OPTIMUM DESIGN

Dr. / Ahmed Nagib Elmekawy

Lecture 2 – Problem Formulation

Design of a can - Summary

Min $f(D,H) = \pi D^2/2 + \pi DH$ (cm^2)

Subject to:

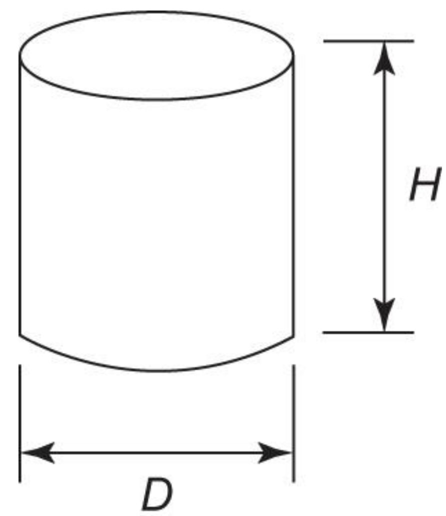
$(\pi D^2/4)H \geq 400$ (cm^3)

$3.5 \leq D$

$D \leq 8$

$8 \leq H$

$H \leq 18$



Problem Formulation

The *importance of properly formulating* a design optimization problem must be stressed because the optimum solution will be only as good as the formulation. For example, if we forget to include a critical constraint in the formulation, the optimum solution will most likely violate it. Also, if we have too many constraints, or if they are inconsistent, there may be no solution for the problem. However, once the problem is properly formulated, good software
is usually available to solve it.
It is important to note that the process of developing a proper formulation for optimum design of practical problems is iterative in itself. Several iterations usually are needed to revise the formulation before an acceptable one is finalized.

Standard Design Optimzation Model

Find $\mathbf{x}^*$ such that

Design Variables

$$MINIMIZE: \quad f(\mathbf{x})$$

Objective function

Subject To :

