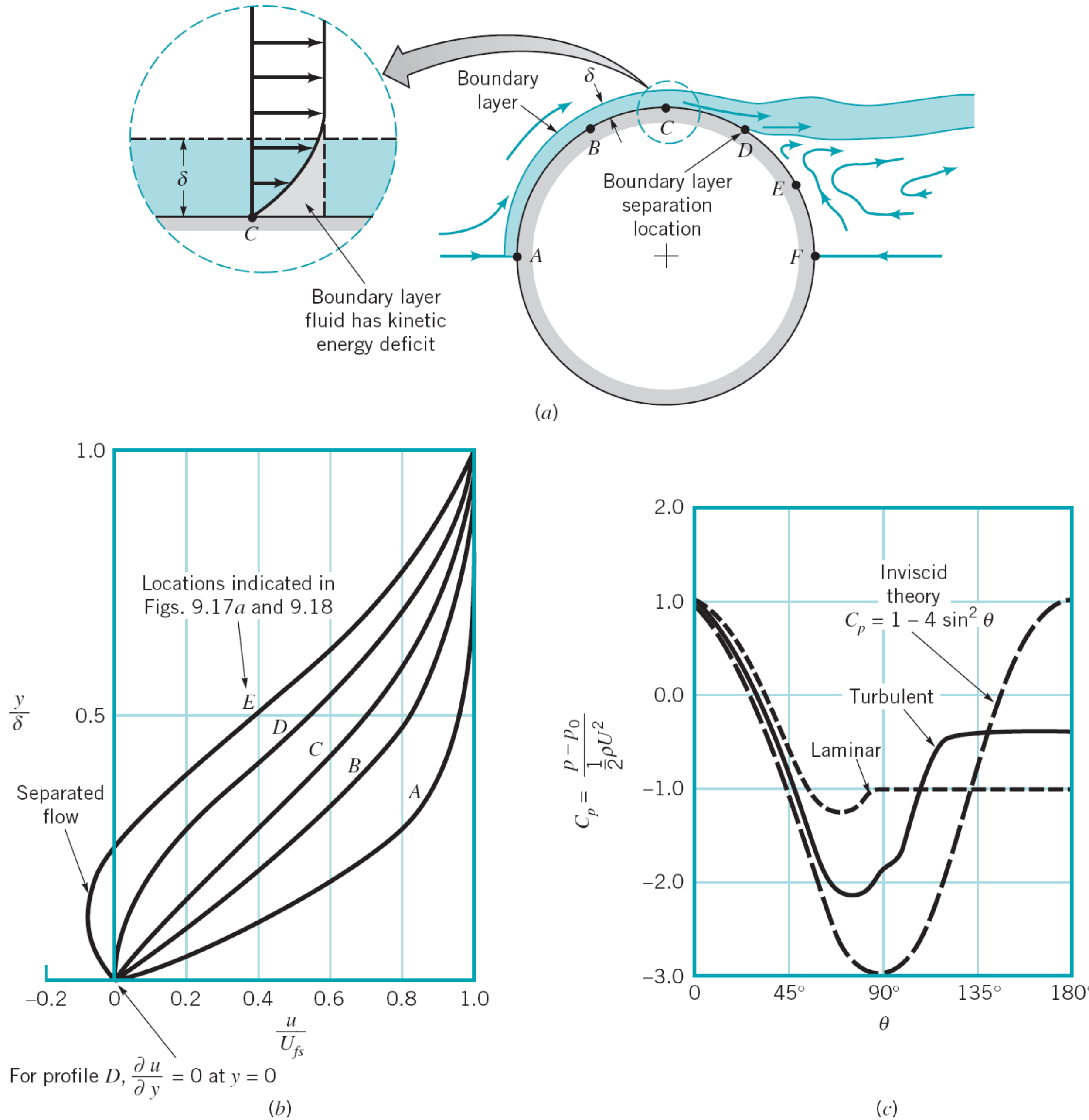


Boundary layer characteristics on a circular cylinder:

(a) boundary layer separation location.

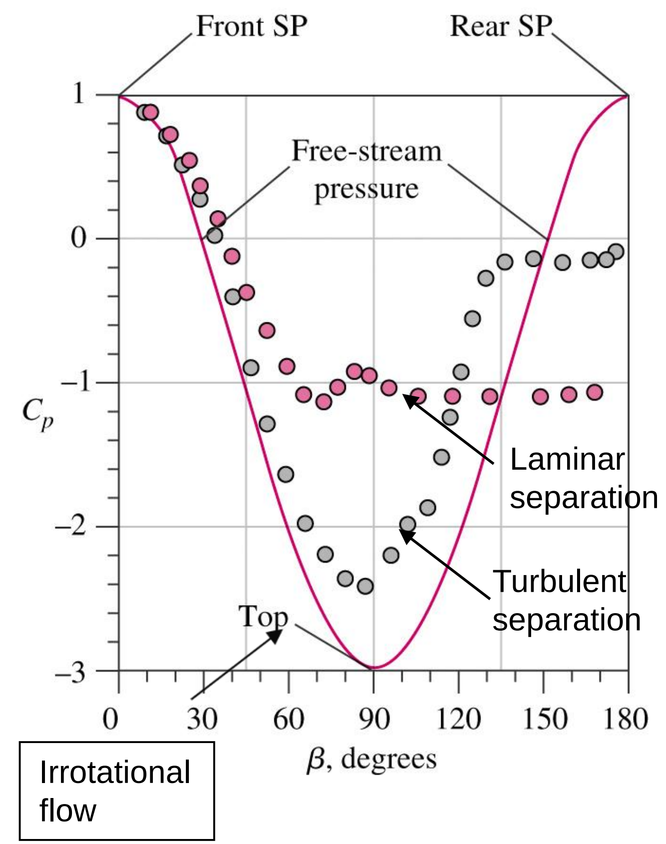
(b) typical boundary layer velocity profiles at various locations on the cylinder,

(c) surface pressure distributions for inviscid flow and boundary layer flow.



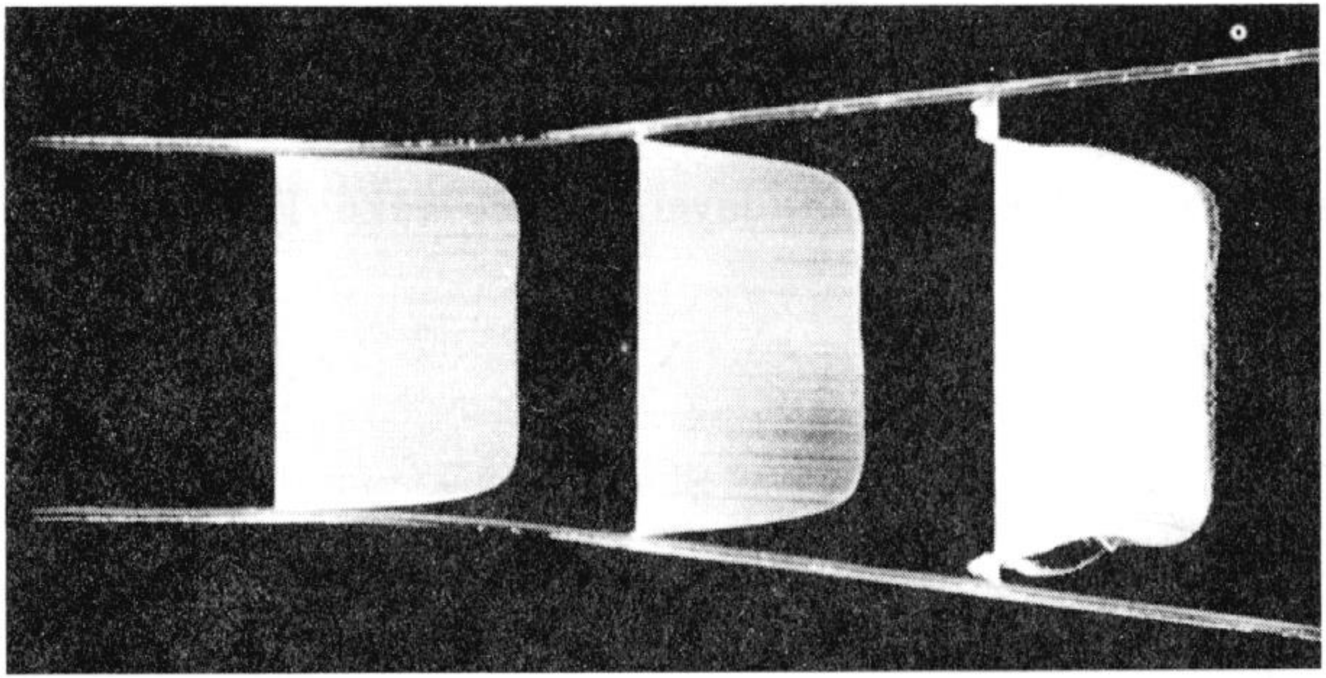
Flow around a Cylinder

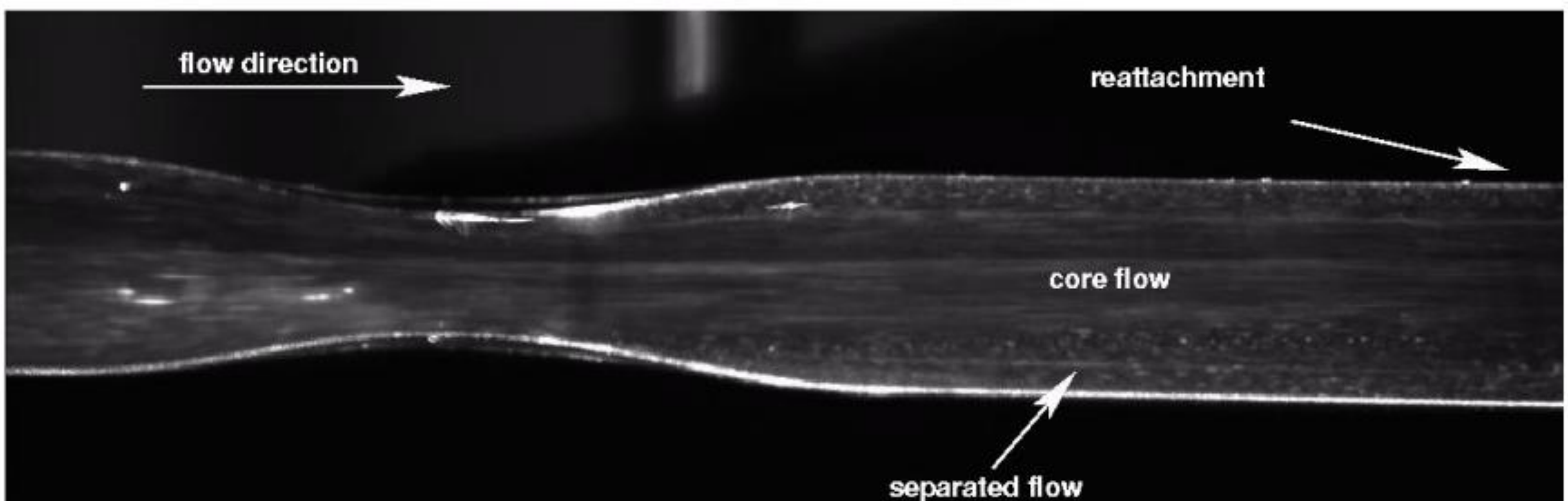
- Integration of surface pressure (which is symmetric in x), reveals that the DRAG is ZERO. This is known as *D'Alembert's Paradox*
  - For the irrotational flow approximation, the drag force on ***any*** non-lifting body of ***any*** shape immersed in a uniform stream is **ZERO**
  - Why?
  - Viscous effects have been neglected. Viscosity and the no-slip condition are responsible for
    - Flow separation (which contributes to pressure drag)
    - Wall-shear stress (which contributes to friction drag)



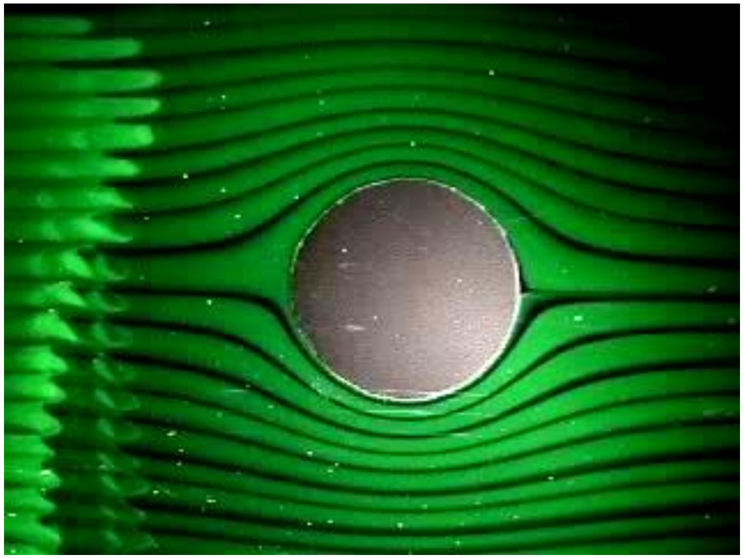
$$C_P = \frac{P - P_\infty}{\rho V^2} = 1 - \frac{V^2}{V_\infty^2}$$

Flow separation in a diffuser with a large angle

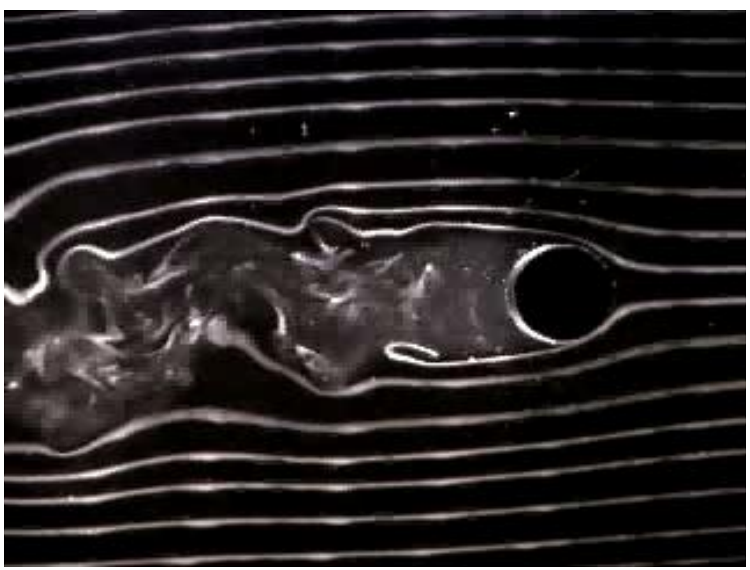




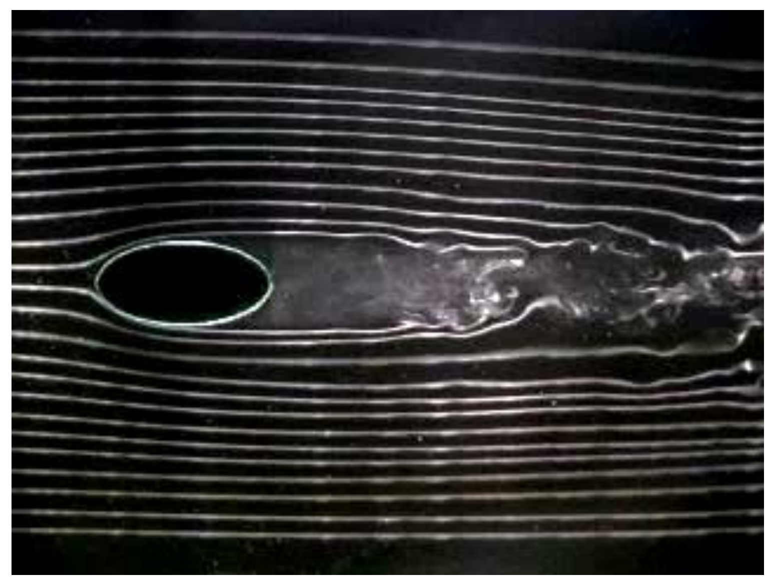
Flow around Cylinders & Ellipsoids



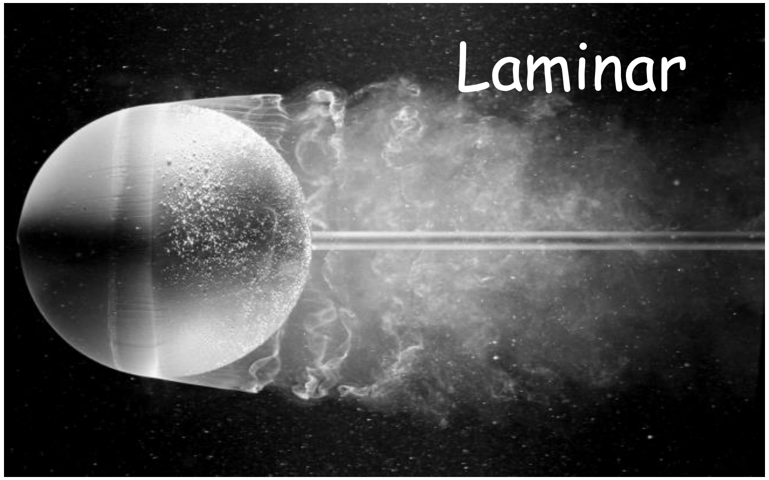
Potential (Ideal) Flow

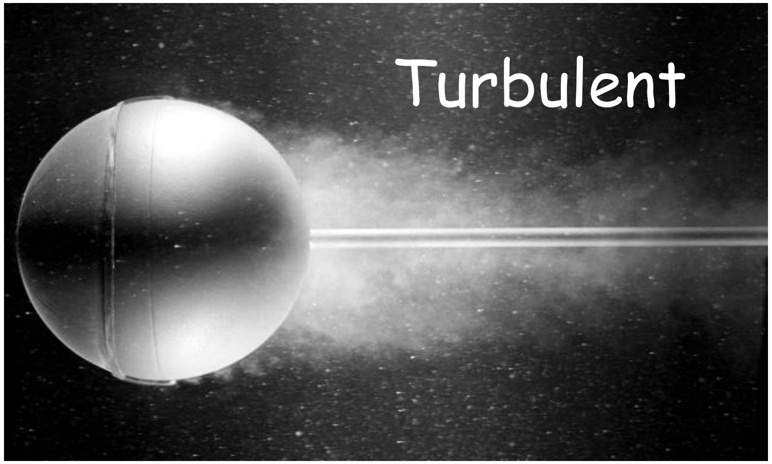


Real Flow

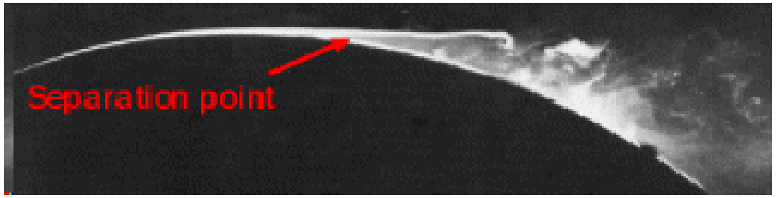


Cylinder and Sphere Drag

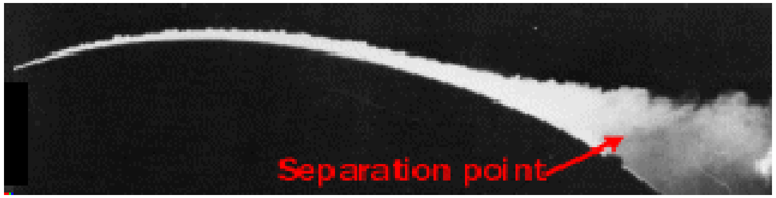




- Flow is strong function of Re.
- Wake narrows for turbulent flow since TBL (turbulent boundary layer) is more resistant to separation due to adverse pressure gradient.
- $\theta_{sep,lam} \approx 80^{o}$
- $\theta_{sep,turb} \approx 140^{o}$



