

Final Exam Formulas Sheet

$$\nabla \cdot \vec{V} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\rho \left(\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \right) = \nabla p + \mu \nabla^2 \vec{V} + \vec{F}_b$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z$$

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V}$$

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

Laminar: $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$, $\frac{\delta^*}{x} = \frac{1.721}{\sqrt{Re_x}}$, $\frac{\Theta}{x} = \frac{0.664}{\sqrt{Re_x}}$, $C_{Df} = \frac{1.328}{Re_l^{1/2}}$

Turbulent: $\frac{\delta}{x} = \frac{0.37}{Re_x^{1/5}}$, $C_{Df} = \frac{0.074}{Re_l^{1/5}}$

Incompressible continuity equation:
$$\frac{1}{r} \frac{\partial(ru_r)}{\partial r} + \frac{1}{r} \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$$

r-component of the incompressible Navier–Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) \\ &= -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \end{aligned}$$

θ-component of the incompressible Navier–Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_\theta}{\partial z} \right) \\ &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \end{aligned}$$

z-component of the incompressible Navier–Stokes equation:

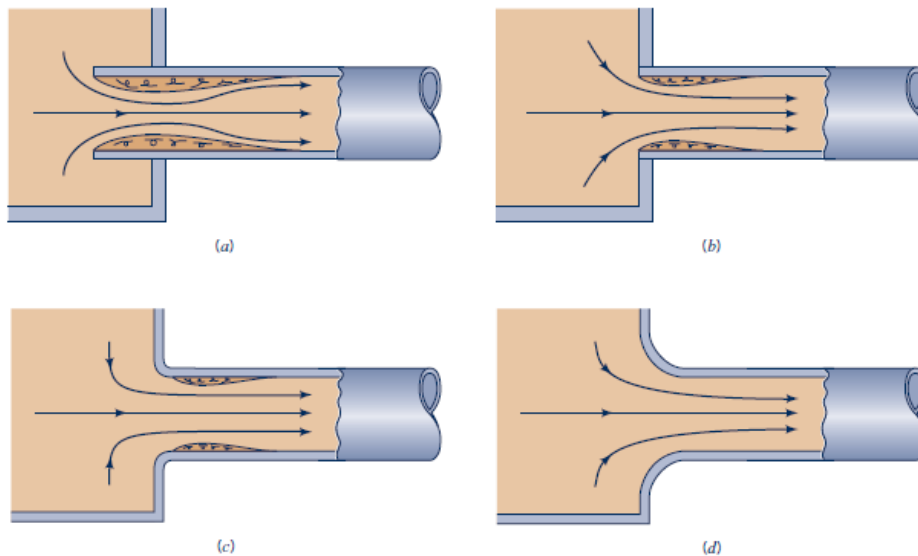
$$\begin{aligned} & \rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) \\ &= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \end{aligned}$$

■ **Table 8.1**

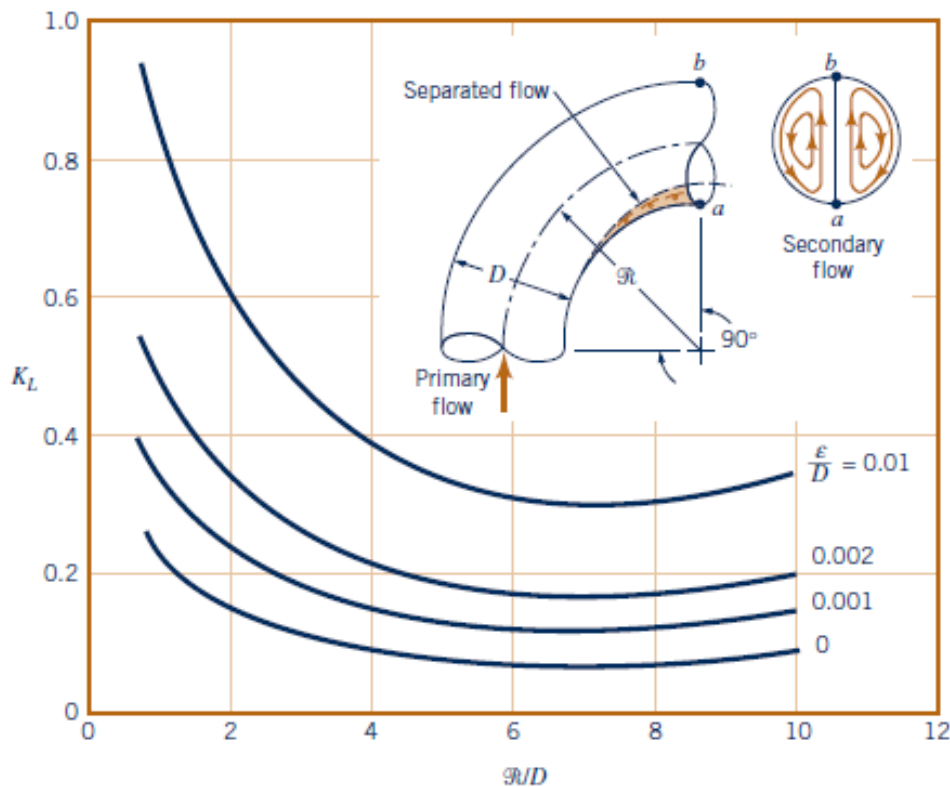
Equivalent Roughness for New Pipes [Adapted from Moody (Ref. 7) and Colebrook (Ref. 8)]

Pipe	Equivalent Roughness, ϵ	
	Feet	Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