$$h_j(\mathbf{x}) = 0 \qquad j = 1 \ldots p$$

$$g_i(\mathbf{x}) \le 0 \qquad i = 1 \ldots m$$

$$x_i^{(L)} \le x_i \le x_i^{(U)} \qquad i = 1 \ldots n$$

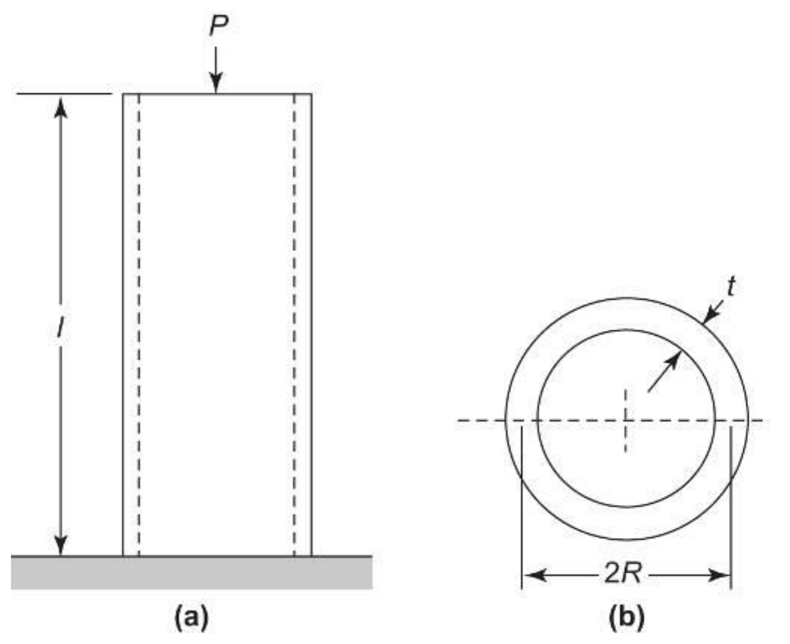
Constraints

Min Weight Column

Step 1. Describe problem

Separate into one-line phrases

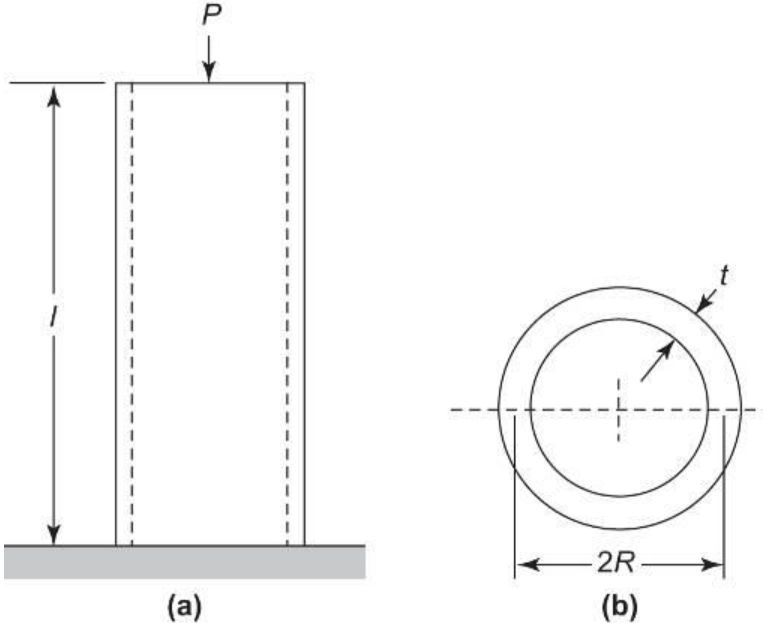
Min mass tubular column
Length l
Supports load P
No buckling
No overstressing
“fixed” end-conditon