Laminar Separation



Turbulent Separation

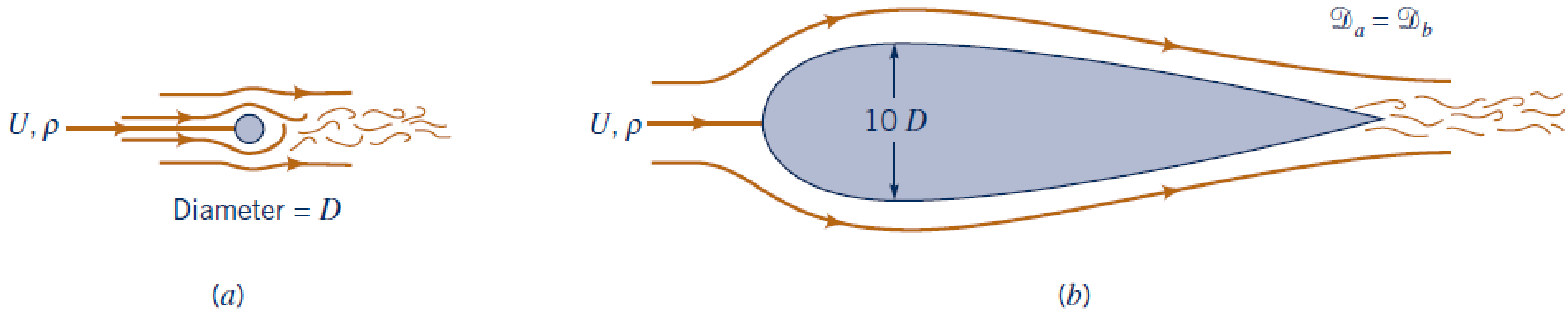
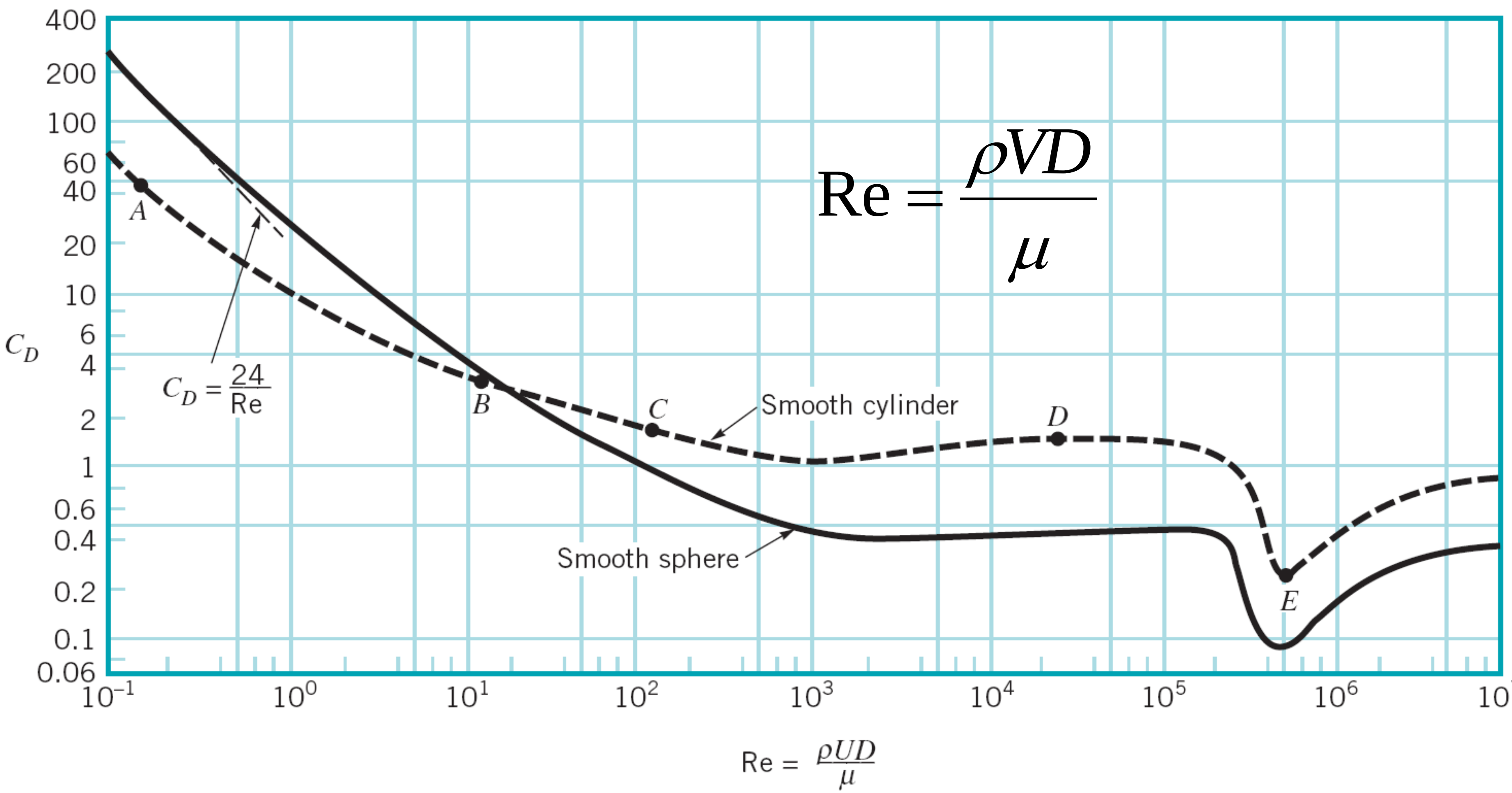


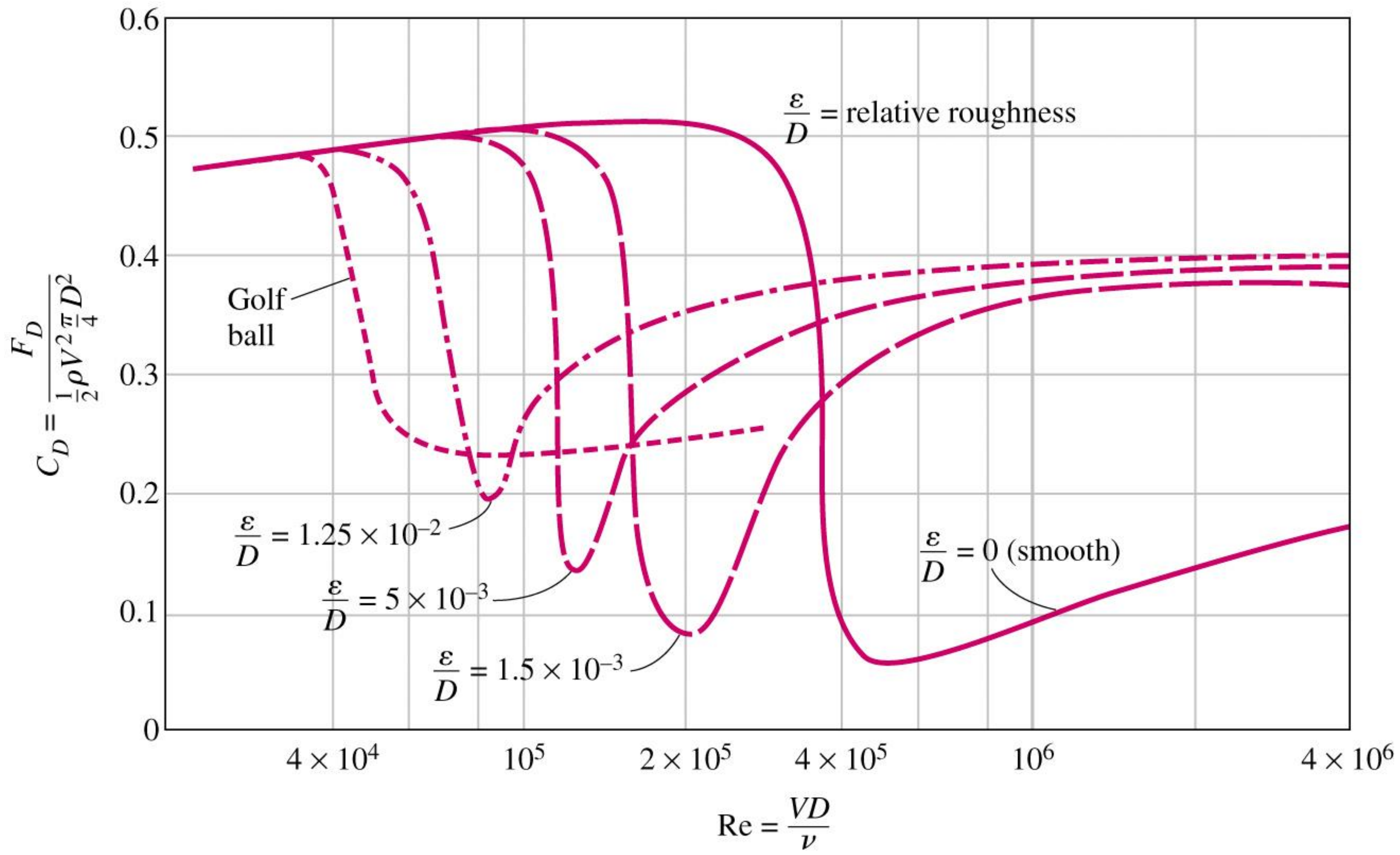
Figure 9.20 Two objects of considerably different size that have the same drag force: (*a*) circular cylinder $C_D = 1.2$; (*b*) streamlined strut $C_D = 0.12$.

Smooth Cylinder and Sphere Drag



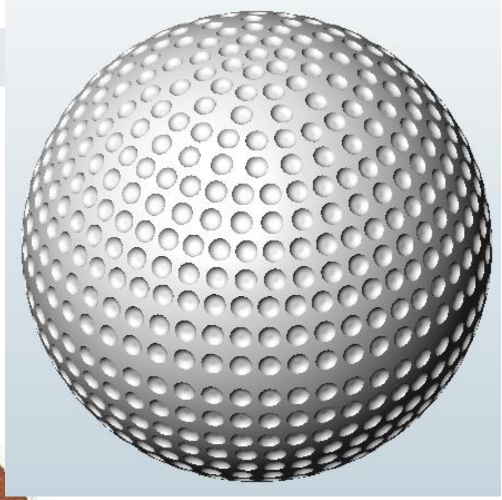
Drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere.

Effect of Surface Roughness



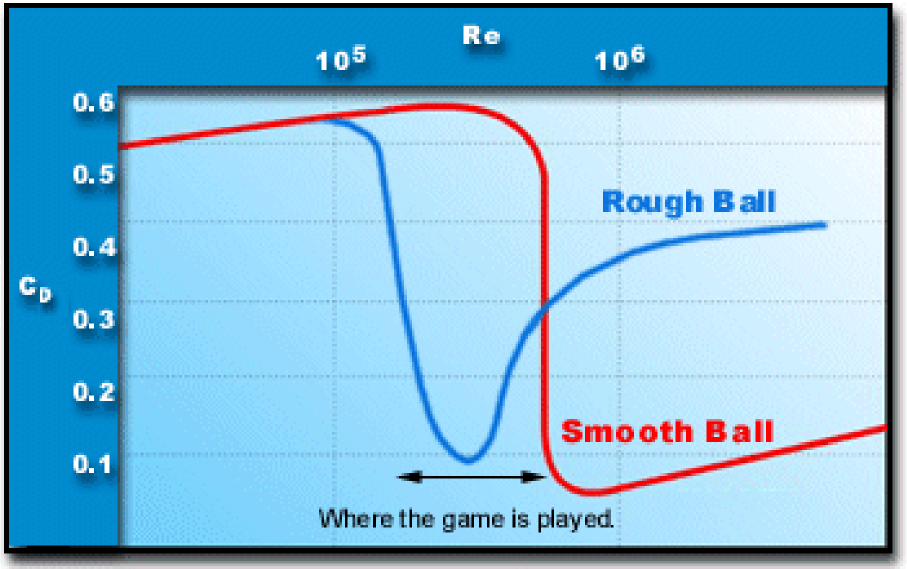
For blunt bodies an increase in ε may decrease C_D by tripping the flow into turbulent at lower Re

Sports Balls



Sports balls

- Many games involve balls designed to use drag reduction brought about by surface roughness.
- Many sports balls have some type of surface roughness, such as the seams on baseballs or cricket balls and the fuzz on tennis balls.

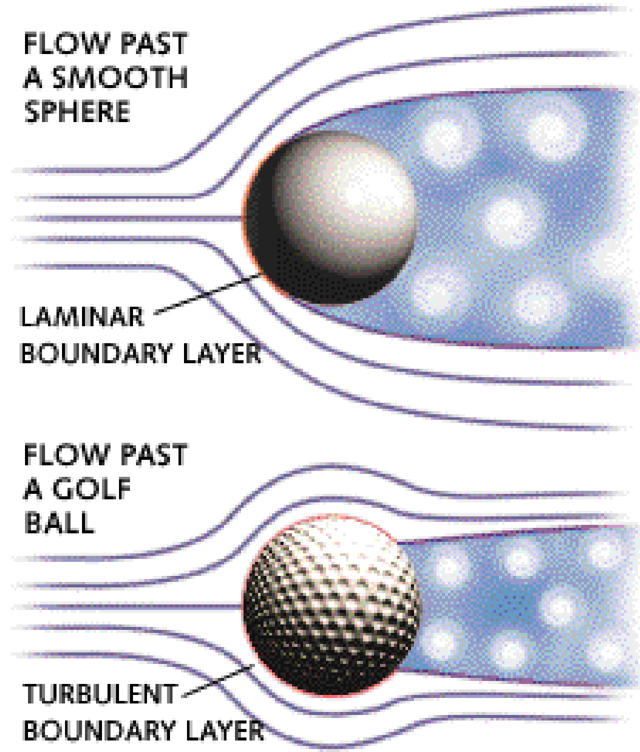


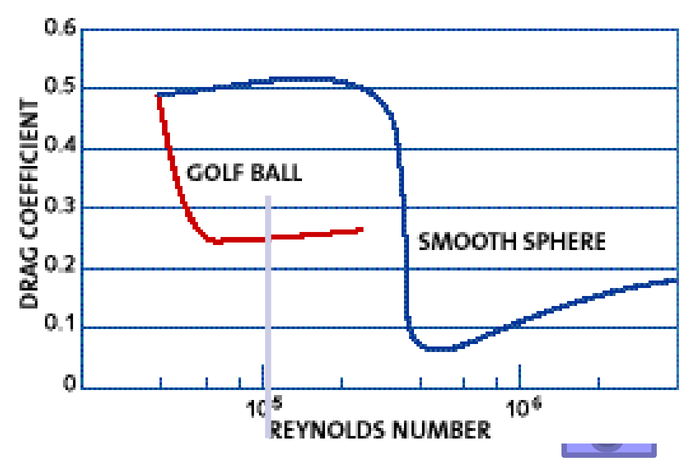
$$\mathrm{Re} = \frac{\rho V D}{\mu}$$

- It is the Reynolds number (not the speed) that determines whether the boundary layer is laminar or turbulent.
- Thus, the larger the ball, the lower the speed at which a rough surface can be of help in reducing the drag.

