Matlab Sheet 3

Plotting in Matlab

1. a. Estimate the roots of the following equation

$$x^3 - 3x^2 + 5x\sin\left(\frac{\pi x}{4} - \frac{5\pi}{4}\right) + 3 = 0$$

by plotting the equation.

b. Use the estimates found in part a to find the roots more accurately with the `fzero` function.

2. Plot columns 2 and 3 of the following matrix A versus column 1. The data in column 1 are time (seconds). The data in columns 2 and 3 are force (newtons).

$$\mathbf{A} = \begin{bmatrix} 0 & -7 & 6 \\ 5 & -4 & 3 \\ 10 & -1 & 9 \\ 15 & 1 & 0 \\ 20 & 2 & -1 \end{bmatrix}$$

3. Many applications use the following "small angle" approximation for the sine to obtain a simpler model that is easy to understand and analyze. This approximation states that $\sin x \approx x$, where x must be in radians. Investigate the accuracy of this approximation by creating three plots. For the first, plot sin x and x versus x for $0 \le x \le 1$. For the second, plot the approximation error sin x - x versus x for $0 \le x \le 1$. For the third, plot the relative error [sin(x) - x]/sin(x) versus x for $0 \le x \le 1$. How small must x be for the approximation to be accurate within 5 percent?

4. You can use trigonometric identities to simplify the equations that appear in many applications. Confirm the identity $\tan(2x) = 2\tan x/(1 - \tan^2 x)$ by plotting both the left and the right sides versus x over the range $0 \le x \le 2\pi$.

5. The following functions describe the oscillations in electric circuits and the vibrations of machines and structures. Plot these functions on the same plot. Because they are similar, decide how best to plot and label them to avoid confusion.

$$x(t) = 10\, e^{-0.5t} \sin(3t + 2)$$
$$y(t) = 7\, e^{-0.4t} \cos(5t - 3)$$

6. In certain kinds of structural vibrations, a periodic force acting on the structure will cause the vibration amplitude to repeatedly increase and decrease with time. This phenomenon, called beating, also occurs in musical sounds. A particular structure's displacement is described by

$$y(t) = \frac{1}{f_1^2 - f_2^2}[\cos(f_2 t) - \cos(f_1 t)]$$

where y is the displacement in mm and t is the time in seconds. Plot y versus t over the range 0 ≤ t ≤ 20 for f_1 = 8 rad/sec and f_2 = 1 rad/sec. Be sure to choose enough points to obtain an accurate plot.

7. Oscillations in mechanical structures and electric circuits can often be

$$y(t) = e^{-t/\tau} \sin(\omega t + \phi)$$

where t is time and ω is the oscillation frequency in radians per unit time. The oscillations have a period of $2\pi/\omega$, and their amplitudes decay in time at a rate determined by τ, which is called the time constant. The smaller τ is, the faster the oscillations die out.

a. Use these facts to develop a criterion for choosing the spacing of the t values and the upper limit on t to obtain an accurate plot of y(t).

(Hint: Consider two cases: $4\tau > 2\pi/\omega$ and $4\tau < 2\pi/\omega$)

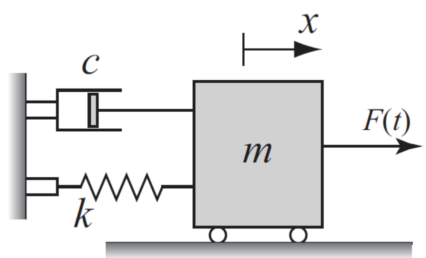
b. Apply your criterion, and plot y(t) for $\tau = 10$, $\omega = \pi$ and $\phi = 2$.

c. Apply your criterion, and plot y(t) for $\tau = 0.1$, $\omega = \pi$ and $\phi = 2$.