Min Weight Column

Step 2. Collect info

Diagram, handbooks, notes



$$mass = density(volume)$$

$$= density(area)length$$

$$area \approx circum(thick)$$

$$area = 2\pi Rt$$

$$mass = \rho(2\pi Rt)l$$

$$\sigma \leq \sigma_a$$

$$\frac{P}{A} = \frac{P}{2\pi Rt} \leq \sigma_a$$

Crush

$$P \leq P_{cr}$$

$$P \leq \frac{\pi^3 E R^3 t}{4l^2}$$

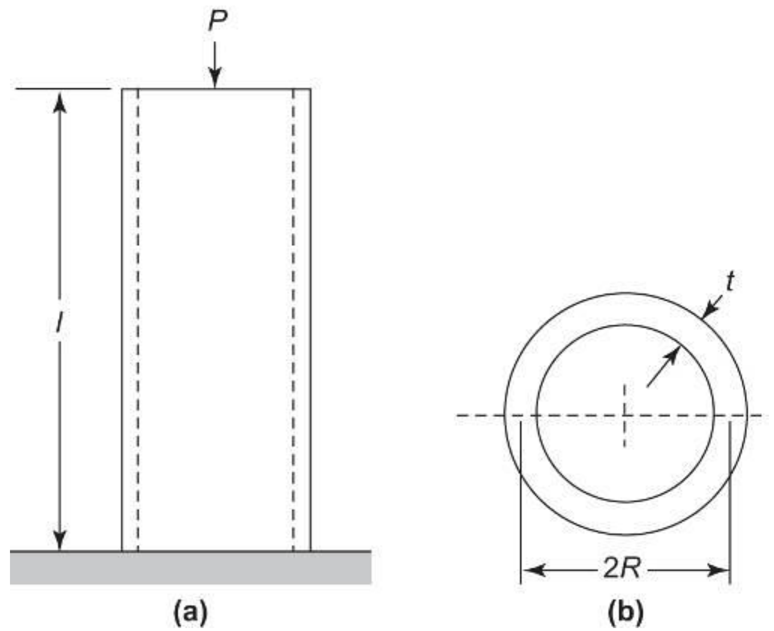
Buckle

Min Weight Column

Step 3. Define DVs

“Form” which influences behavior

Name, symbol, units



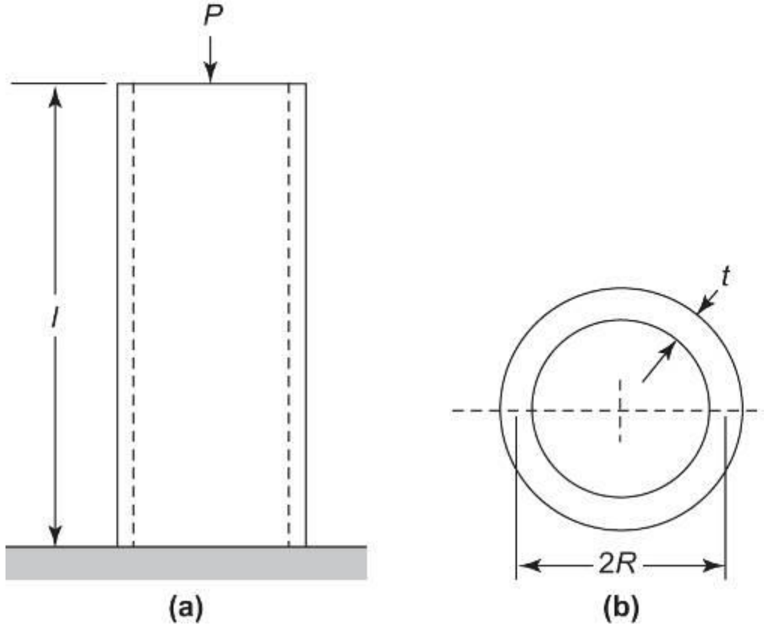
$$\mathbf{x} = [x_1 . x_2] = [R, t]$$

Name	Radius	Thickness
Symbol	R	t
Units	inches	Inches

Min Weight Column

Step 4. Determine objective function performance criterion, f(DVs)

$$MIN \quad f(\mathbf{x}) = \rho(2\pi R t)l$$



Min Weight Column

Step 5. Formulate constraints

Laws of nature, man, economics

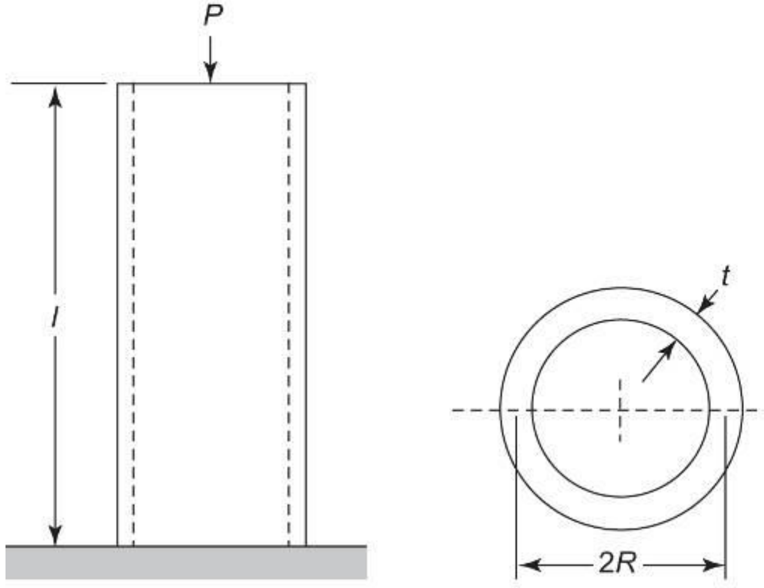
Failure modes

$$\frac{P}{2\pi Rt} \leq \sigma_a$$

$$P \leq \frac{\pi^3 E R^3 t}{4l^2}$$

$$R_{\min} \leq R \leq R_{\max}$$

$$t_{\min} \leq t \leq t_{\max}$$



(a) (b)

PDPs for this problem?