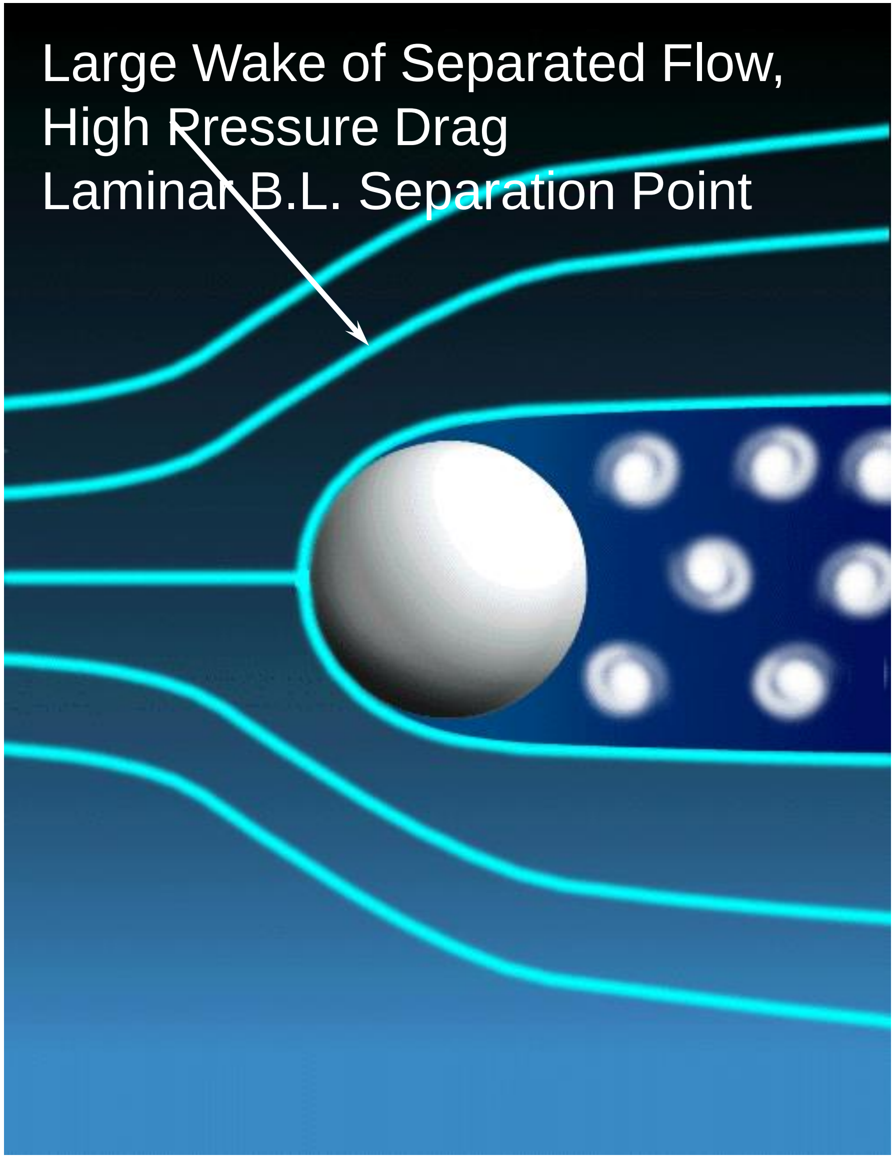
Drag on a Golf Ball

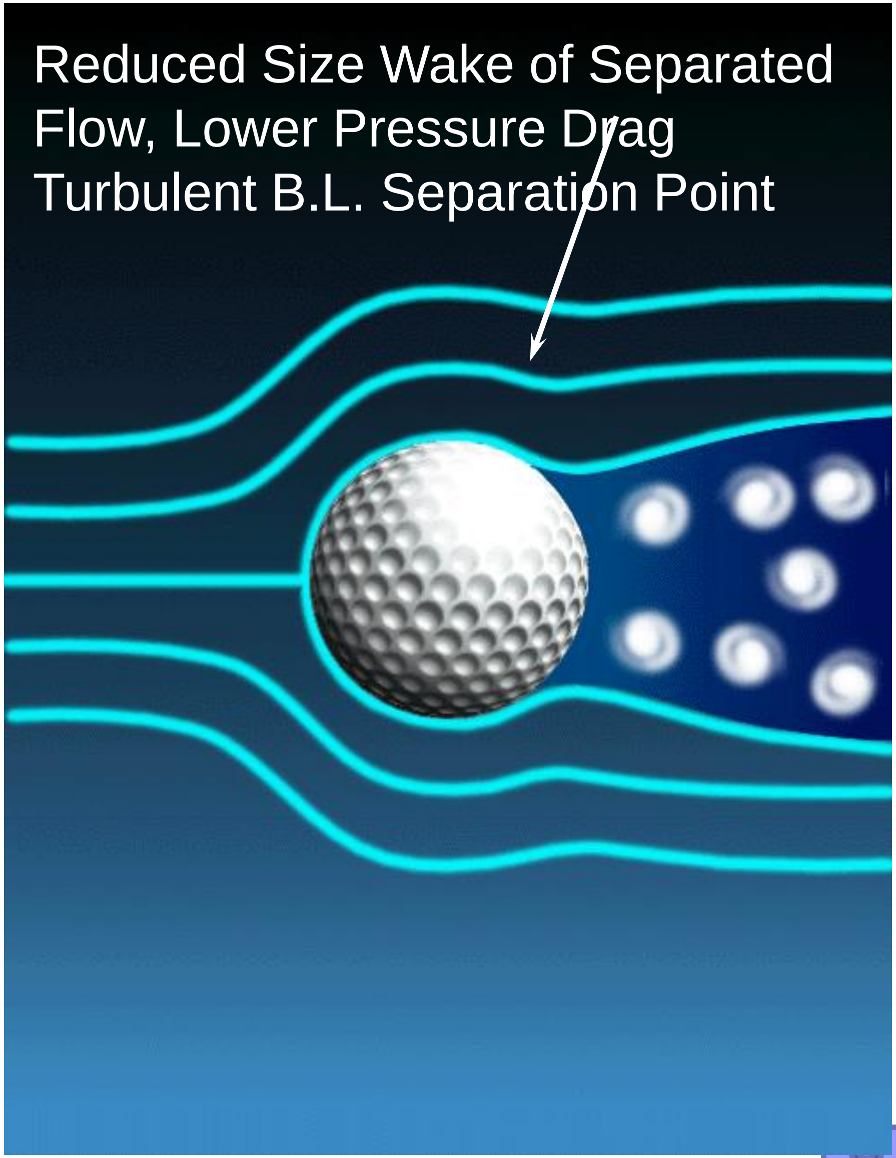
- Drag on a golf ball comes mainly from pressure drag.
- The only practical way of reducing pressure drag is to design the ball so that the point of separation moves back further on the ball.
- The golf ball's dimples increase the turbulence in the boundary layer, increase the inertia of the boundary layer, and delay the onset of separation.
- The Reynolds number where the boundary layer begins to become turbulent with a golf ball is 40,000
- A non-dimpled golf ball would really hamper Tiger Woods' long game
- Why not use this for aircraft or cars?

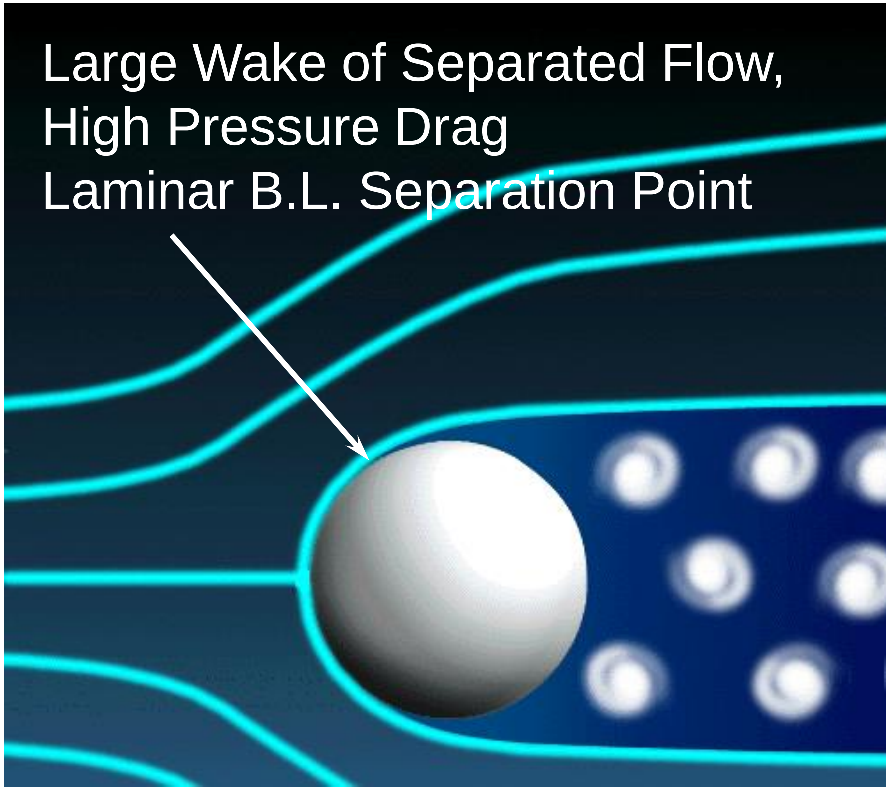


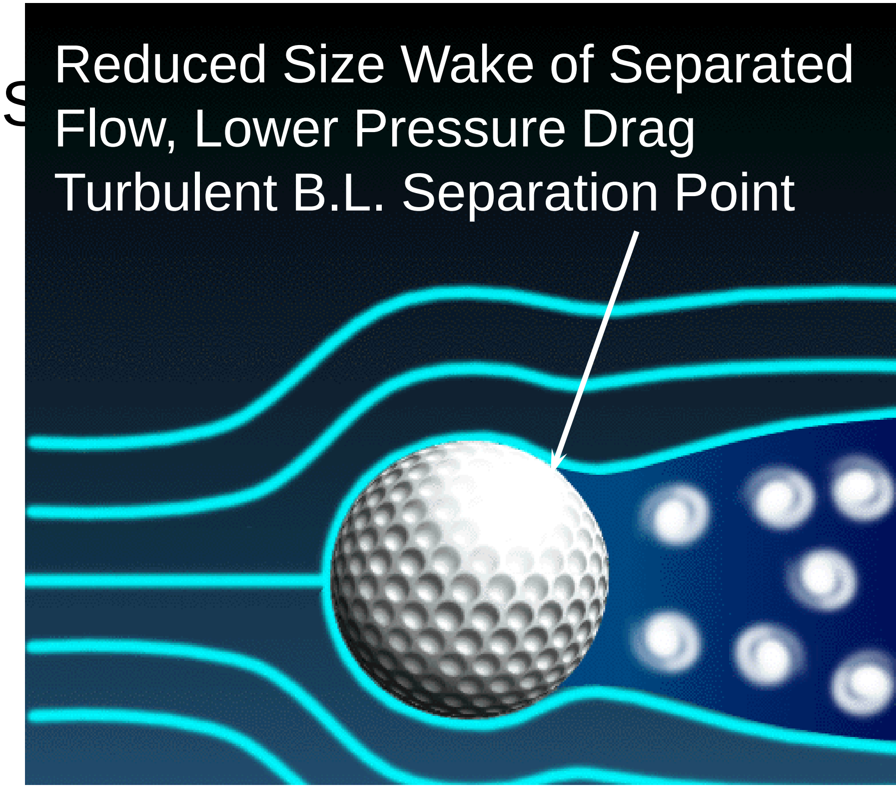


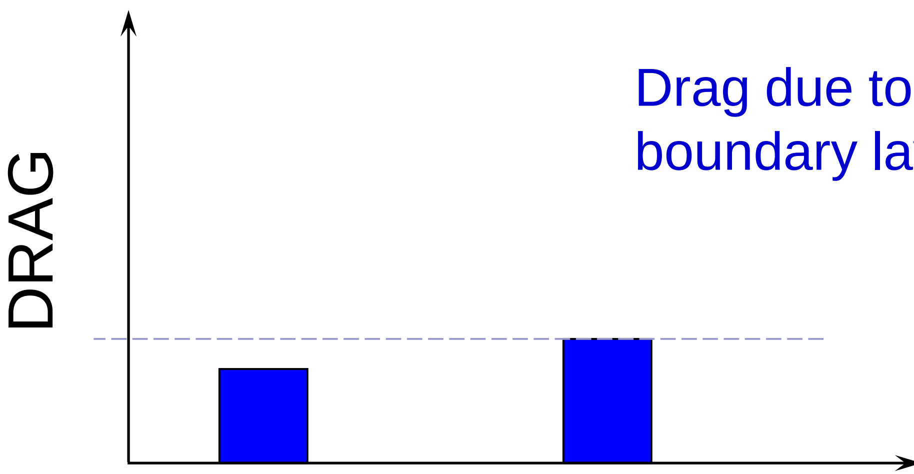
GOLF BALL AERODYNAMICS



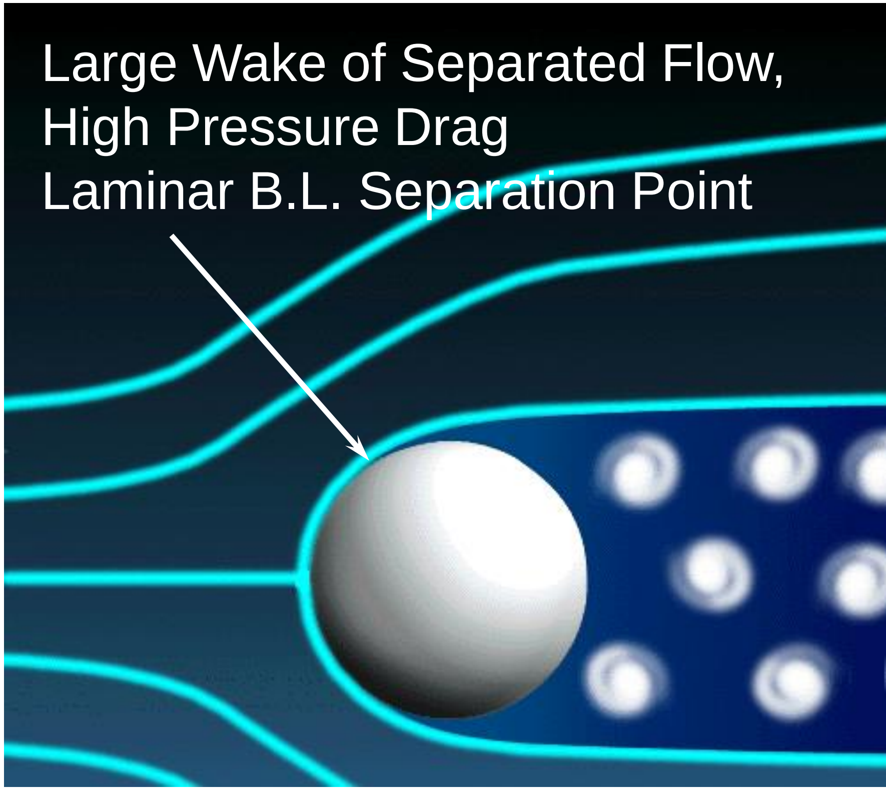


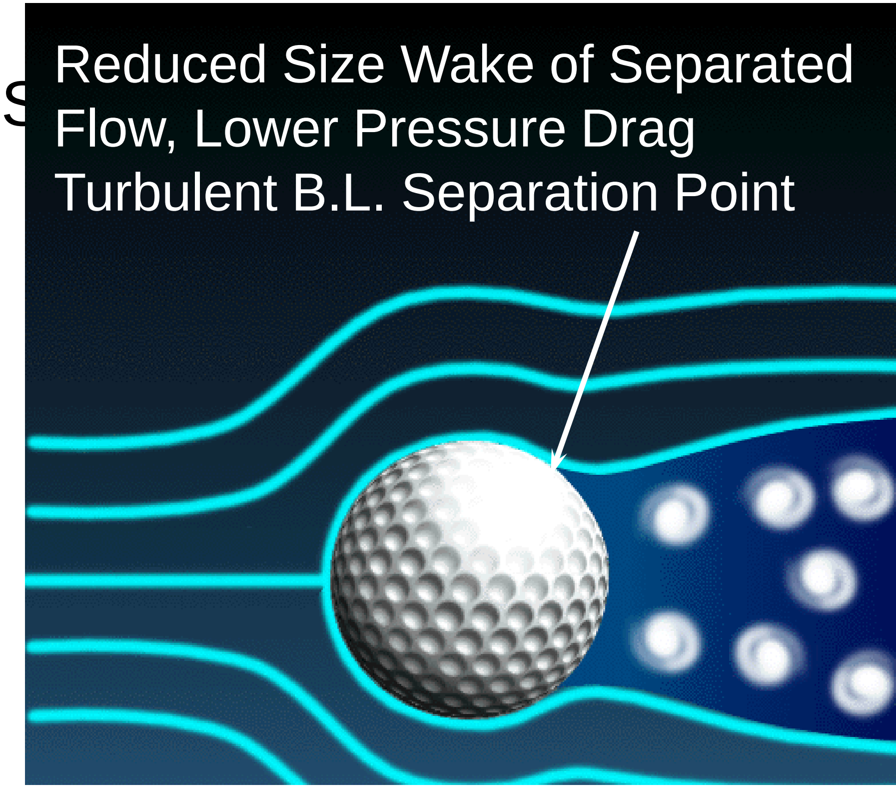


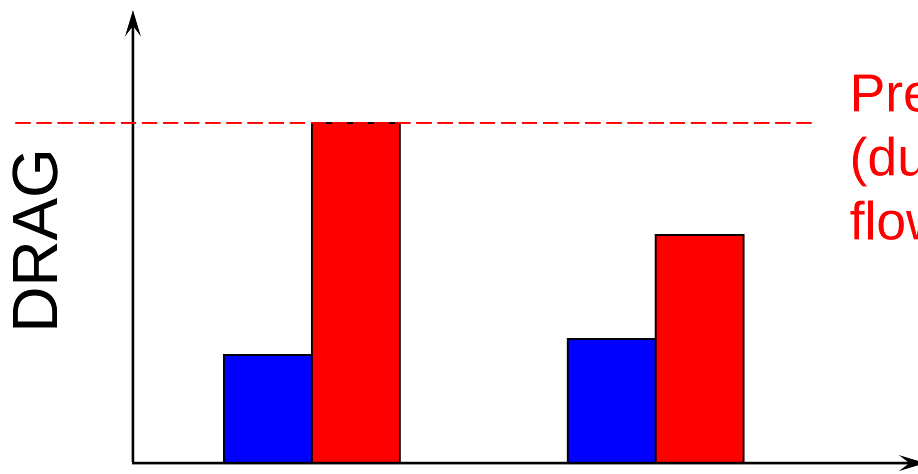




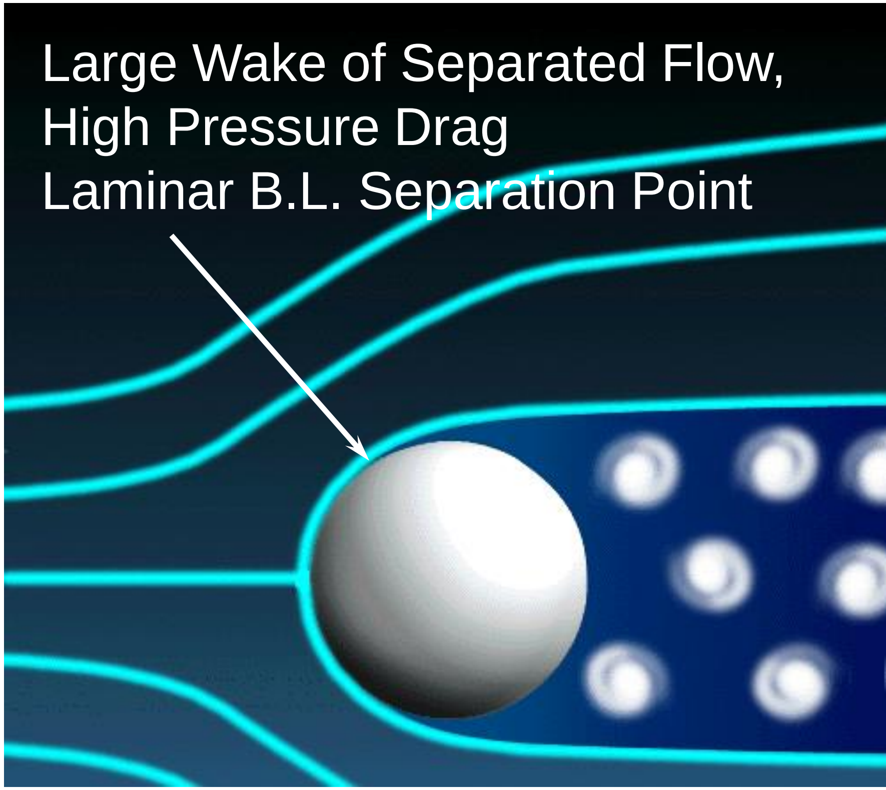
Drag due to viscous
boundary layer skin friction

