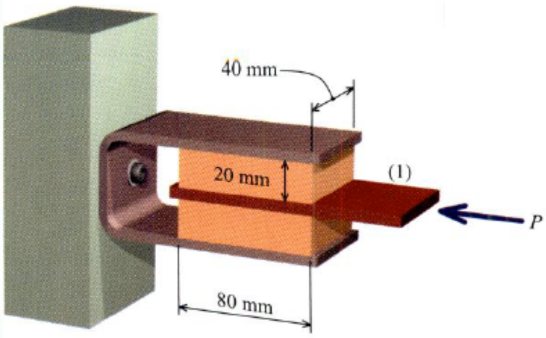
Midterm Answer

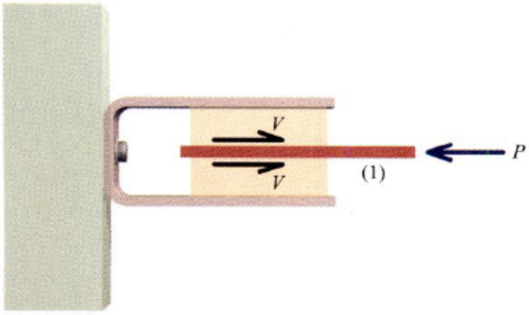
QUESTION ONE Answer:



Two blocks of rubber, each 80 mm long by 40 mm wide by 20 mm thick, are bonded to a rigid support mount and to a movable plate (1). When a force of P = 2,800 N is applied to the assembly, plate (1) deflects 8 mm horizontally. Determine the shear modulus G of the rubber used for the blocks.

Plan the Solution

Hooke's Law expresses the relationship between shear stress and shear strain [Eq. (3.5)]. The shear stress can be determined from the applied load P and the area of the rubber blocks that contact the movable plate (1). Shear strain is an angular measure, which can be determined from the horizontal deflection of plate (1) and the thickness of the rubber blocks. Shear modulus G is computed from the shear stress divided by the shear strain.

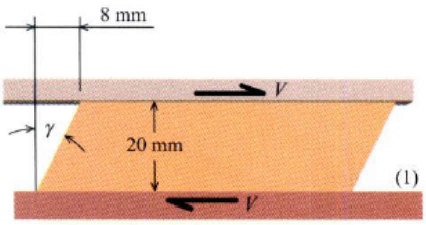


SOLUTION

Consider a free-body diagram of movable plate (1). Each rubber block provides a shear force that opposes the applied load P. From equilibrium, the sum of forces in the horizontal direction is

$$\Sigma F_x = 2V - P = 0$$

$$\therefore V = P/2 = (2{,}800\text{ N})/2 = 1{,}400\text{ N}$$



Next, consider a free-body diagram of the upper rubber block in its deflected position. The shear force V acts on a surface that is 80 mm long and 40 mm wide. Therefore, the shear stress τ in the rubber block is

$$\tau = \frac{1{,}400\text{ N}}{(80\text{ mm})(40\text{ mm})} = 0.4375\text{ MPa}$$

The 8-mm horizontal deflection causes the block to skew as shown. The angle γ (measured in radians) is the shear strain:

$$\tan\gamma = \frac{8\text{ mm}}{20\text{ mm}} \qquad \therefore \gamma = 0.3805\text{ rad}$$

The shear stress τ, the shear modulus G, and the shear strain γ are related by Hooke's Law:

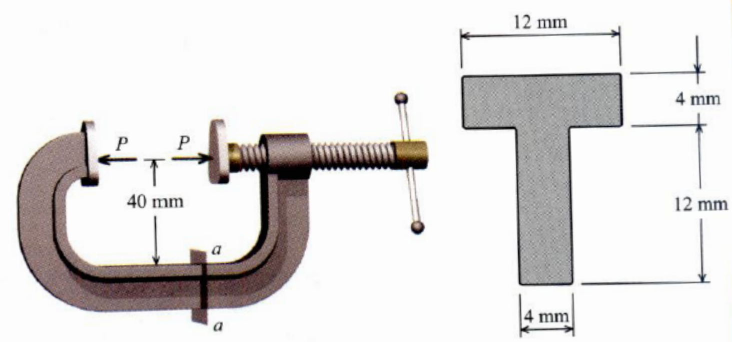
$$\tau = G\gamma$$

Therefore, the shear modulus G of the rubber used for the blocks is

$$G = \frac{\tau}{\gamma} = \frac{0.4375\text{ MPa}}{0.3805\text{ rad}} = 1.150\text{ MPa} \qquad \textbf{Ans.}$$

QUESTION TWO Answer:

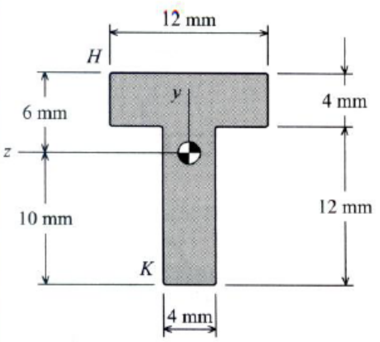
The C-clamp shown is made of an alloy that has a yield strength of 324 MPa in either tension or compression. Determine the allowable clamping force that the clamp can exert if a factor of safety of 3.0 is required.