Min Weight Column - Summary

$$MIN \quad f(\mathbf{x}) = \rho(2\pi Rt)l$$

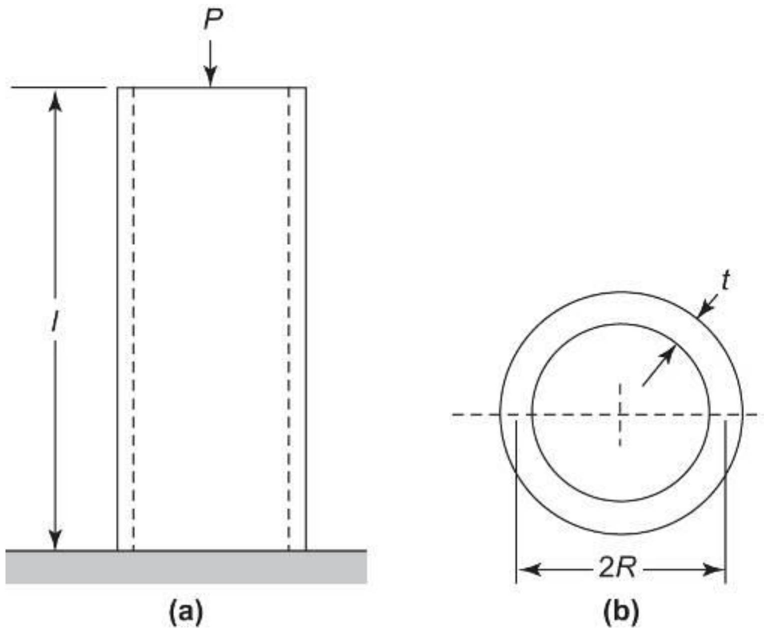
Subject to:

$$\frac{P}{2\pi Rt} \le \sigma_a$$

$$P \le \frac{\pi^3 E R^3 t}{4l^2}$$

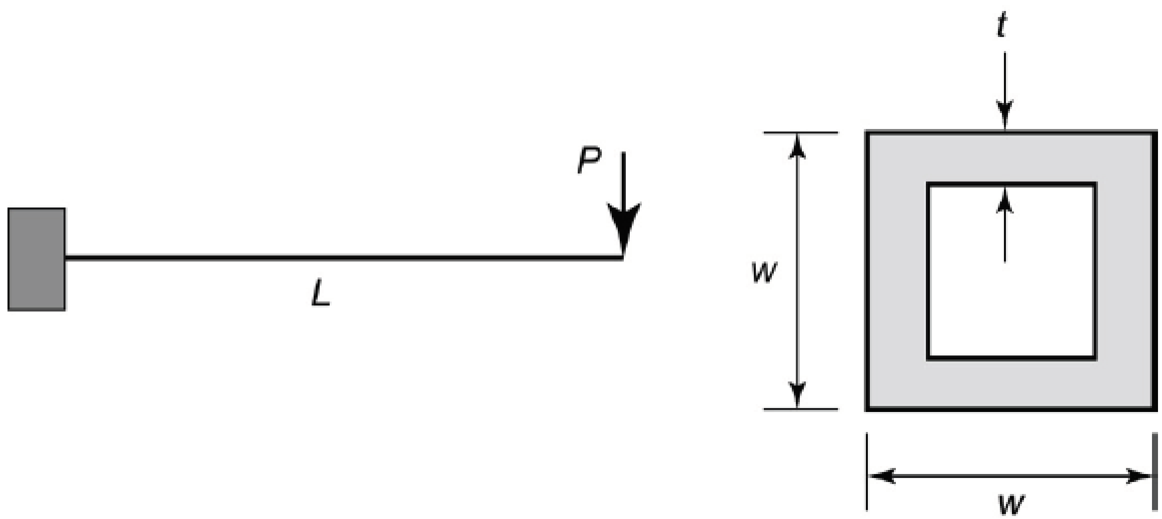
$$R_{\min} \le R \le R_{\max}$$

$$t_{\min} \le t \le t_{\max}$$



How many equality or inequality eqns?

Design of a Cantilever Beam



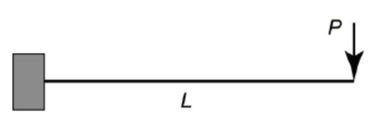
Design of a Cantilever Beam

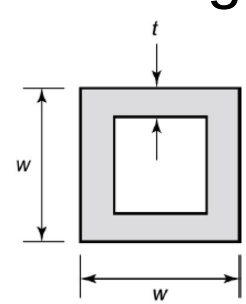
Step 1. Describe problem (restate w/bullets)