8. The vibrations of the body of a helicopter due to the periodic force applied by the rotation of the rotor can be modeled by a frictionless spring-mass-damper system subjected to an external periodic force. The position $x(t)$ of the mass is given by the equation:

$$x(t) = \frac{2f_o}{\omega_n^3 - \omega^3} \sin\left(\frac{\omega_n - \omega}{2}\ t\right) \sin\left(\frac{\omega_n + \omega}{2}\ t\right)$$

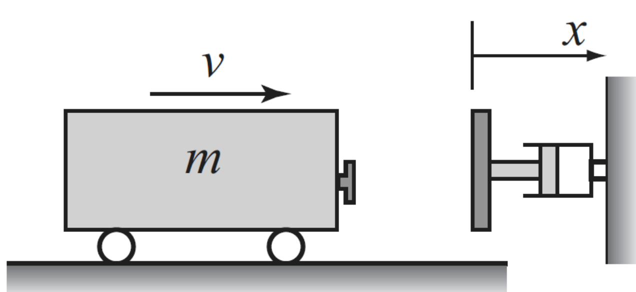
where $F(t) = F_0\ \sin \omega t$, and $f_0 = F_0/m$, ω is the frequency of the applied force, and ω_n is the natural frequency of the helicopter. When the value of ω is close to the value of ω_n, the vibration consists of fast oscillation with slowly changing amplitude called beat. Use $f_0 = 12$ N/kg, ω_n=10 rad/s, and ω=12 rad/s to plot $x(t)$ as a function of t for $0 \le t \le 10$ s.



9. A railroad bumper is designed to slow down a rapidly moving railroad car. After a 20,000 kg railroad car traveling at 20 m/s engages the bumper, its displacement x (in meters) and velocity v (in m/s) as a function of time t (in seconds) is given by:

$$x(t) = 4.219(e^{-1.58t} - e^{-6.32t}) \quad \text{and} \quad v(t) = 26.67e^{-6.32t} - 6.67e^{-1.58t}$$

Plot the displacement and the velocity as a function of time for $0 \le t \le 4$ s. Make two plots on one page.



10. The curvilinear motion of a particle is defined by the following parametric equations:

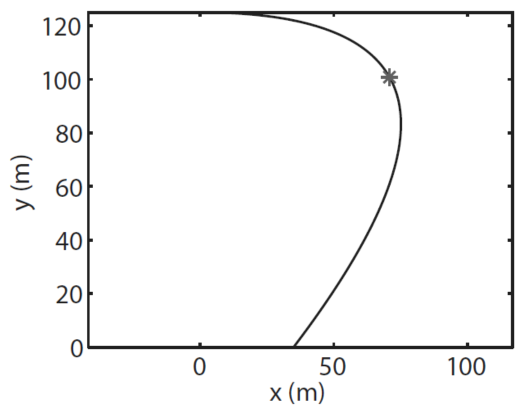
$$x = 52t - 9t^2 \text{ m and } y = 125t - 5t^2 \text{m}$$

The velocity of the particle is given by

$$v = \sqrt{v_x^2 + v_y^2}$$

, where $v_x = \frac{dx}{dt}$ and $v_y = \frac{dy}{dt}$.

For 0 ≤ t ≤ 5 s make one plot that shows the position of the particle (y versus x), and a second plot (on the same page) of the velocity of the particle as a function of time. In addition, by using MATLAB's min function, determine the time at which the velocity is the lowest, and the corresponding position of the particle. Using an asterisk marker, show the position of the particle in the first plot. For time use a vector with spacing of 0.1 s.

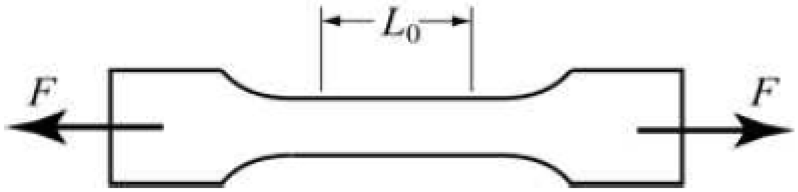


11. The demand for water during a fire is often the most important factor in the design of distribution storage tanks and pumps. For communities with populations less than 200,000, the demand Q (in gallons/min) can be calculated by:

$$Q = 1020\sqrt{P}(1 - 0.01\sqrt{p})$$

where P is the population in thousands. Plot the water demand Q as a function of the population P (in thousands) for 0 ≤ P ≤ 200. Label the axes and provide a title for the plot.