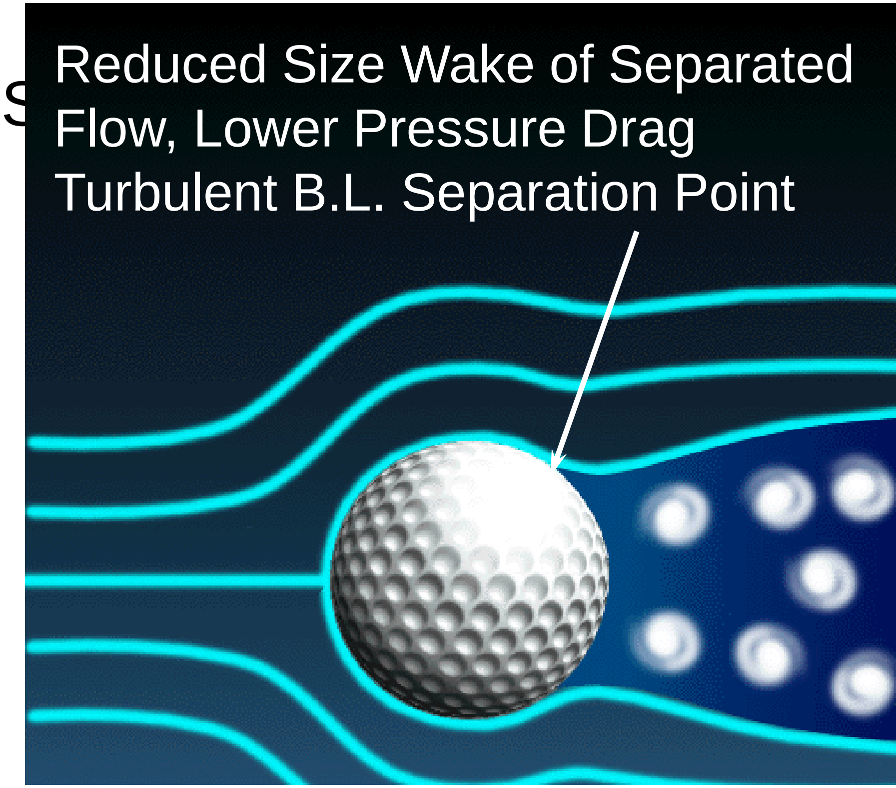


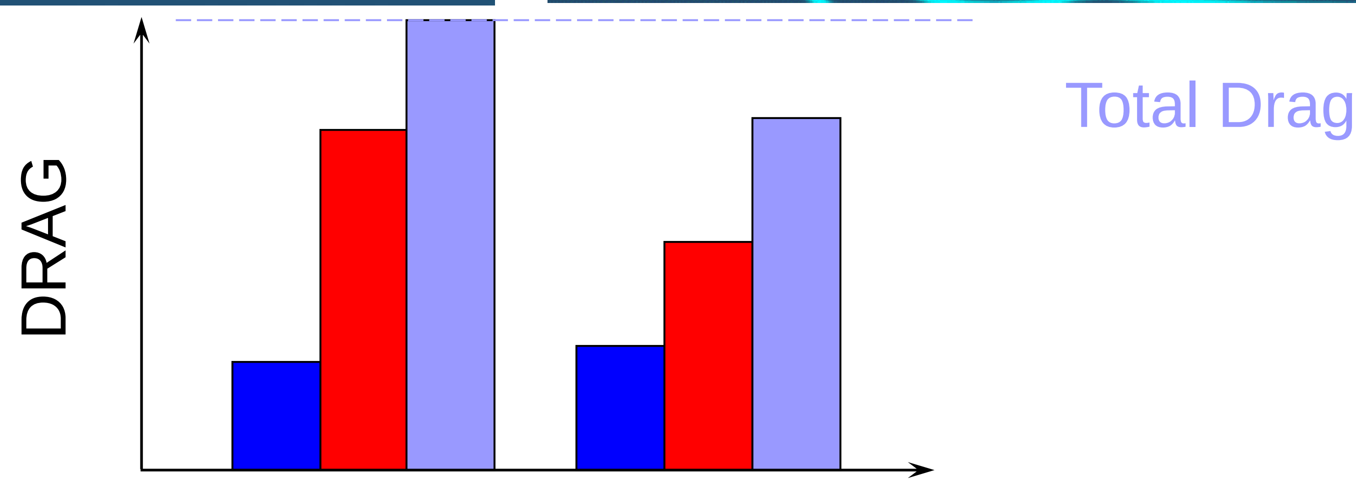




Pressure drag
(due to viscous
flow separation, wake)

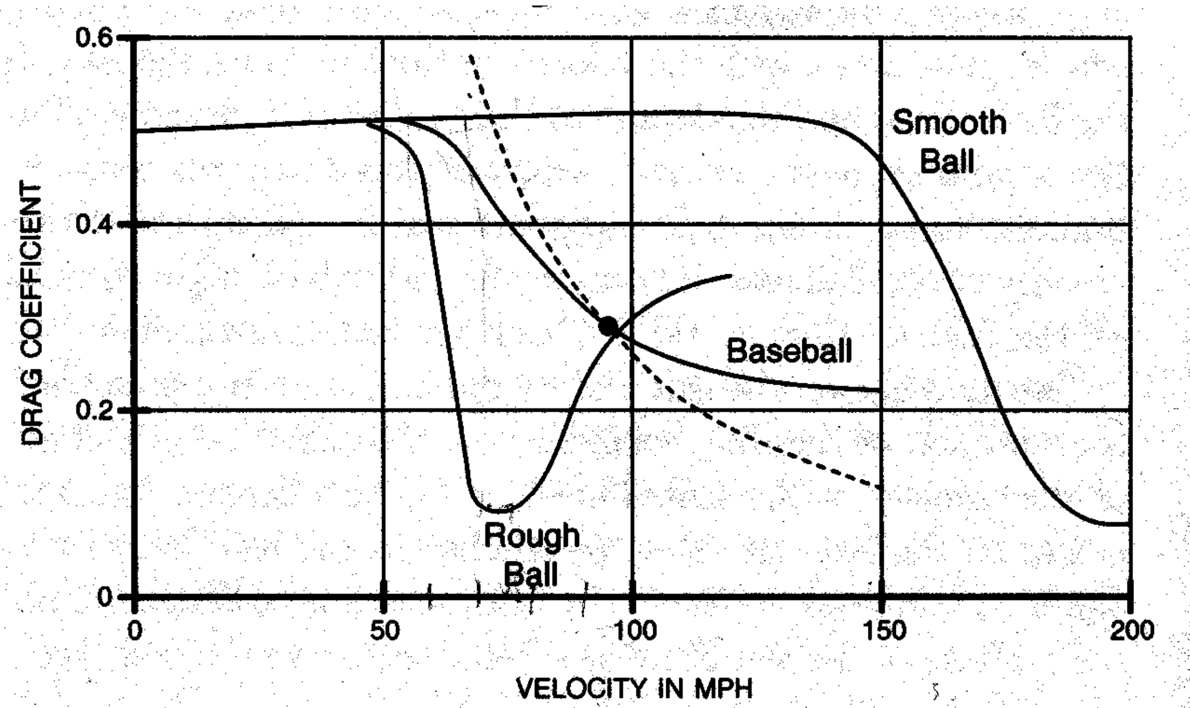

Large Wake of Separated Flow,
High Pressure Drag
Laminar B.L. Separation Point


Reduced Size Wake of Separated
Flow, Lower Pressure Drag
Turbulent B.L. Separation Point


DRAG
Total Drag

Baseball

- At the velocities of 50 to 130 mph dominant in baseball the air passes over a smooth ball in a highly resistant flow.
- Turbulent flow does not occur until nearly 200 mph for a smooth ball
- A rough ball (say one with raised stitches like a baseball) induces turbulent flow
- A baseball batted 400 feet would only travel 300 feet if it was smooth.



Adverse pressure gradients (Tennis ball)

- Separation of the boundary layers occurs whenever the flow tries to decelerate quickly, that is whenever the pressure gradient is positive (adverse pressure gradient-pressure increases).
- In the case of the tennis ball, the flow initially accelerates on the upstream side of the ball, while the local pressure decreases in accord with Bernoulli's equation.

- Near the top of the ball the local external pressure increases and the flow should decelerate as the pressure field is converted to kinetic energy.
- However, because of viscous losses, not all kinetic energy is recovered and the flow reverses around the separation point.

F
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Faster, Faster, Faster

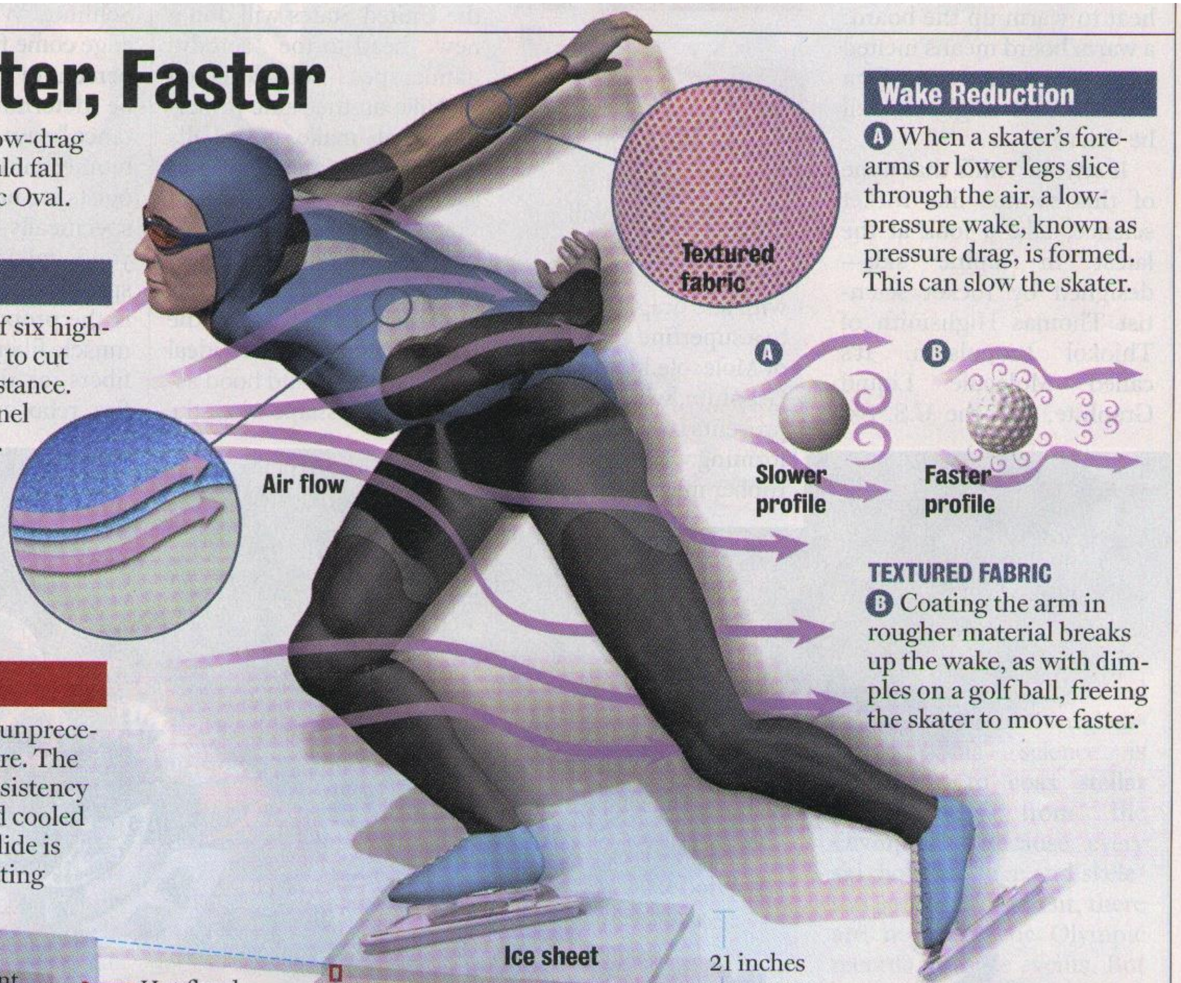
Thanks to faster ice and new, low-drag skating suits, many records could fall this month at the Utah Olympic Oval.

THE SPEED SUIT

Racing With the Wind

Nike's Swift Skin suit is made of six high-tech fabrics strategically placed to cut down on friction and wind resistance. Engineers employed wind-tunnel tests to optimize aerodynamics.

SEAM PLACEMENT The suit's stitches are aligned along the paths of air flow to prevent drag.



Wake Reduction

A When a skater's forearms or lower legs slice through the air, a low-pressure wake, known as pressure drag, is formed. This can slow the skater.

TEXTURED FABRIC
B Coating the arm in rougher material breaks up the wake, as with dimples on a golf ball, freeing the skater to move faster.

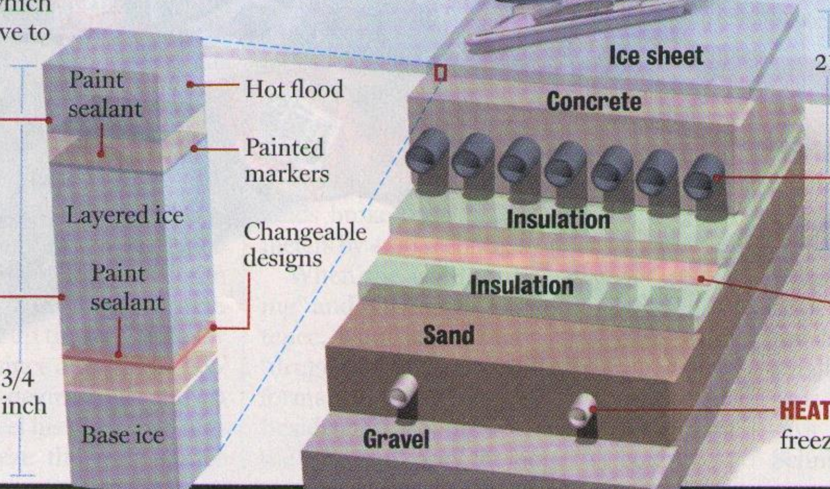
THE NEW RINK

Custom-Tailored Ice

The Utah Olympic Oval allows unprecedented control of ice temperature. The ice can be heated to a softer consistency for traction in shorter races, and cooled harder for longer ones, where glide is needed. The secret is in eliminating trapped air bubbles, which make ice less responsive to temperature changes.

TOP LAYERS The top sheet is applied as hot water, which contains less dissolved air.

MIDDLE LAYERS The middle sheet is built up from numerous thin layers, which freeze faster, before air can be trapped.



Beneath the Ice

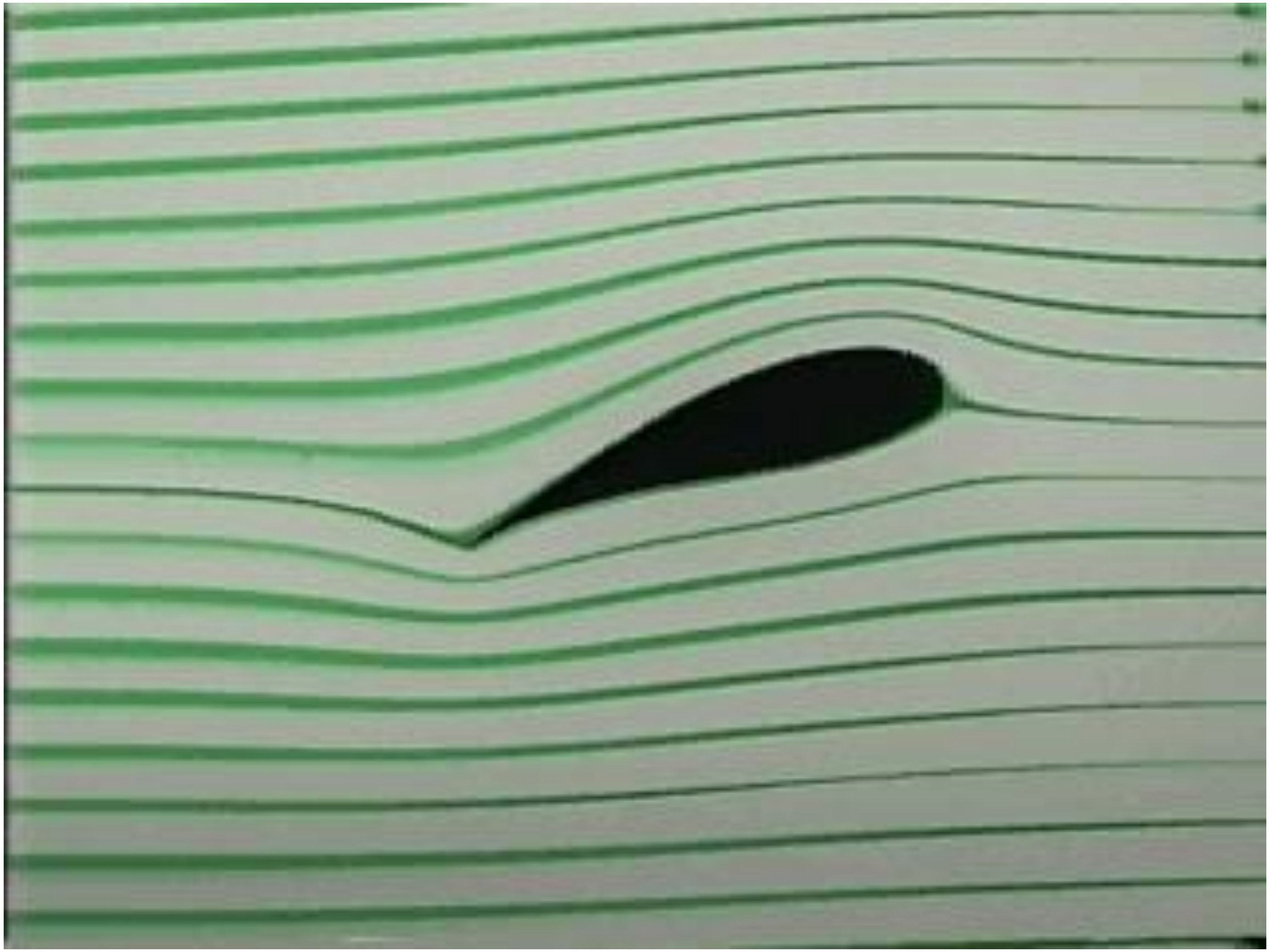
REFRIGERATION PIPES Thirty-three miles in all, they circulate chilled salt water to cool the ice sheet.

LUBRICANT It helps buffer against expansions and contractions that could cause damage to the rink.

HEATING TUBES They keep the base from freezing, which could crack the concrete.

SOURCES: RICK MACDONALD, NIKE; DAVE LOVEROCK, JET ICE; JIM GULCZYNSKI, PETTIT NATIONAL ICE CENTER; ALAN RINDLISBACHER, LAYTON CONSTRUCTION; POPULAR SCIENCE.