- hollow square cross- section
- load of 20 kN at its end
- The beam, made of steel, is 2 m Long
- the material should not fail under the action of the load
- deflection of the free end should be no more than1 cm.
- The width-to-thickness ratio for the beam should be no more than 8 to avoid local buckling of the walls.
- A minimum-mass beam is desired. The width and thickness of the beam must be within the following limits:

$$60 \le \text{width} \le 300 \text{ mm}$$

$$3 \le \text{thickness} \le 15 \text{ mm}$$





Design of a Cantilever Beam

Step 2. Data and Information Collection

The information needed for this problem includes expressions for bending and shear stresses, and the expression for the deflection of the free end.

$$A = w^2 - (w-2t)^2 = 4t(w-t), \mathrm{mm}^2$$

$$I = \frac{1}{12} w \times w^3 - \frac{1}{12}(w-2t) \times (w-2t)^3 = \frac{1}{12} w^4 - \frac{1}{12}(w-2t)^4, \mathrm{mm}^4$$

$$\mathrm{Q} = \frac{1}{2} w^2 \times \frac{w}{4} - \frac{1}{2}(w-2t)^2 \times \frac{(w-2t)}{4} = \frac{1}{8} w^3 - \frac{1}{8}(w-2t)^3, \mathrm{mm}^3$$

$$M = PL, \mathrm{N/mm}$$

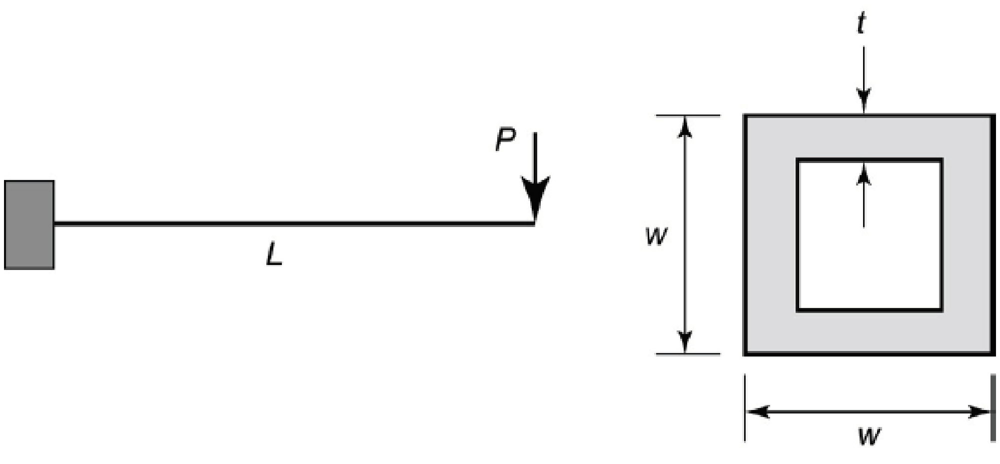
$$V = P, \mathrm{N}$$

Design of a Cantilever Beam

Step 3. Define DVs

w = outside width (depth) of the section, mm

t = wall thickness, mm

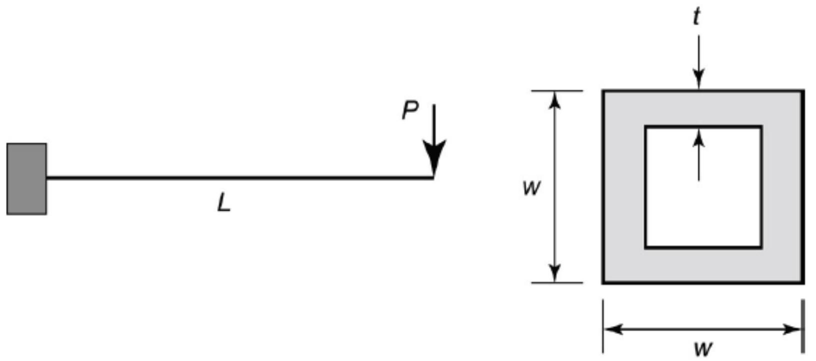


Design of a Cantilever Beam

Step 4. Determine objective function

- The objective is to design a minimum-mass cantilever beam. Since the mass is proportional to the cross-sectional area of the beam, the objective function for the problem is taken as the cross-sectional area which is to be minimized:

$$f(w,t) = A = 4t(w-t),\ \mathrm{mm}^2$$



Design of a Cantilever Beam

Step 5. Formulate constraints

Bending stress constraint: $\sigma \le \sigma_a$

$$\frac{PLw}{2I} \le \sigma_a$$

Shear stress constraint: $\tau \le \tau_a$

$$\frac{PQ}{2It} \le \tau_a$$

Deflection constraint: $q \le q_a$

$$\frac{PL^3}{3EI} \le q_a$$

Width–thickness restriction: $\dfrac{w}{t} \le 8$

$$w \le 8t$$

Design of a Cantilever Beam

Step 5. Formulate constraints

Dimension restrictions:

$$60 \le w, \text{ mm}; \;\; w \le 300, \text{ mm}$$

$$3 \le t, \text{ mm}; \;\; t \le 15, \text{ mm}$$

Design Variables

- Generally, the design variables should be independent of each other. If they are not, there must be some equality constraints between them (explained later).
- A minimum number of design variables is required to properly formulate a design optimization problem.
- As many independent parameters as possible should be designated as design variables at the problem formulation phase. Later on, some of these variables can be assigned fixed numerical values.
- A numerical value should be given to each identified design variable to determine if a trial design of the system is specified.

Optimization Criterion

There can be many feasible designs for a system, and some are better than others. The question is how do we quantify this statement and designate a design as better than another. For this, we must have a criterion that associates a number with each design. This way, the merit of a given design is specified. The criterion must be a scalar function whose numerical value can be obtained once a design is specified; that is, it must be a function of the design variable vector x. Such a criterion is usually called an objective function for the optimum design problem, and it needs to be maximized or minimized depending on problem requirements.

Optimization Criterion

It is emphasized that a valid objective function must be influenced directly or indirectly by the variables of the design problem; otherwise, it is not a meaningful objective function.

The selection of a proper objective function is an important decision in the design process.

Some common objective functions are cost

Optimization Criterion

Some common objective functions are

- cost (to be minimized)
- profit (to be maximized)
- weight (to be minimized)
- energy expenditure (to be minimized)
- ride quality of a vehicle (to be maximized).

Optimization Criterion

In many situations, an obvious objective function can be identified. For example, we always want to minimize the cost of manufacturing goods or maximize return on investment. In some situation

ns, two or more objective functions may be identified. For example,

we may want to minimize the weight of a structure and at the same time minimize the deflection or stress at a certain point. These are called multi-objective design optimization problems For some design problems, it is not obvious what the objective function should be or how it should be expressed in terms of the design variables. Some insight and experience may be needed to identify a proper objective function for a particular design problem.

Optimization Criterion

For example, consider the optimization of a passenger car. What are the design variables? What is the objective function, and what is its functional form in terms of the design variables?

This is a practical design problem that is quite complex. Usually, such problems are divided into several smaller subproblems and each one is formulated as an optimum design problem.

For example, design of a passenger car can be divided into a number of optimization subproblems involving the trunk lid, doors, side panels, roof, seats, suspension system, transmission system, chassis, hood, power plant, bumpers, and so on. Each subproblem is now manageable and can be formulated as an optimum design problem.

Constrains

Most realistic systems must be designed and fabricated with the given resources and must meet performance requirements. For example, structural members should not fail under normal operating loads. The natural frequencies of a structure must be different from the operating frequency of the machine it supports; otherwise, resonance can occur and cause catastrophic failure. Members must fit into the available space, and so on.

These constraints, as well as others, must depend on the design variables, since only then do their values change with different trial designs; that is, a meaningful constraint must be a function of at least one design variable.

Constrains

Linear and Nonlinear Constraints

Many constraint functions have only first-order terms in design variables. These are called linear constraints.

Linear-programming problems have only linear constraints and objective functions.

More general problems have nonlinear objective function and/or constraint functions. These are called nonlinear-programming problems.

Constrains

Feasible Design

The design of a system is a set of numerical values assigned to the design variables (ie, a particular design variable vector x). Even if this design is absurd (eg, negative radius) or inadequate in terms of its function, it can still be called a design. Clearly, some designs are useful and others are not. A design meeting all requirements is called a feasible design (acceptable or workable). An infeasible design (unacceptable) does not meet one or more of the requirements.