12. A In a typical tension test a dog bone shaped specimen is pulled in a machine.



During the test, the force F needed to pull the specimen and the length L of a gauge section are measured. This data is used for plotting a stress-strain diagram of the material. Two definitions, engineering and true, exist for stress and strain. The engineering stress and strain are defined by

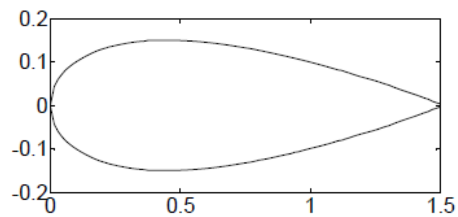
$\sigma_e = \frac{F}{A_0}$ and $\varepsilon_e = \frac{L - L_o}{L_o}$ where L_o and A_o are the initial gauge length and the initial cross-sectional area of the specimen, respectively. The true stress σ_t and strain ε_t are defined by $\sigma_t = \frac{F}{A_0}\frac{L}{L_o}$ and $\varepsilon_t = \ln\frac{L}{L_o}$.

The following are measurements of force and gauge length from a tension test with an aluminum specimen. The specimen has a round cross section with radius 6.4 mm (before the test). The initial gauge length is L_o=25 mm. Use the data to calculate and generate the engineering and true stress-strain curves, both on the same plot. Label the axes and use a legend to identify the curves. Units: When the force is measured in newtons (N) and the area is calculated in m^2, the unit of the stress is pascals (Pa).

F (N)	0	13,031	21,485	31,963	34,727	37,119	37,960	39,550
L (mm)	25.4	25.474	25.515	25.575	25.615	25.693	25.752	25.978
F (N)	40,758	40,986	41076	41,255	41,481	41,564		
L (mm)	26.419	26.502	26.600	26.728	27.130	27.441		

13. The shape of a symmetrical four-digit NACA airfoil is described by the equation

$$y = \pm\frac{tc}{0.2}\left[0.2969\sqrt{\frac{x}{c}} - 0.1260\frac{x}{c} - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1015\left(\frac{x}{c}\right)^4\right]$$



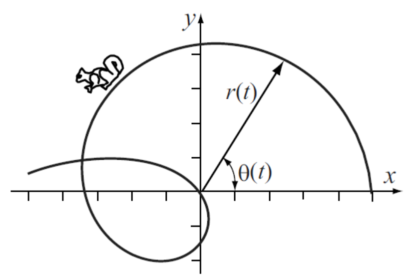
where c is the cord length and t is the maximum thickness as a fraction of the cord length (tc= maximum thickness). Symmetrical four-digit NACA airfoils are designated NACA 00XX, where XX is 100t (i.e., NACA 0012 has t=0.12). Plot the shape of a NACA 0020 airfoil with a cord length of 1.5 m.

14. The position as a function of time of a squirrel running on a grass field is given in polar coordinates by:

$$r(t) = 25 + 30[1 - e^{\sin(0.07t)}] \text{ m}$$

$$\theta(t) = 2\pi(1 - e^{-0.2t})$$

Plot the trajectory (position) of the squirrel for $0 \le t \le 20$ s.



15. Obtain the surface and contour plots for the function

$$z = x^2 - 2xy + 4y^2$$

showing the minimum at $x = y = 0$.

16. A square metal plate is heated to 80°C at the corner corresponding to $x = y = 1$. The temperature distribution in the plate is described by

$$T = 80\, e^{-(x-1)^2}\ e^{-3(y-1)^2}$$

Obtain the surface and contour plots for the temperature. Label each axis. What is the temperature at the corner corresponding to $x = y = 0$?