Streamlines

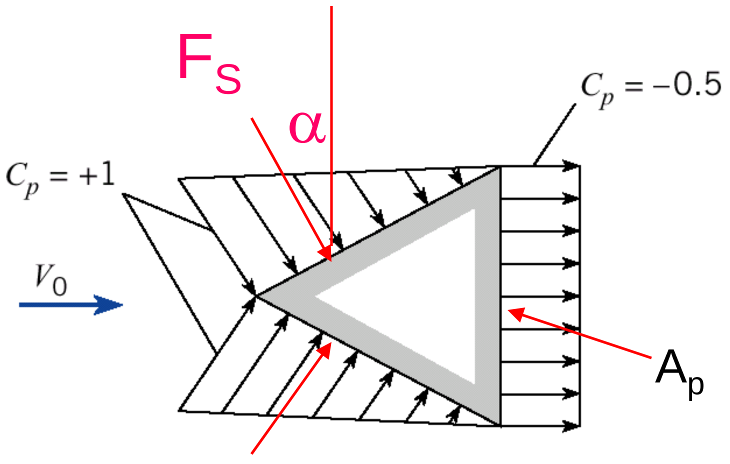


Example-1

- Given: Pressure distribution is shown, flow is left to right.

$$C_p = \frac{p}{\rho V^2/2}$$

- Find: Find C_D
- Solution: C_D is based on the projected area of the block from the direction of flow. Force on downstream face is:



$$\left(F_D\right)\big|_{Drag} = C_p(\rho V^2/2)\ A_p = 0.5 A_p(\rho V^2/2)$$

The total force on each side face is:

$$F_S = C_p(\rho V^2/2) A_p = 0.5 A_p(\rho V^2/2)$$

The drag force on one face is:

$$\left(F_S\right)\big|_{Drag} = F_S \sin\alpha = 0.5 A_p(\rho V^2/2) * 0.5$$

The total drag force is:

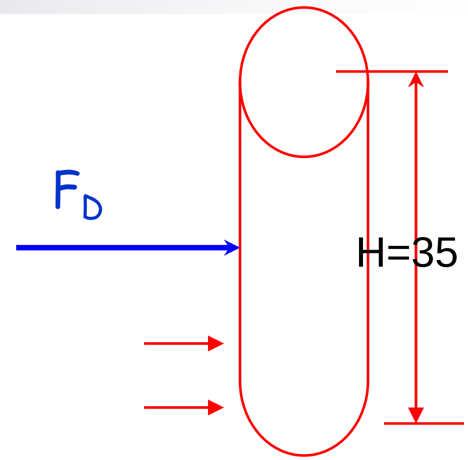
$$F_{Drag} = 2(F_S)\big|_{Drag} + (F_D)\big|_{Drag}$$

$$= 2*(0.5A_p(\rho V^2/2)*0.5) + 0.5A_p(\rho V^2/2) = A_p(\rho V^2/2) = C_D A_p(\rho V^2/2)$$

Coefficient of Drag is: $C_D = 1$

Example-2

- Given: Flag pole, 35 m high, 10 cm diameter, in 25-m/s wind, P_{atm} = 100 kPa, T=20°C
- Find: Moment at bottom of flag pole
- Solution:



Air properties at 20°C

$$\nu = 1.51x10^{-5}\, m^2/s,\ \rho = 1.20\, kg/m^3$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{25*0.1}{1.51x10^{-5}} = 1.66x10^5$$

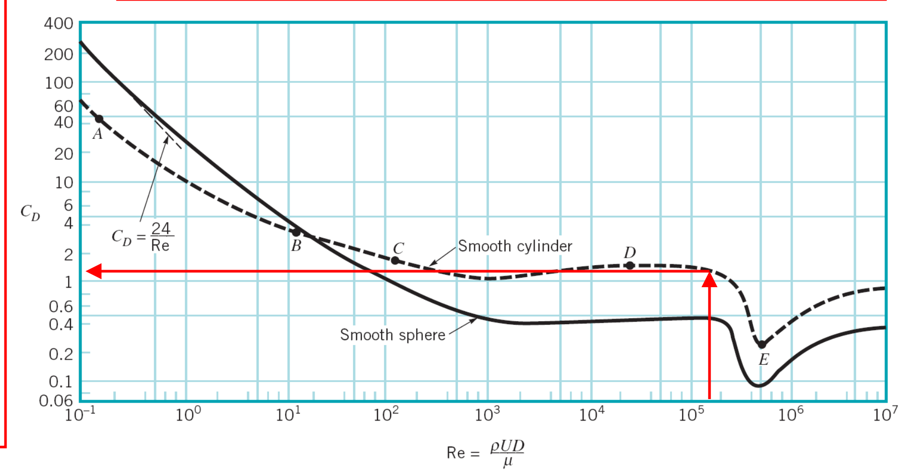
$$C_D = 1.1 \text{ (Fig. 9–21)}$$

$$F_D = C_D A_p \rho \frac{V^2}{2}$$

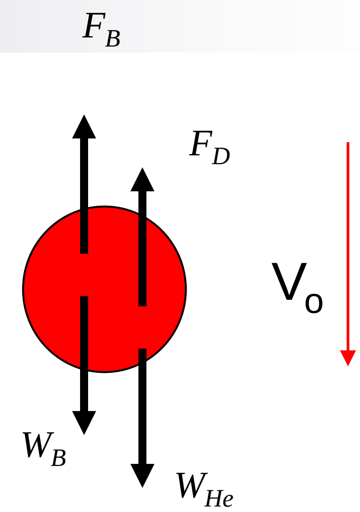
$$M = F_D \frac{H}{2} = C_D A_p \rho \frac{V_o^2}{2} \frac{H}{2}$$

$$= 1.1*0.10*35*1.2*\frac{25^2}{2}*\frac{35}{2}$$

$$= 23\, kN \cdot m$$



Example-3



- Given: Spherical balloon 2-m diameter, filled with helium at std conditions. Empty weight = 3 N.
- Find: Velocity of ascent/descent.
- Solution:

$$\sum F_y = 0 = F_B + F_D - W_B - W_{He}$$

$$= \gamma_{air}\frac{\pi}{6}D^3 + F_D - 3 - \gamma_{He}\frac{\pi}{6}D^3$$

$$F_D = 3 - (\gamma_{air} - \gamma_{He})\frac{\pi}{6}D^3$$

$$= 3 - \gamma_{air}(1 - \frac{287}{2{,}077})\frac{\pi}{6}2^3$$

$$= -1.422\,N$$

$$F_D = C_D A_p \rho \frac{V_o^2}{2}$$

$$V_o = \sqrt{\frac{2F_D}{C_D A_p \rho}} = \sqrt{\frac{2*1.422}{C_D(\pi/4)*2^2*1.225}} = \sqrt{\frac{0.739}{C_D}}$$

Iteration 1: Guess C_D= 0.4 ??

$$V_o = \sqrt{\frac{0.739}{0.4}} = 1.36\,m/s$$

Check Re

$$\text{Re} = \frac{VD}{\nu} = \frac{1.36*2}{1.46x10^{-5}} = 1.86x10^5$$

Iteration 2: Chart Re $\rightarrow$ C_D= 0.42

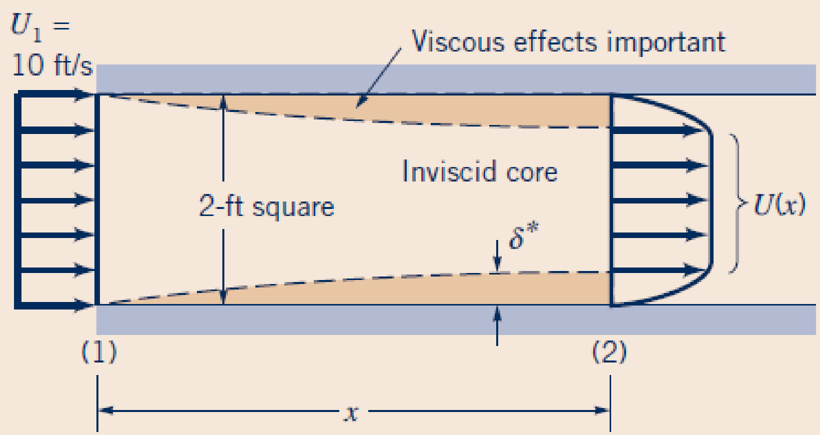
$$V_o = \sqrt{\frac{0.739}{0.42}} = 1.33\,m/s$$

Example 9.3

GIVEN Air flowing into a 2-ft-square duct with a uniform velocity of 10 ft/s forms a boundary layer on the walls as shown in Fig. E9.3*a*. The fluid within the core region (outside the boundary layers) flows as if it were inviscid. From advanced calculations it is determined that for this flow the boundary layer displacement thickness is given by

$$\delta^* = 0.0070(x)^{1/2} \qquad (1)$$

where δ^* and x are in feet.



FIND Determine the velocity $U = U(x)$ of the air within the duct but outside of the boundary layer.

Example 9.3 - Solution

If we assume incompressible flow (a reasonable assumption because of the low velocities involved), it follows that the volume flowrate across any section of the duct is equal to that at the entrance (i.e., $Q_1 = Q_2$). That is,

$$U_1 A_1 = 10 \text{ ft/s } (2 \text{ ft})^2 = 40 \text{ ft}^3/\text{s} = \int_{(2)} u \, dA$$

According to the definition of the displacement thickness, δ^*, the flowrate across section (2) is the same as that for a uniform flow with velocity U through a duct whose walls have been moved inward by δ^*. That is,

$$40 \text{ ft}^3/\text{s} = \int_{(2)} u \, dA = U(2 \text{ ft} - 2\delta^*)^2 \qquad \textbf{(2)}$$

By combining Eqs. 1 and 2 we obtain

$$40 \text{ ft}^3/\text{s} = 4U(1 - 0.0070x^{1/2})^2$$

or

$$U = \frac{10}{(1 - 0.0070x^{1/2})^2} \text{ ft/s} \qquad \textbf{(Ans)}$$

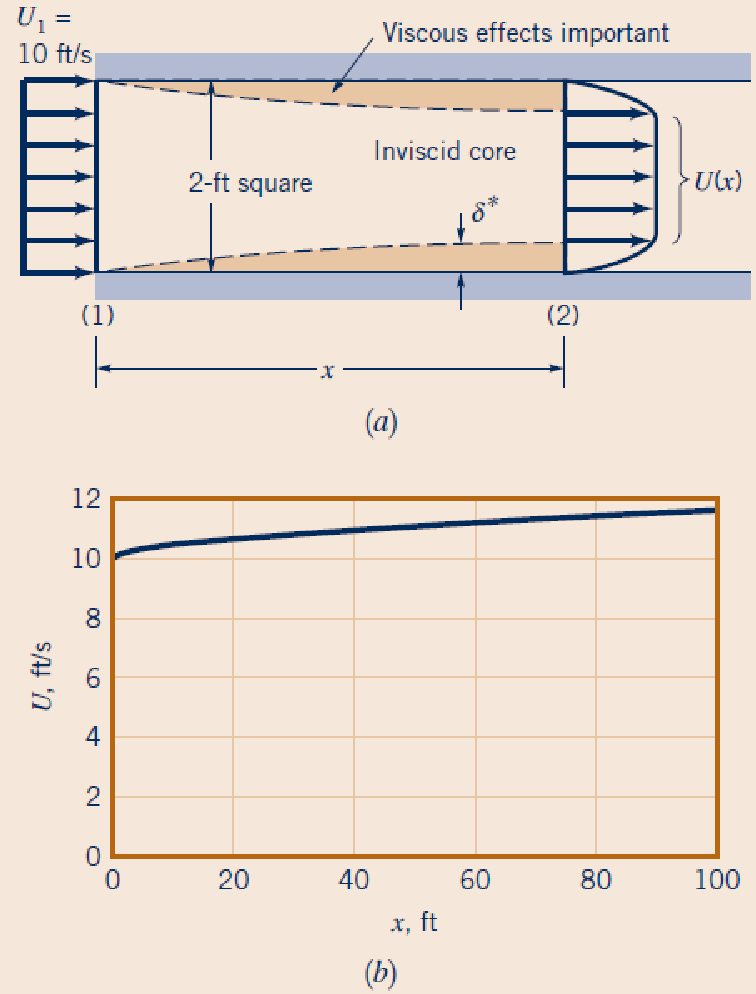


Figure E9.3

Example 9.3 - Solution

COMMENTS Note that U increases in the downstream direction. For example, as shown in Fig. E9.3*b*, $U = 11.6$ ft/s at $x = 100$ ft. The viscous effects that cause the fluid to stick to the walls of the duct reduce the effective size of the duct, thereby (from conservation of mass principles) causing the fluid to accelerate. The pressure drop necessary to do this can be obtained by using the Bernoulli equation (Eq. 3.7) along the inviscid streamlines from section (1) to (2). (Recall that this equation is not valid for viscous flows within the boundary layer. It is, however, valid for the inviscid flow outside the boundary layer.) Thus,

$$p_1 + \tfrac{1}{2}\rho U_1^2 = p + \tfrac{1}{2}\rho U^2$$

Hence, with $\rho = 2.38 \times 10^{-3}$ slugs/ft^3 and $p_1 = 0$ we obtain

$$\begin{aligned} p &= \frac{1}{2}\rho\,(U_1^2 - U^2) \\ &= \frac{1}{2}(2.38 \times 10^{-3}\ \text{slugs/ft}^3) \\ &\quad \times \left[(10\ \text{ft/s})^2 - \frac{10^2}{(1 - 0.0079x^{1/2})^4}\ \text{ft}^2/\text{s}^2\right] \end{aligned}$$

or

$$p = 0.119\left[1 - \frac{1}{(1 - 0.0070x^{1/2})^4}\right]\ \text{lb/ft}^2$$

For example, $p = -0.0401$ lb/ft^2 at $x = 100$ ft.

If it were desired to maintain a constant velocity along the centerline of this entrance region of the duct, the walls could be displaced outward by an amount equal to the boundary layer displacement thickness, δ^*.

Boundary Layer

Another boundary layer thickness definition, the boundary layer momentum thickness, θ is often used when determining the drag on an object. Again because of the velocity deficit, in the boundary layer, the momentum flux across section b–b is less than that across section a–a. This deficit in momentum flux for the actual boundary layer flow on a plate of width *b* is given by

$$\int \rho u(U - u)\, dA = \rho b \int_0^\infty u(U - u)\, dy$$

which by definition is the momentum flux in a layer of uniform speed U and thickness θ. That is,

$$\rho b U^2 \Theta = \rho b \int_0^\infty u(U - u)\, dy$$

$$\Theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Laminar Boundary Layer

Prandtl/Blasius Boundary Layer Solution

for steady, two-dimensional laminar flows with negligible gravitational effects, incompressible flow:

Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

By solving Navier–Stokes equatıons

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Prandtl/Blasius Boundary Layer Solution

The appropriate boundary conditions are that the fluid velocity far from the body is the upstream velocity and that the fluid sticks to the solid body surfaces. Although the mathematical problem is well-posed, no one has obtained an analytical solution to these equations for flow past any shaped body! Currently much work is being done to obtain numerical solutions to these governing equations for many flow geometries.

By using boundary layer concepts introduced in the previous sections, Prandtl was able to impose certain approximations 1valid for large Reynolds number flows2 and thereby to simplify the governing equations. In 1908, H. Blasius 11883–19702, one of Prandtl's students, was able to solve these simplified equations for the boundary layer flow past a flat plate parallel to the flow. A brief outline of this technique and the results are presented below. Additional details may be found in the literature (Refs. 1–32 in Munson Book Chapter 9).

Prandtl/Blasius Boundary Layer Solution

Since the boundary layer is thin, it is expected that the component of velocity normal to the plate is much smaller than that parallel to the plate and that the rate of change of any parameter across the boundary layer should be much greater than that along the flow direction. That is,

$$v \ll u \quad \text{and} \quad \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$$

Prandtl/Blasius Boundary Layer Solution

$$u^* = \frac{u}{U} \qquad x^* = \frac{x}{L}$$

$$\frac{\partial u}{\partial x} = \frac{\partial(u^*U)}{\partial(x^*L)} = \frac{U}{L}\frac{\partial u^*}{\partial x^*}$$

اي term فيه ⊛ بالعامل بتاعه بواحد ⟸

$\boxed{1}$ ⟸ ال term $\left(\frac{L}{\delta}\right)^2$ قيمته كبيرة جداً يعني هنهمل term

ال Convection بالنسبة لل Pressure

$\boxed{2}$ ⟸ ال term $\left(\frac{\nu}{UL}\right) \ll \left(\frac{\nu}{UL}\right)\left(\frac{L}{\delta}\right)^2$

$\left(\frac{\nu}{UL}\right)\frac{\partial^2 v^*}{\partial x^{*2}}$ ال term ده مهمل

بالنسبة لل term ال

$\left(\frac{\nu}{UL}\right)\left(\frac{L}{\delta}\right)^2\frac{\partial^2 v^*}{\partial y^{*2}}$

$\boxed{3}$ ⟸ $\frac{\nu}{UL}\left(\frac{L}{\delta}\right)^2\frac{\partial^2 v^*}{\partial y^{*2}}$ ب $\ll$ ال pressure term

ال term ده مهم

Prandtl/Blasius Boundary Layer Solution

$$\cancel{u^* \frac{\partial v^*}{\partial x^*}}\ \boxed{1} + \cancel{v^* \frac{\partial v^*}{\partial y^*}}\ \boxed{1} = -\left(\frac{L}{\delta}\right)^2 \frac{\partial p^*}{\partial y^*} + \cancel{\left(\frac{\nu}{UL}\right)\frac{\partial^2 v^*}{\partial x^{*2}}}\ \boxed{2} + \cancel{\left(\frac{\nu}{UL}\right)\left(\frac{L}{\delta}\right)^2 \frac{\partial^2 v^*}{\partial y^{*2}}}\ \boxed{3}$$

$\boxed{1}$

$$\therefore \frac{\partial p^*}{\partial y^*} \simeq 0$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{-dP^*}{dx^*} + \cancel{\left(\frac{\nu}{UL}\right)\frac{\partial^2 u^*}{\partial x^{*2}}} + \underbrace{\left(\frac{\nu}{UL}\right)\left[\frac{L}{\delta}\right]^2 \frac{\partial^2 u^*}{\partial y^{*2}}}$$

$\Longleftarrow$ مهملة بالنسبة ل

$\Longleftarrow$ ل inertia يلزم تعامل

لأن high Re بحيث مؤثرة جداً

$$\frac{1}{Re}\left(\frac{L}{\delta}\right)^2 \sim 1 \Longrightarrow \frac{L}{\delta} \sim \sqrt{Re}$$

$$\boxed{\delta \sim \frac{L}{\sqrt{Re}}} \Longrightarrow \boxed{\frac{\delta}{L} \sim \frac{1}{\sqrt{Re}}}$$

Prandtl/Blasius Boundary Layer Solution

With these assumptions it can be shown that the governing equations reduce to the following boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

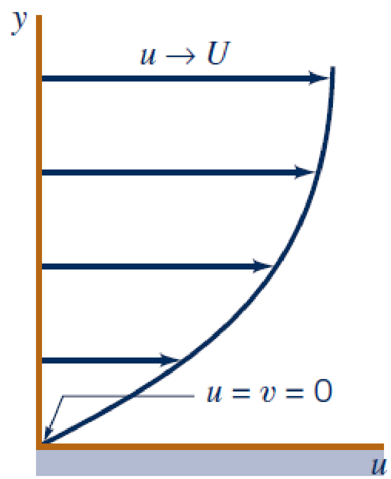
$$\frac{\partial p}{\partial y} = 0$$

Prandtl/Blasius Boundary Layer Solution

Boundary conditions

$$u = v = 0 \quad \text{on} \quad y = 0$$

$$u \to U \quad \text{as} \quad y \to \infty$$



No Exact Solution is available for those equations

Prandtl/Blasius Boundary Layer Solution

It can be argued that in dimensionless form the boundary layer velocity profiles on a flat plate should be similar regardless of the location along the plate. That is,

$$\frac{u}{U} = g\left(\frac{y}{\delta}\right)$$

$$\delta \sim \left(\frac{\nu x}{U}\right)^{1/2}$$

Prandtl/Blasius Boundary Layer Solution

The final solution is

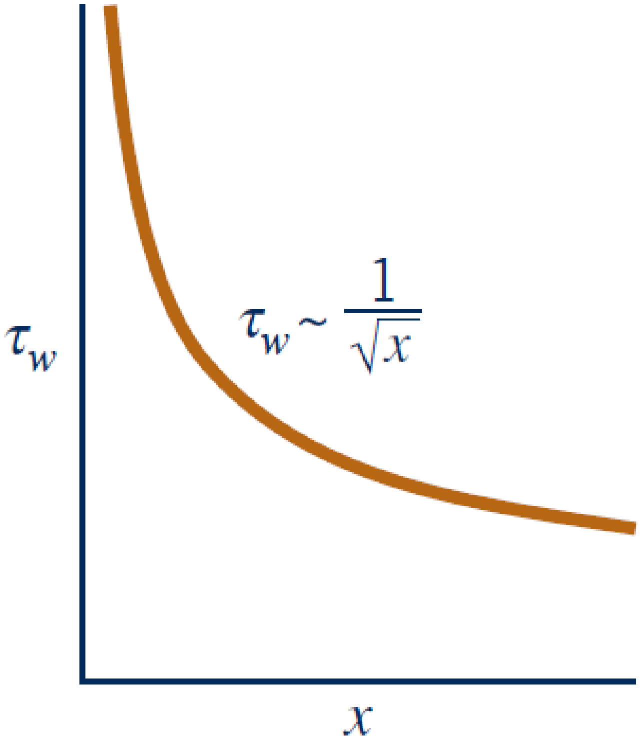
$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

$$\frac{\delta^*}{x} = \frac{1.721}{\sqrt{\mathrm{Re}_x}} \qquad \frac{\Theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

$$\tau_w = 0.332 U^{3/2} \sqrt{\frac{\rho\mu}{x}}$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \qquad c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