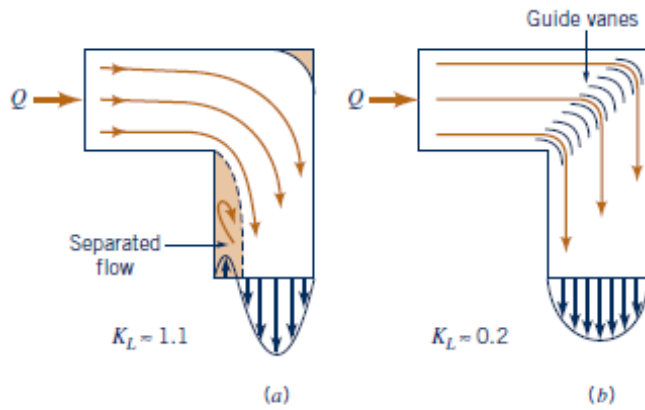
$$\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right)$$



■ **Figure 8.22** Entrance flow conditions and loss coefficient (Data from Refs. 28, 29).
 (a) Reentrant, $K_L = 0.8$, (b) sharp-edged, $K_L = 0.5$, (c) slightly rounded, $K_L = 0.2$ (see Fig. 8.24),
 (d) well-rounded, $K_L = 0.04$ (see Fig. 8.24).



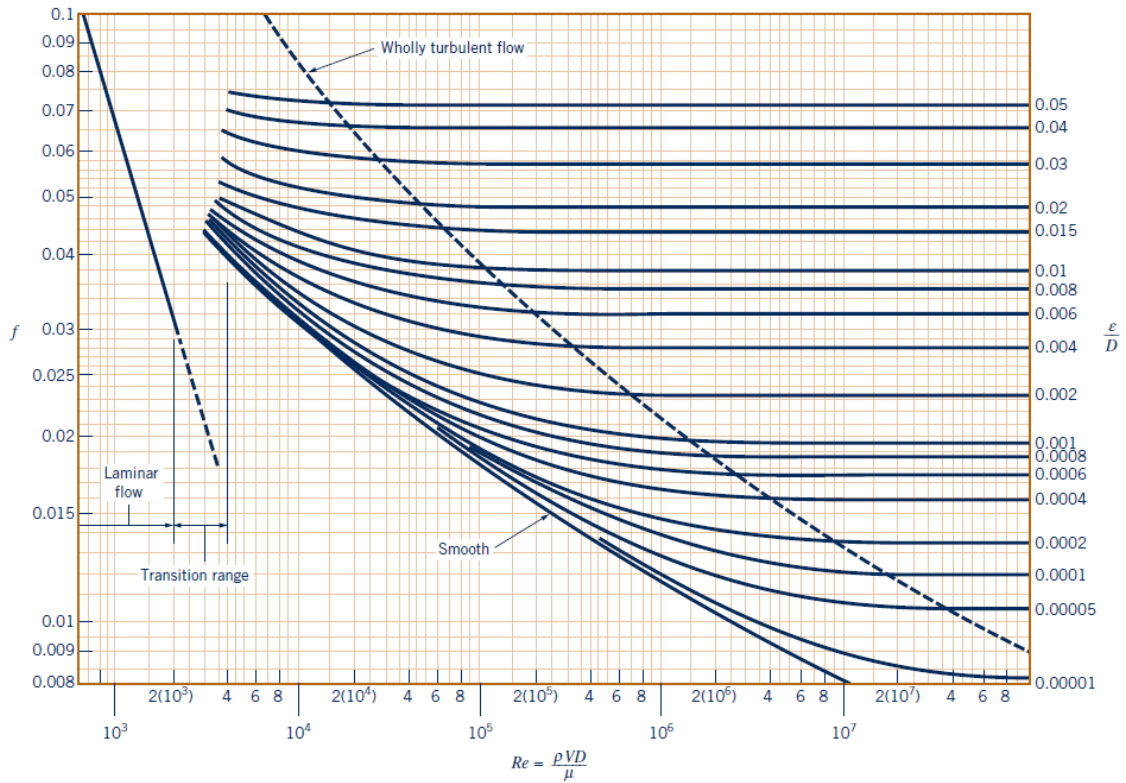
■ **Figure 8.30** Character of the flow in a 90° bend and the associated loss coefficient (Ref. 5).



■ **Figure 8.31** Character of the flow in a 90° mitered bend and the associated loss coefficient: (a) without guide vanes, (b) with guide vanes.

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	
	0.08
*e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210





■ **Figure 8.20** Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)

$$\text{Pump Power} = \frac{\Delta P Q}{\eta}$$

$$Z = \frac{1}{\left[1 + \left(\frac{P_{avg} 344400(10)^{1.785G}}{T_f^{3.825}} \right) \right]}$$

$$P_{avg} = \frac{2}{3} \left(\frac{P_1^3 - P_2^3}{P_1^2 - P_2^2} \right)$$

$$Q = 1.1494 \times 10^{-3} \left(\frac{T_b}{P_b} \right) \left[\frac{(P_1^2 - P_2^2)}{G T_f L Z f} \right]^{0.5} D^{2.5}$$

$$V = 14.7349 \left(\frac{P_b}{T_b} \right) \left(\frac{Z T}{P} \right) \left(\frac{Q_b}{D^2} \right)$$

$$V_{max} = 100 \sqrt{\frac{Z R T}{29 G P}}$$

$$t = \frac{l V_0}{2 g H_e} \ln \frac{V_0 + V}{V_0 - V}$$

$$V_0 = \sqrt{\frac{2 g d H_0}{f l}}$$