Cross section *a–a*.

Plan the Solution

The location of the centroid for the tee-shaped cross section must be determined at the outset. Once the centroid has been located, the eccentricity e of the clamping force P can be determined and the equivalent force and moment acting on section a–a can be established. Expressions for the combined axial and bending stresses, written in terms of the unknown P, can be set equal to the allowable normal stress. From these expressions, the maximum allowable clamping force can be determined.



Section Properties

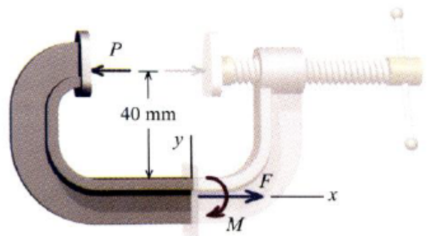
The centroid for the tee-shaped cross section is located as shown in the sketch on the left. The cross-sectional area is $A = 96\ \text{mm}^2$, and the moment of inertia about the z centroidal axis can be calculated as $I_z = 2{,}176\ \text{mm}^4$.

Allowable Normal Stress

The alloy used for the clamp has a yield strength of 324 MPa. Since a factor of safety of 3.0 is required, the allowable normal stress for this material is 108 MPa.

Internal Force and Moment

A free-body diagram cut through the clamp at section a–a is shown. The internal axial force F is equal to the clamping force P. The internal bending moment M is equal to the clamping force P times the eccentricity e between the centroid of section a–a and the line of action of P, which is $e = 40\ \text{mm} + 6\ \text{mm} = 46\ \text{mm}$.



Axial Stress

On section a–a, the internal force F (which is equal to the clamping force P) produces a normal stress of

$$\sigma_{\text{axial}} = \frac{F}{A} = \frac{P}{A} = \frac{P}{96\ \text{mm}^2}$$

This normal stress is uniformly distributed over the entire cross section. By inspection, the axial stress is tension.

Bending Stress

Since the $\perp$ shape is not symmetrical about its z axis, the bending stress on section a–a at the top of the flange (point H) will be different from the bending stress at the bottom of the stem (point K). At point H, the bending stress can be expressed in terms of the clamping force P as

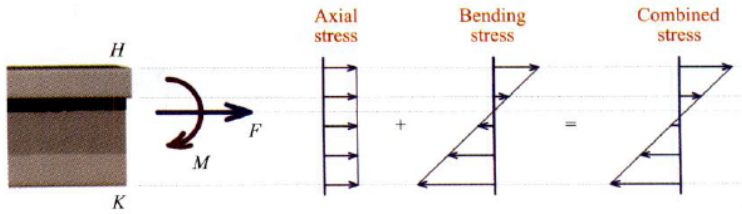
$$\sigma_{\text{bend},H} = \frac{My}{I_t} = \frac{P(46\ \text{mm})(6\ \text{mm})}{2{,}176\ \text{mm}^4} = \frac{P}{7.88406\ \text{mm}^2}$$

By inspection, the bending stress at point H will be tension.

The bending stress at point K can be expressed as

$$\sigma_{\text{bend},K} = \frac{My}{I_t} = \frac{P(46\ \text{mm})(10\ \text{mm})}{2{,}176\ \text{mm}^4} = \frac{P}{4.73043\ \text{mm}^2}$$

By inspection, the bending stress at point K will be compression.



Combined Stress at H

The combined stress at point H can be expressed in terms of the unknown clamping force P as

$$\sigma_{\text{comb},H} = \frac{P}{96\ \text{mm}^2} + \frac{P}{7.88406\ \text{mm}^2} = P\left[\frac{1}{96\ \text{mm}^2} + \frac{1}{7.88406\ \text{mm}^2}\right] = \frac{P}{7.28572\ \text{mm}^2}$$

Note that the axial and bending stress expressions are added since both are tension stresses. This expression can be set equal to the allowable normal stress to obtain one possible value for P:

$$\boxed{\frac{P}{7.28572\ \text{mm}^2} \leq 108\ \text{MPa} = 108\ \text{N/mm}^2 \qquad \therefore P \leq 787\ \text{N}} \tag{a}$$

Combined Stress at K

The combined stress at point K is the sum of a tension axial stress and a compression bending stress:

$$\sigma_{\text{comb},K} = \frac{P}{96\ \text{mm}^2} - \frac{P}{4.73043\ \text{mm}^2} = P\left[\frac{1}{96\ \text{mm}^2} - \frac{1}{4.73043\ \text{mm}^2}\right] = -\frac{P}{4.97560\ \text{mm}^2}$$

The negative sign indicates that the combined stress at K is a compression normal stress. A second possible value for P can be derived from the following expression. The negative signs can be omitted here because we are interested only in the magnitude of P.

$$\boxed{\frac{P}{4.97560\ \text{mm}^2} \leq 108\ \text{MPa} = 108\ \text{N/mm}^2 \qquad \therefore P \leq 537\ \text{N}} \tag{b}$$

Controlling Clamping Force

The allowable clamping force is the lesser of the two values obtained from Eqs. (a) and (b). For this clamp, the maximum allowable clamping force is $P = 537$ N. **Ans.**