Prandtl/Blasius Boundary Layer Solution



Prandtl/Blasius Boundary Layer Solution

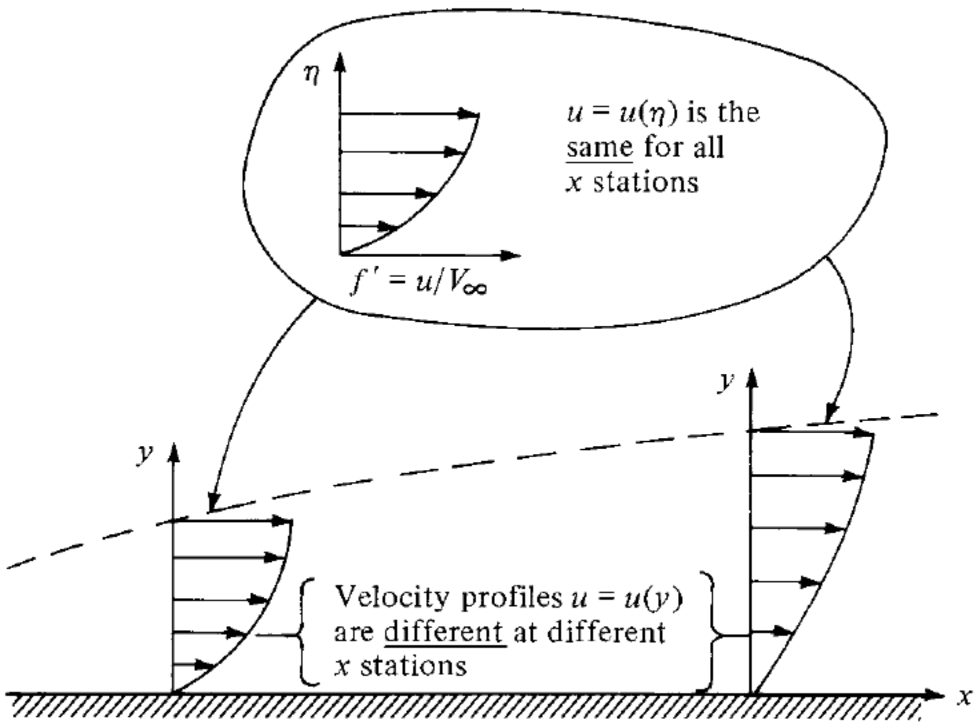


Figure 18.3 Velocity profiles in physical and transformed space, demonstrating the meaning of self-similar solutions.

$$\eta = (U/\nu x)^{1/2} y$$

The dimensionless boundary layer profile

$$u/U = f'(\eta)$$

obtained by numerical solution (termed the Blasius solution), is sketched in Fig. 9.10*a* and is tabulated in Table 9.1

Laminar Flow along a Flat Plate (the Blasius Solution)

$\eta = y(U/\nu x)^{1/2}$	$f'(\eta) = u/U$	η	$f'(\eta)$
0	0	3.6	0.9233
0.4	0.1328	4.0	0.9555
0.8	0.2647	4.4	0.9759
1.2	0.3938	4.8	0.9878
1.6	0.5168	5.0	0.9916
2.0	0.6298	5.2	0.9943
2.4	0.7290	5.6	0.9975
2.8	0.8115	6.0	0.9990
3.2	0.8761	∞	1.0000

$$\eta = (U/\nu x)^{1/2} y$$

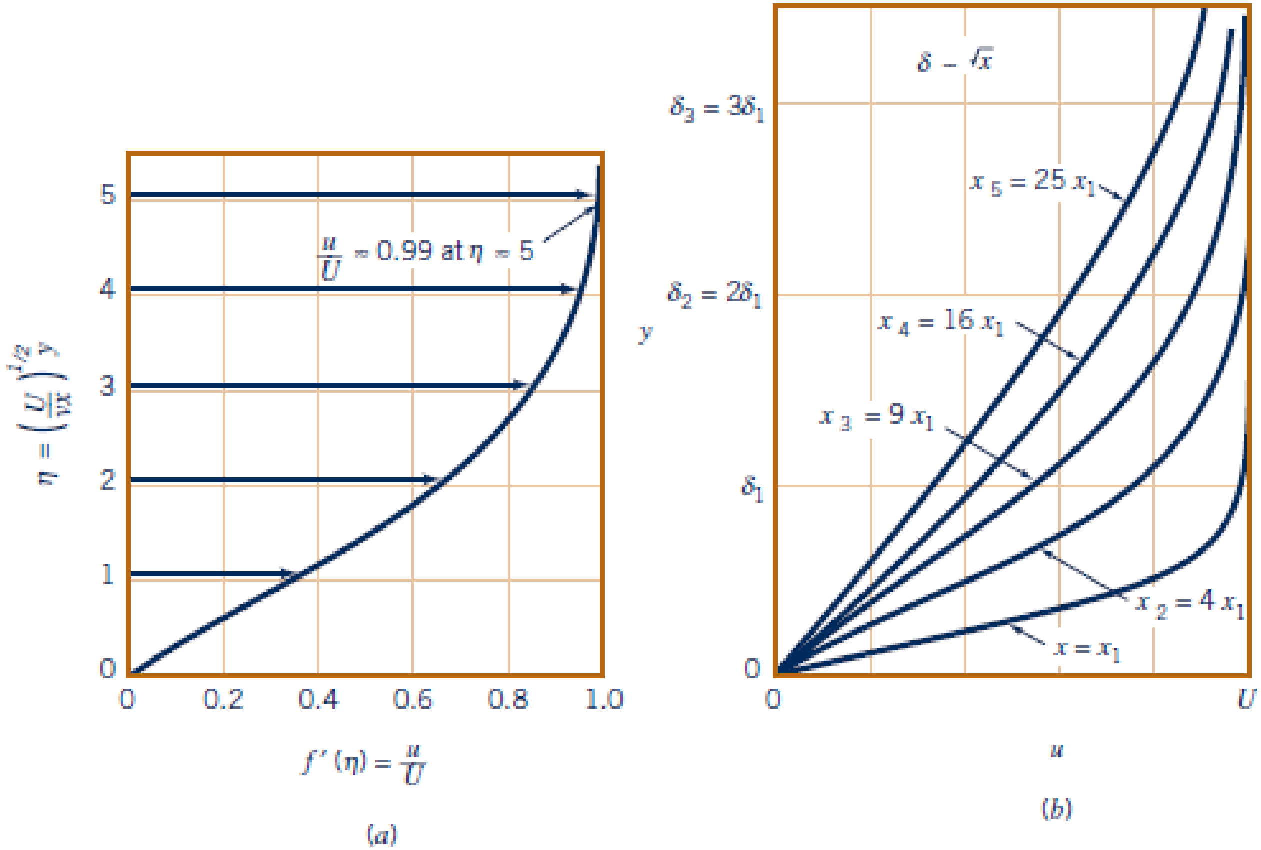


Figure 9.10 Blasius boundary layer profile: (*a*) boundary layer profile in dimensionless form using the similarity variable η, (*b*) similar boundary layer profiles at different locations along the flat plate.

Momentum Integral Boundary Layer Equation for a Flat Plate

Flat Plate Momentum Integral Results for Various Assumed Laminar Flow Velocity Profiles

Profile Character	$\delta \mathrm{Re}_x^{1/2}/x$	$c_f \mathrm{Re}_x^{1/2}$	$C_{Df} \mathrm{Re}_\ell^{1/2}$
a. Blasius solution	5.00	0.664	1.328
b. Linear $u/U = y/\delta$	3.46	0.578	1.156
c. Parabolic $u/U = 2y/\delta - (y/\delta)^2$	5.48	0.730	1.460
d. Cubic $u/U = 3(y/\delta)/2 - (y/\delta)^3/2$	4.64	0.646	1.292
e. Sine wave $u/U = \sin[\pi(y/\delta)/2]$	4.79	0.655	1.310

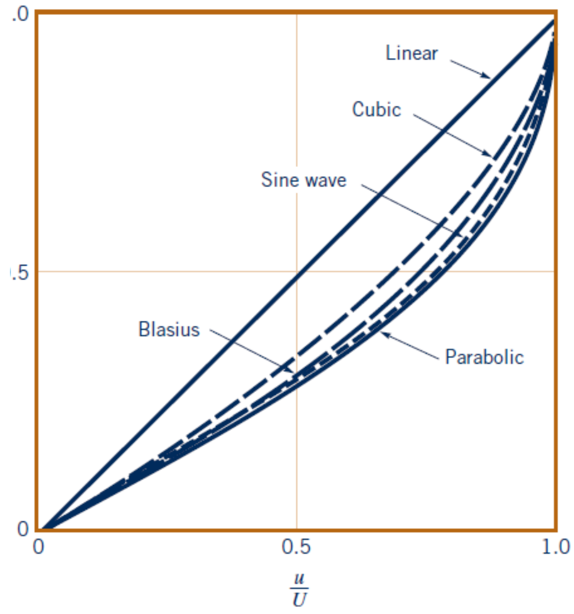


Figure 9.12 Typical approximate boundary layer profiles used in the momentum integral equation.

Turbulent Boundary Layer

Turbulent Boundary Layer

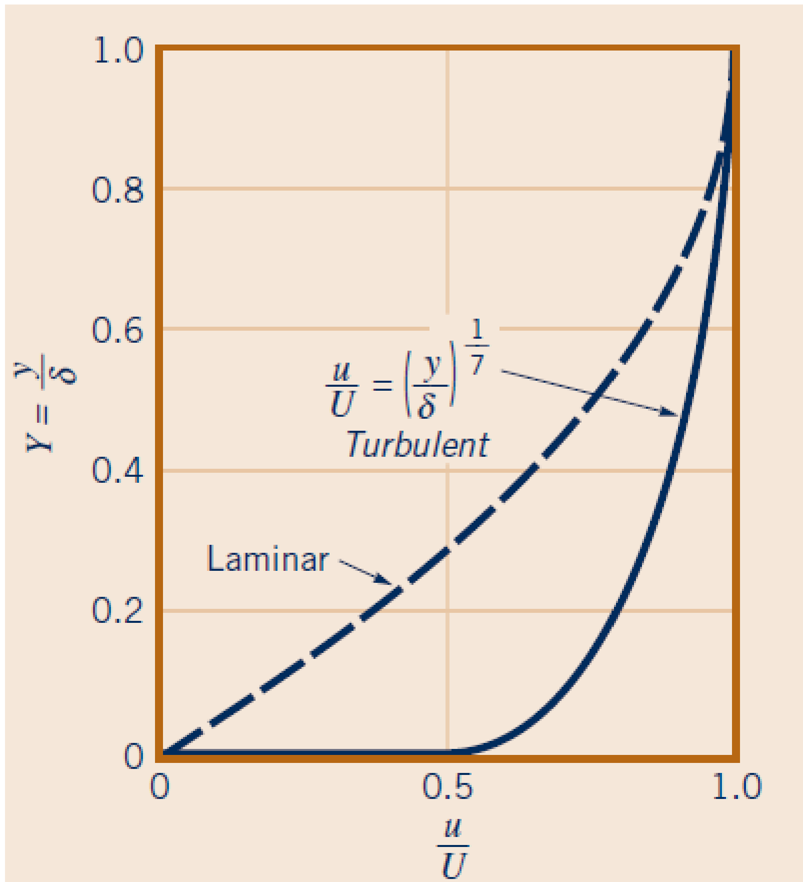
Experimental measurements have shown that the time-averaged velocity for a turbulent boundary layer on a flat plate may be represented by the 1/7th power law:

$$\frac{u}{u_e} = \left(\frac{y}{\delta}\right)^{1/7}$$

$$\delta = \frac{0.37x}{\mathrm{Re}_x^{1/5}}$$

$$C_f = \frac{0.074}{\mathrm{Re}_c^{1/5}}$$

Turbulent Boundary Layer



$$\frac{u}{u_e} = \left(\frac{y}{\delta}\right)^{1/7}$$

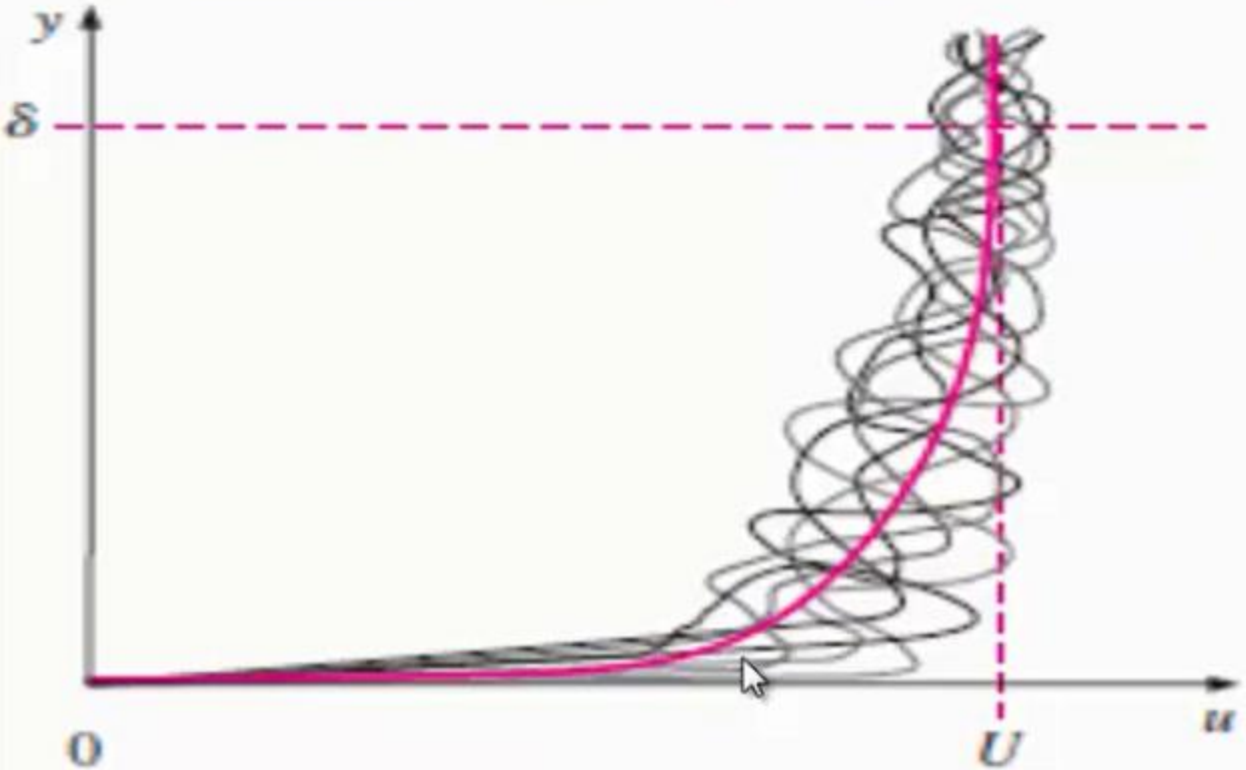
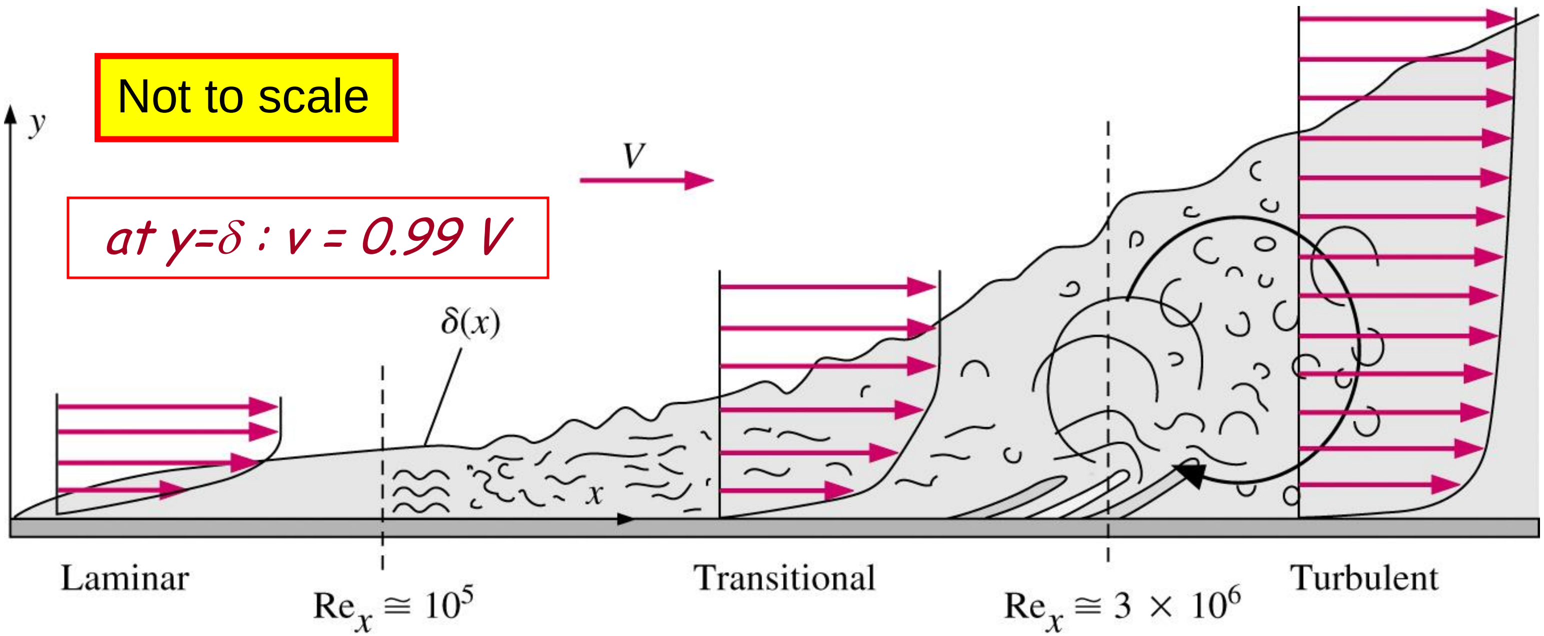
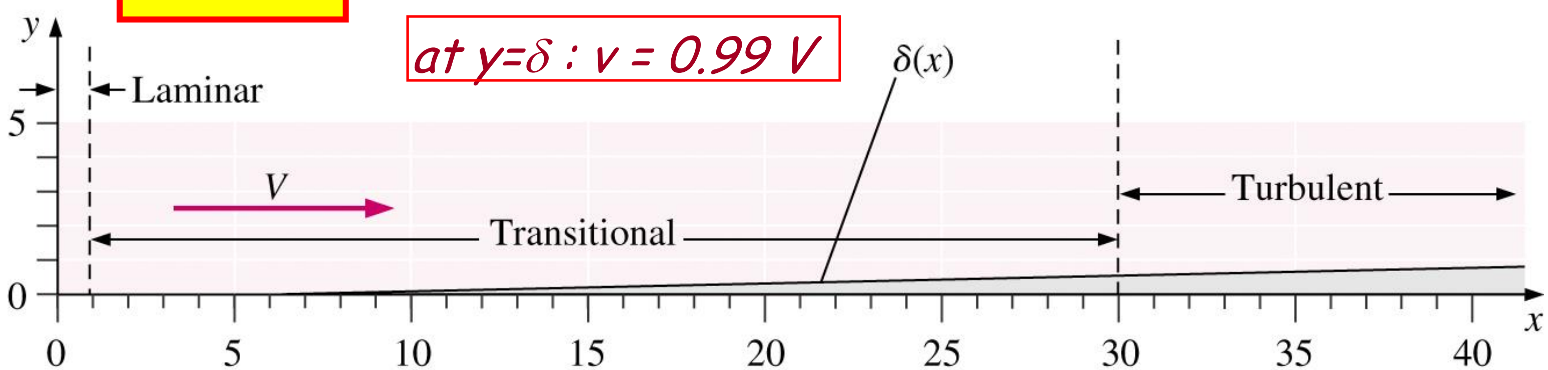
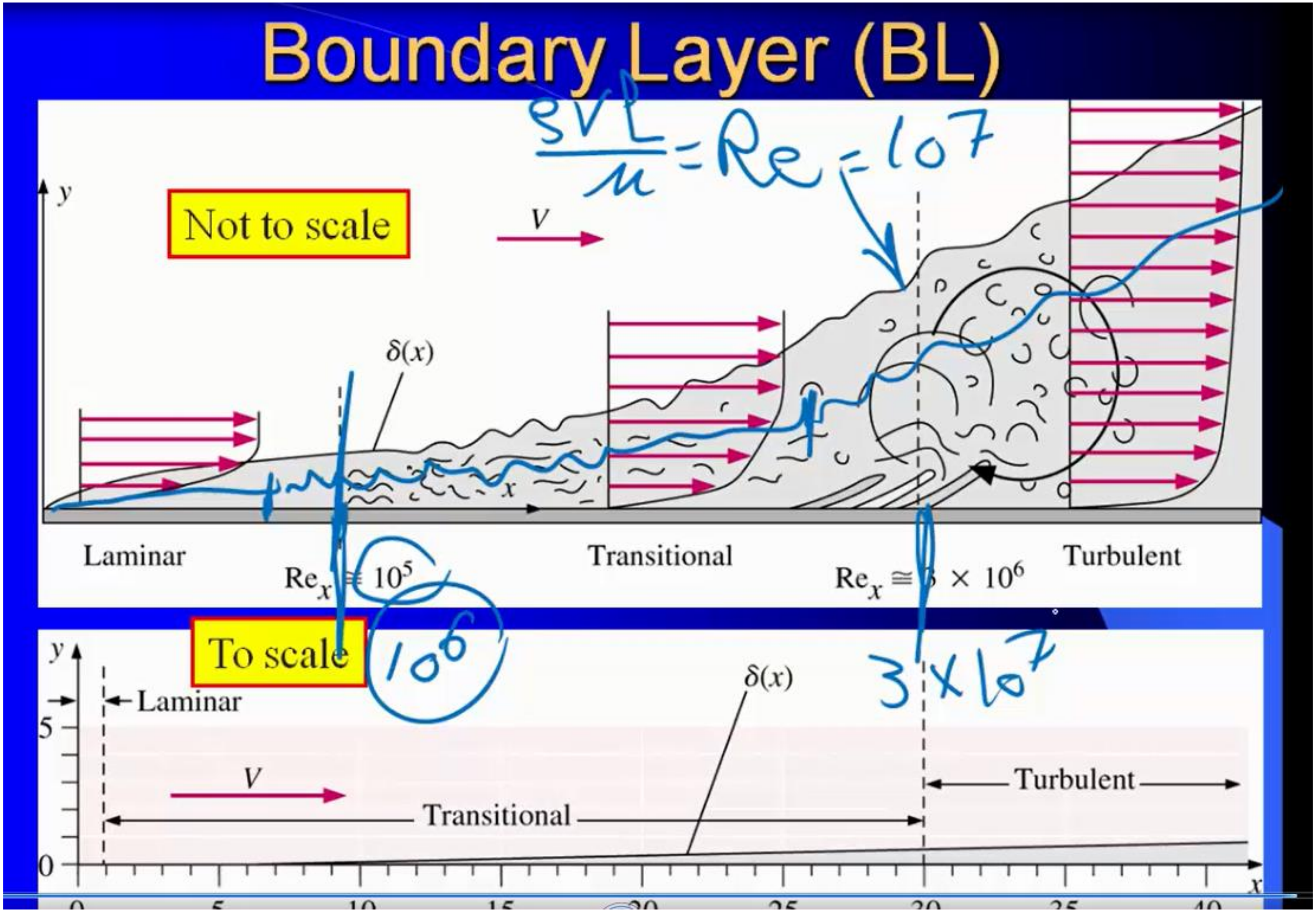


FIGURE 10–112
Illustration of the unsteadiness of a turbulent boundary layer; the thin, wavy black lines are instantaneous profiles, and the thick blue line is a long time-averaged profile.

Boundary Layer on a Flat Plate







Boundary Layer (BL)
Not to scale
V
y
δ(x)
x
Laminar
Transitional
Turbulent
Re_x ≅ 10^5
Re_x ≅ 3 × 10^6
To scale
Laminar
V
Transitional
Turbulent
δ(x)
ρVL/μ = Re = 10^7
10^6
3 × 10^7

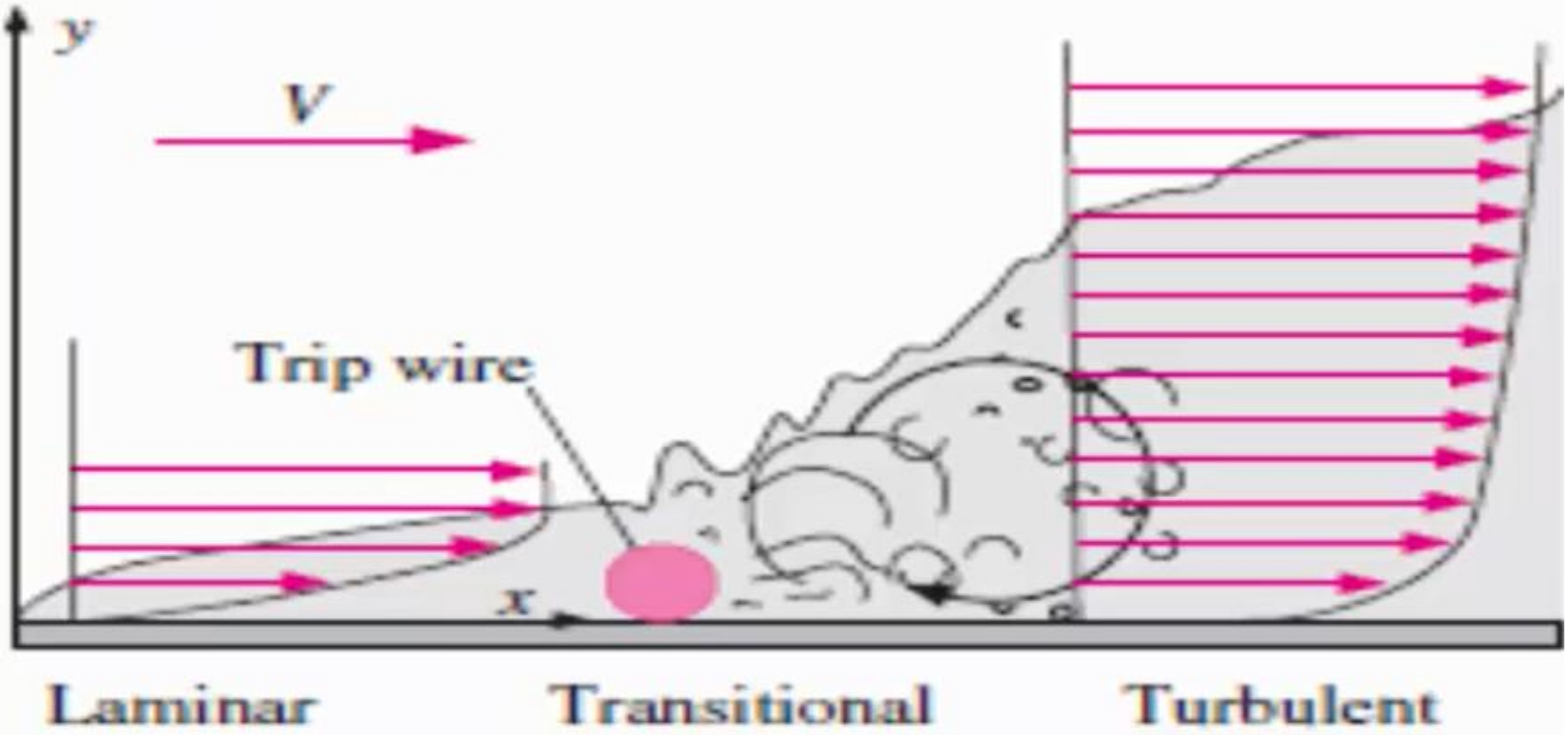


FIGURE 10–83

A trip wire is often used to initiate early transition to turbulence in a boundary layer (not to scale).

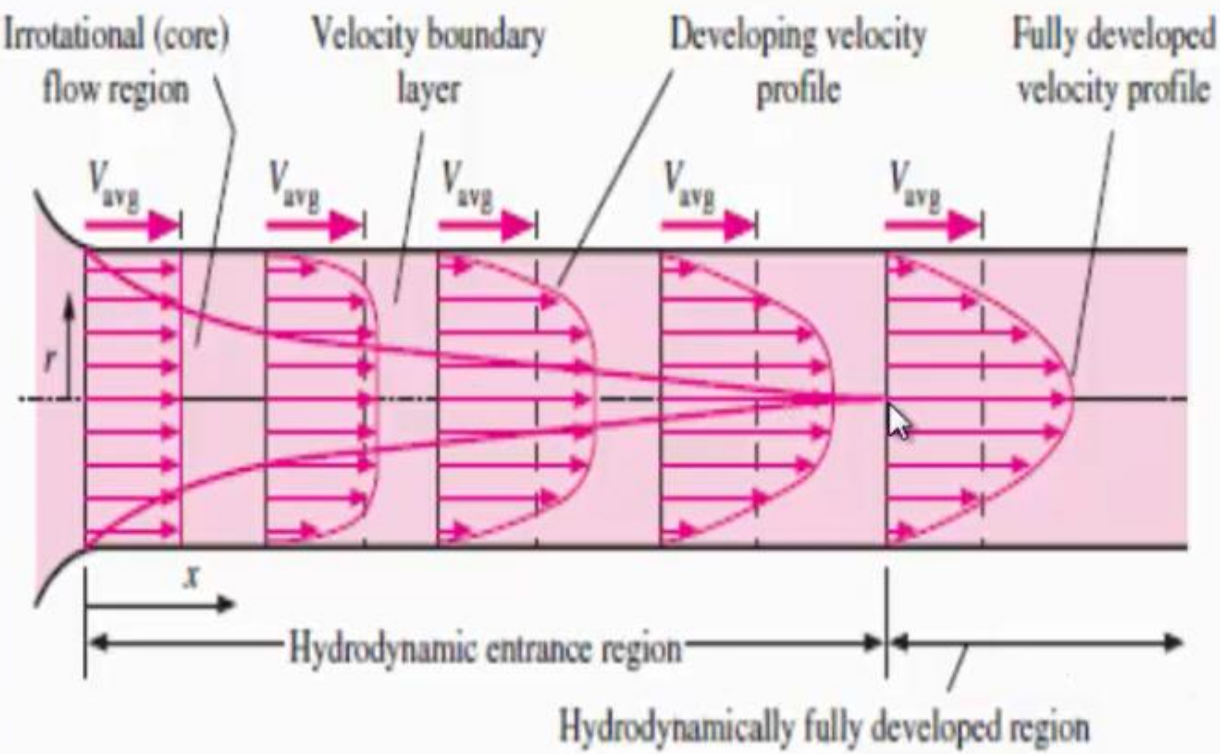


FIGURE 8–8

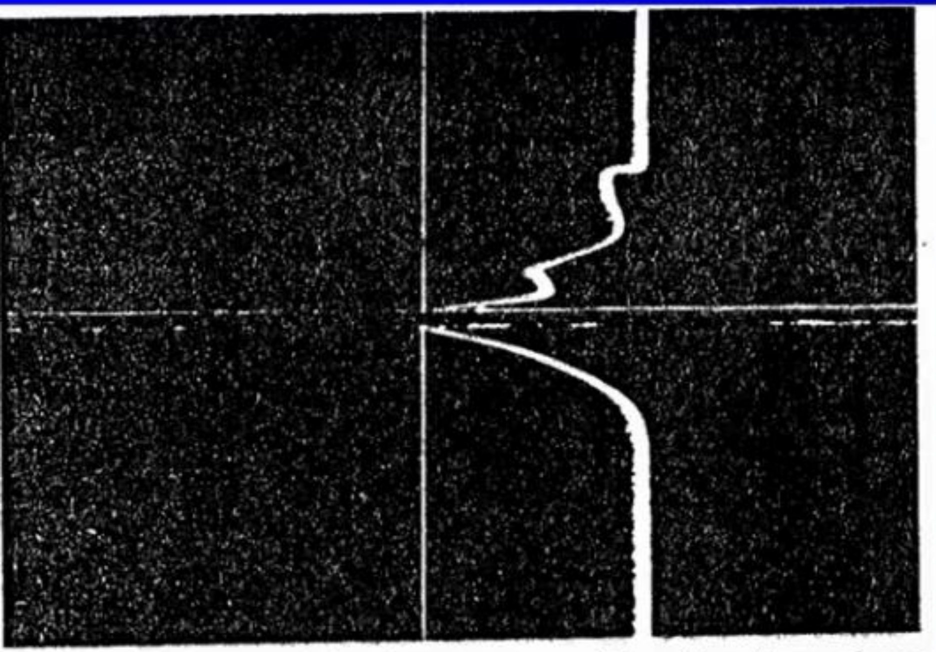
The development of the velocity boundary layer in a pipe. (The developed average velocity profile is parabolic in laminar flow, as shown, but somewhat flatter or fuller in turbulent flow.)

In real-life engineering flows, transition to turbulent flow usually occurs more abruptly and much earlier (at a lower value of Re_x) than the values given for a smooth flat plate with a calm free stream.

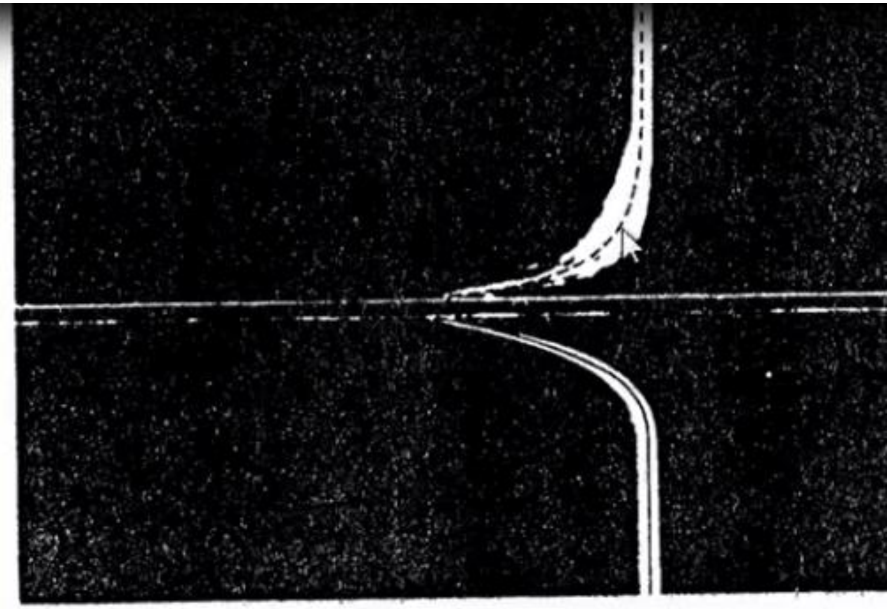
Factors such as roughness along the surface, free-stream disturbances, acoustic noise, flow unsteadiness, vibrations, and curvature of the wall contribute to an **earlier transition location.**

Because of this, an *engineering critical Reynolds number* of **$Rex_{cr} = 5 * 10^5$** is often used to determine whether **a boundary layer is most likely laminar** (**$Rex < Rex_{cr}$**) or **most likely turbulent ($Rex > Rex_{cr}$).**

Turbulent Boundary Layer



19. Instantaneous displacement profiles for flow along a thin plate. The boundary layer on the upper surface has been made turbulent, while the flow along the lower surface is laminar.



20a. The upper boundary layer is turbulent; the lowe laminar. Superposition of many instantaneous veloci profiles suggests mean velocity profiles.

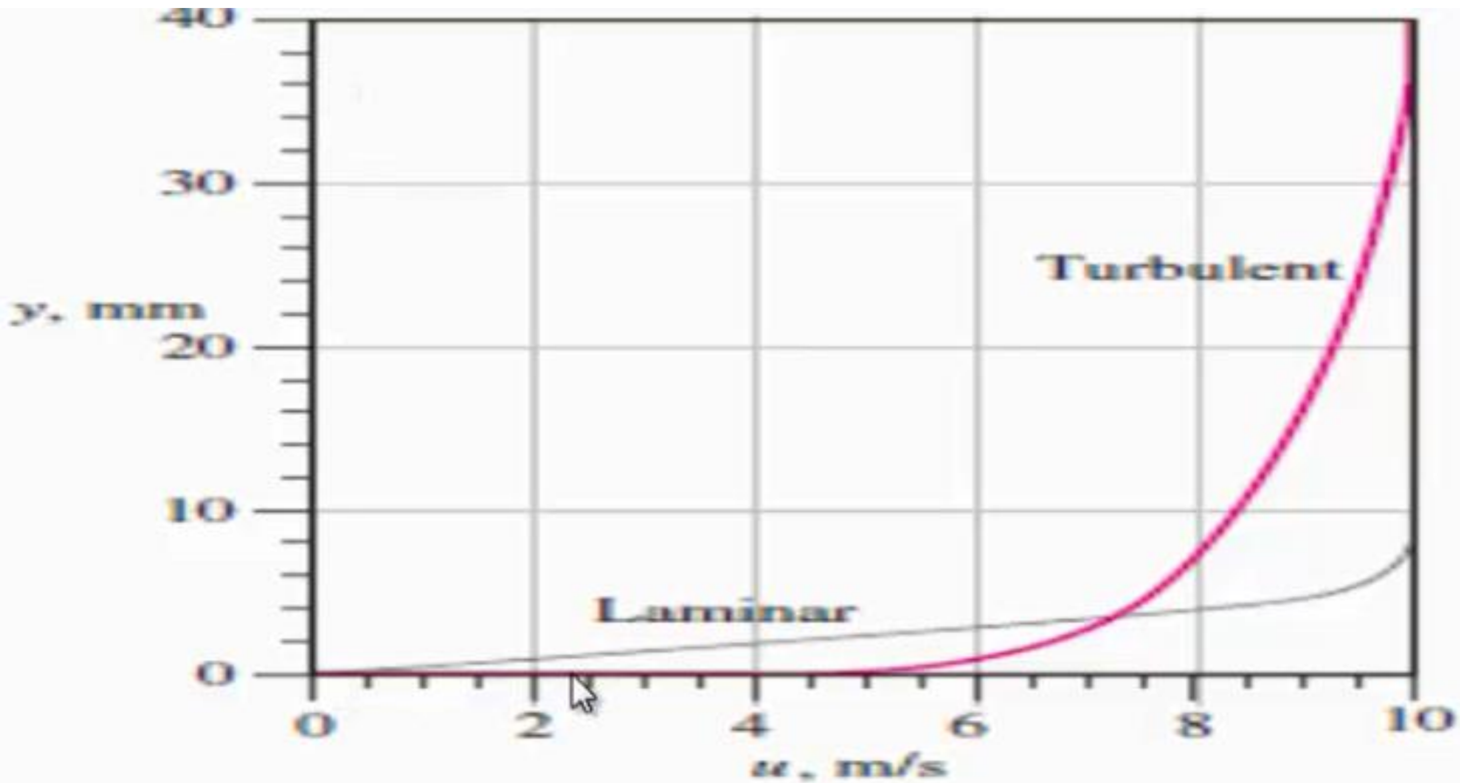


FIGURE 10–115

Comparison of laminar and turbulent flat plate boundary layer profiles in physical variables at the same x-location. The Reynolds number is $Re_x = 1.0 \times 10^6$.

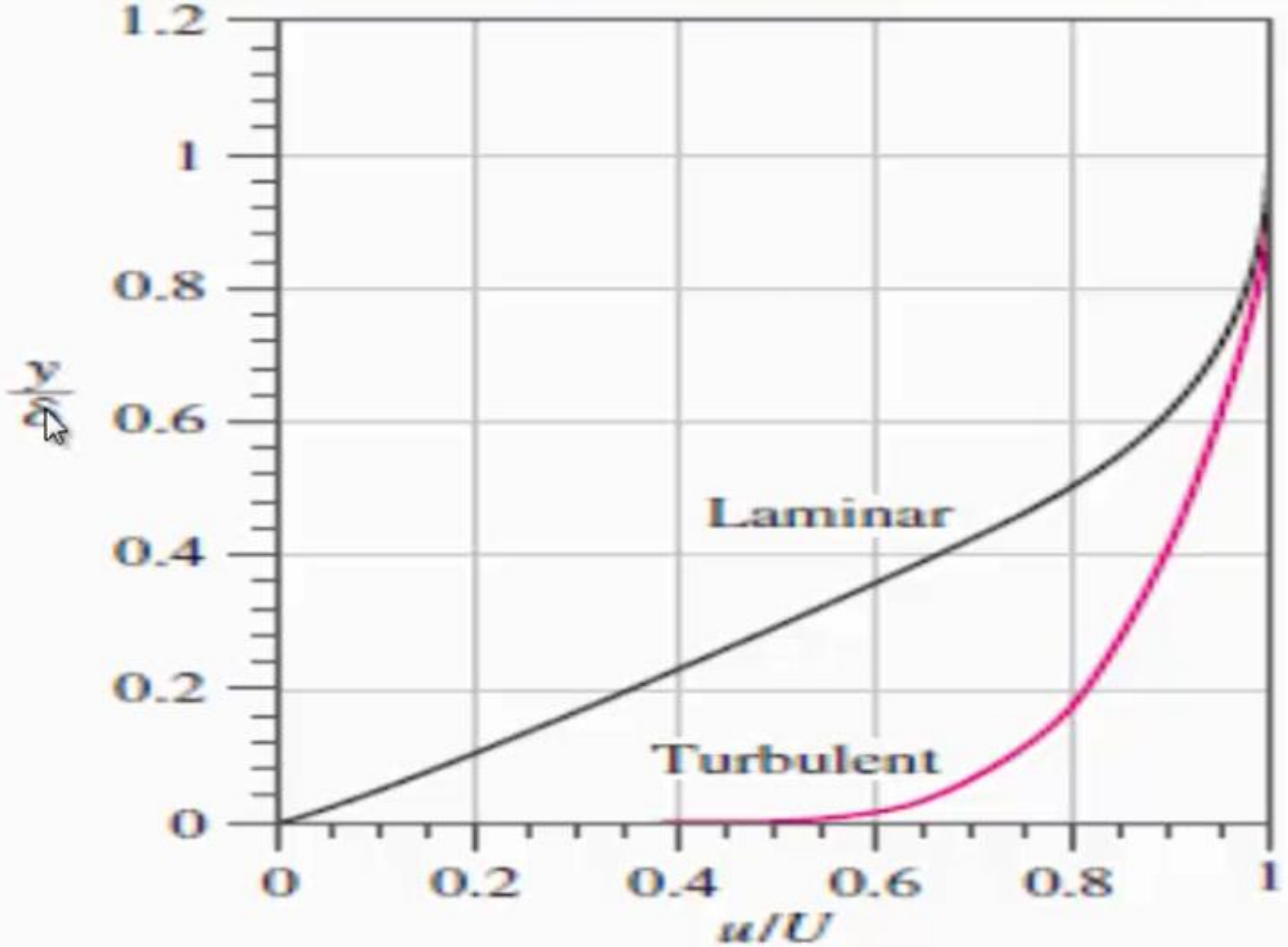


FIGURE 10–113
Comparison of laminar and turbulent flat plate boundary layer profiles, nondimensionalized by boundary layer thickness.

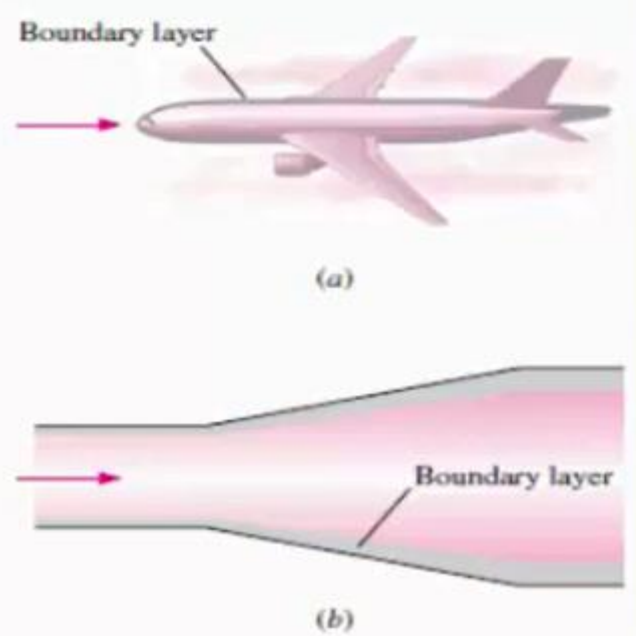
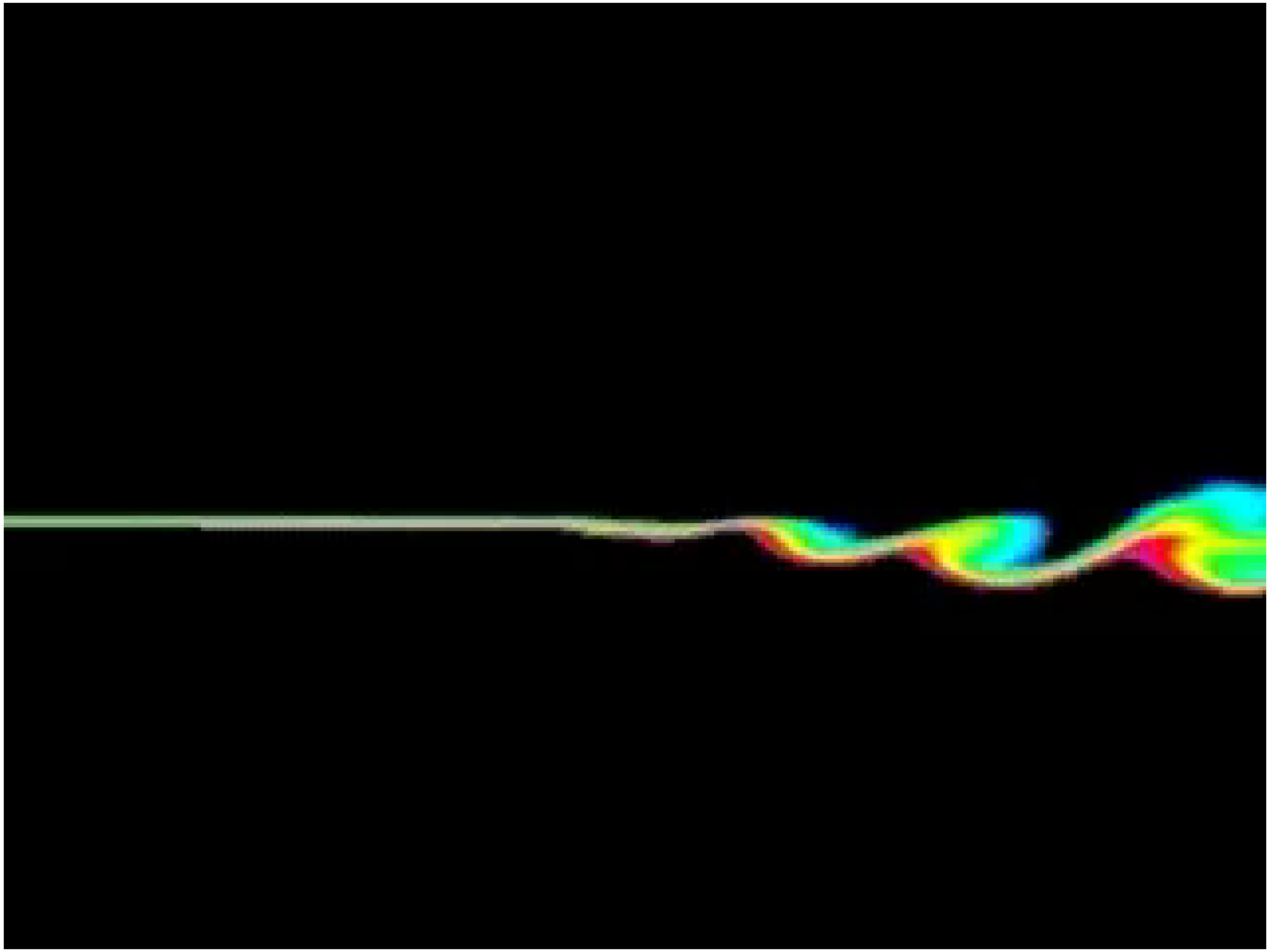
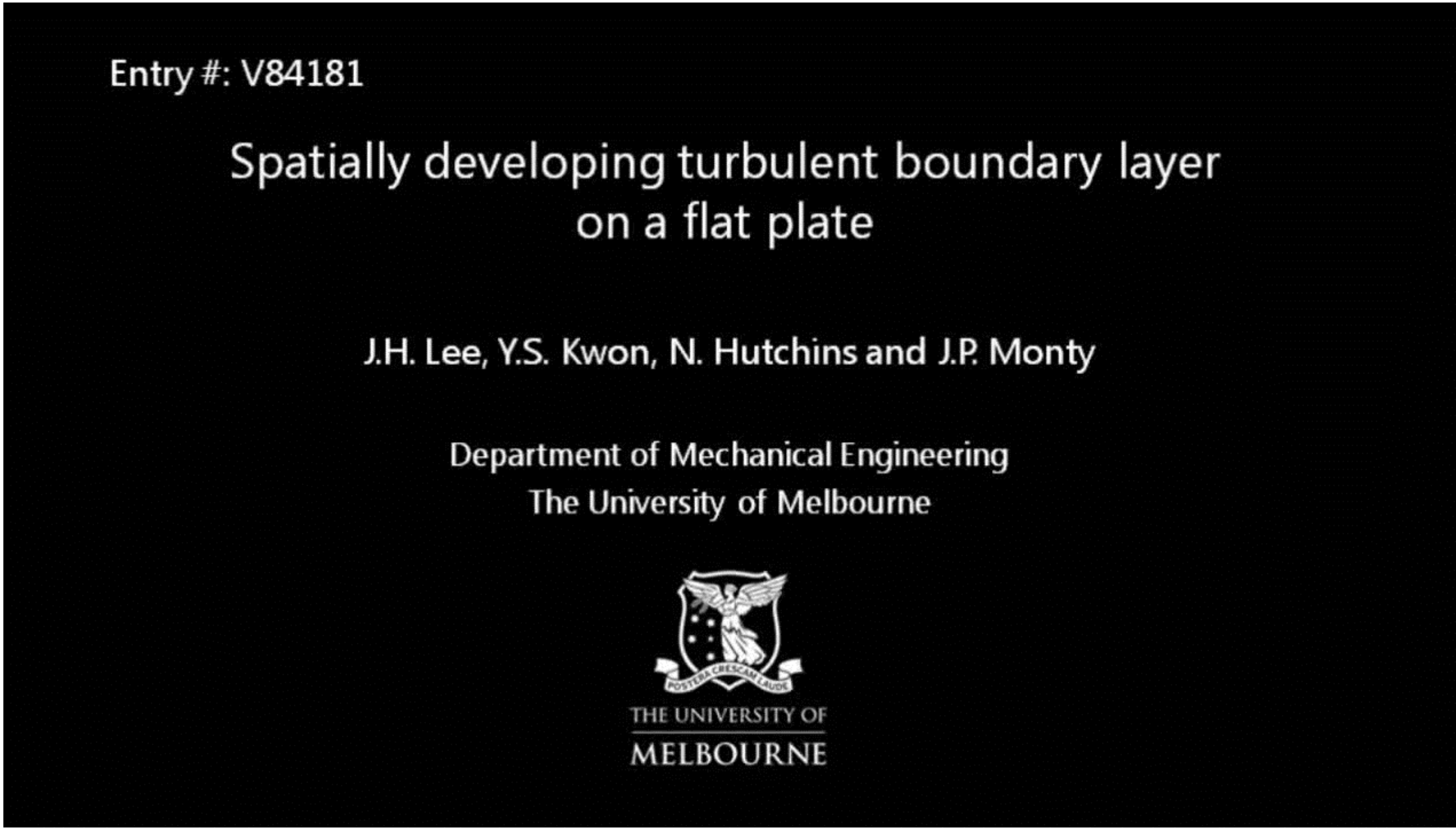


FIGURE 10–120
Boundary layers with nonzero pressure gradients occur in both external flows and internal flows: (*a*) boundary layer developing along the fuselage of an airplane and into the wake, and (*b*) boundary layer growing on the wall of a diffuser (boundary layer thickness exaggerated in both cases).

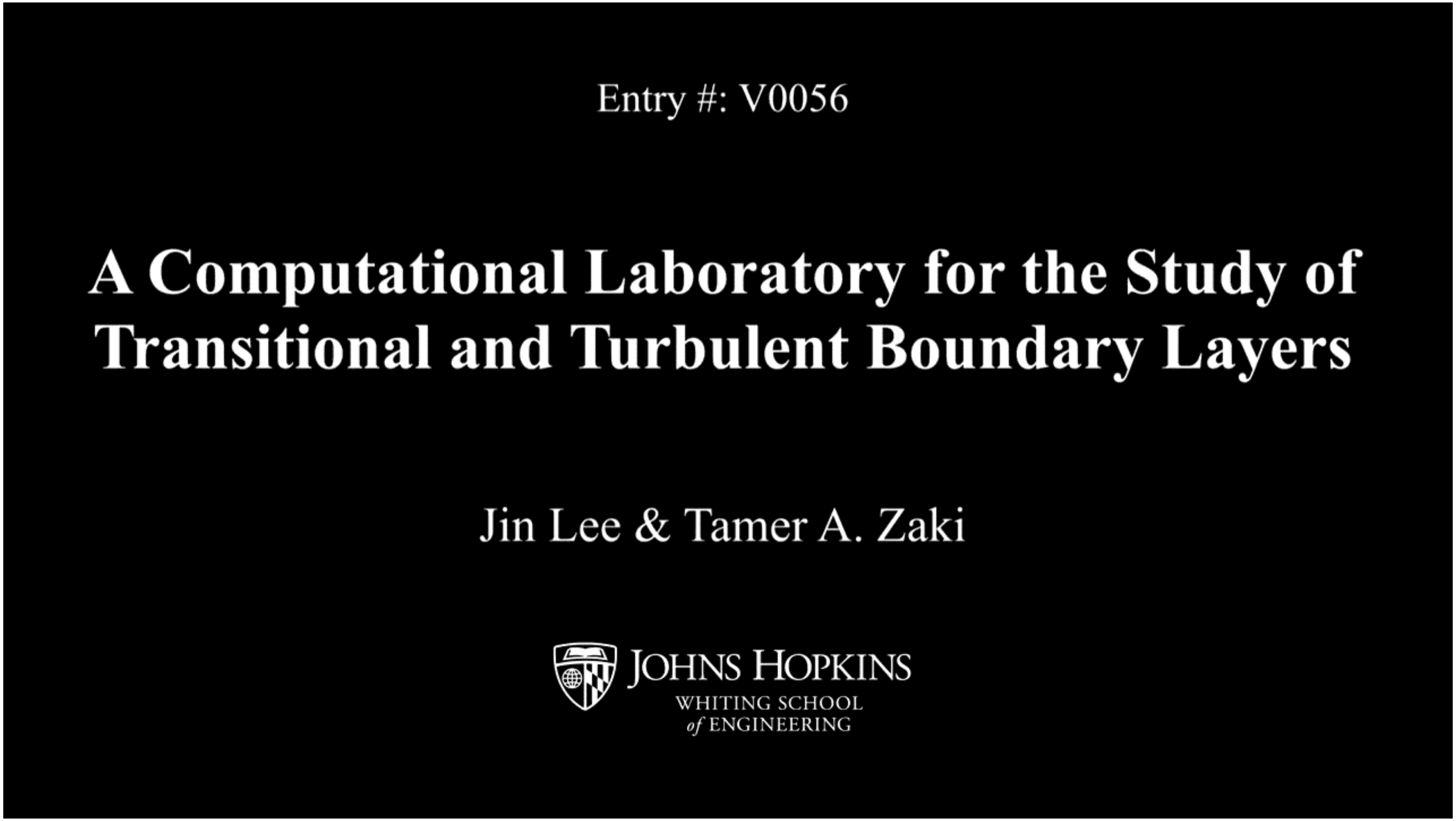
Boundary Layer transition from laminar to turbulent



Transition


Entry #: V84181
Spatially developing turbulent boundary layer
on a flat plate
J.H. Lee, Y.S. Kwon, N. Hutchins and J.P. Monty
Department of Mechanical Engineering
The University of Melbourne
POSTERA CRESCAM LAUDE
THE UNIVERSITY OF
MELBOURNE

Transition


Entry #: V0056
A Computational Laboratory for the Study of Transitional and Turbulent Boundary Layers
Jin Lee & Tamer A. Zaki
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