Constrains

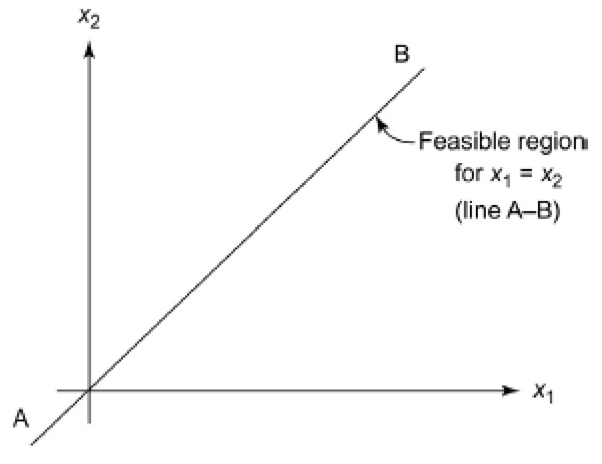
Equality and Inequality Constraints

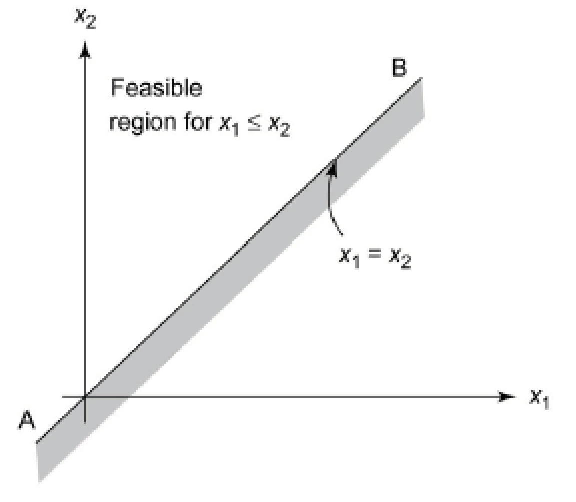
Design problems may have equality as well as inequality constraints. The problem description should be studied carefully to determine which requirements need to be formulated as equalities and which ones as inequalities. For example, a machine component may be required to move precisely by Δ to perform the desired operation, so we must treat this as an equality constraint. A feasible design must satisfy precisely all equality constraints. Also, most design problems have inequality constraints.

Constrains

Equality and Inequality Constraints

To illustrate the difference between equality and inequality constraints, we consider a constraint written in both equality and inequality forms. Fig. shows the equality constraint $x1 = x2$. Feasible designs with respect to the constraint must lie on the straight line A–B.
However, if the constraint is written as an inequality $x1 \leq x2$, the feasible region is much larger, as shown in Fig. Any point on the line A–B or above it gives a feasible design.





Satisfy an equality constraint?

An **equality constraint** is satisfied iff $h(\mathbf{x}) = 0$

For example, let $\mathbf{x}^{(1)} = [1, 1]^T$

$$h(\mathbf{x}) = x_1 - x_2 = 0$$

$$h(\mathbf{x}) = 1 - 1 = 0$$

$$h(\mathbf{x}) = 0$$

Satisfy an inequality constraint

An **inequality constraint** is satisfied iff $g(\mathbf{x}) \le 0$,
i.e. when <u>either</u> $g(\mathbf{x}) < 0$ or $g(\mathbf{x}) = 0$.

Case A: $g(\mathbf{x}) < 0$

For example, let $\mathbf{x}^{(1)} = [1, 1]^T$, then for

$$g(\mathbf{x}) = x_1 + x_2 - 5 \le 0$$

$$g(\mathbf{x}) = 1 + 1 - 5 = -3$$

$$g(\mathbf{x}) = -3 < 0$$

When $g(\mathbf{x}) < 0$ the constraint is said to be INACTIVE or NONBINDING

<u>Active</u> inequality constraint

Case B: $g(\mathbf{x}) = 0$

For example let $\mathbf{x}^{(2)} = [2.5, 2.5]^T$, then for

$$g(\mathbf{x}) = x_1 + x_2 - 5 \leq 0$$

$$g(\mathbf{x}) = 2.5 + 2.5 - 5 = 0$$

$$g(\mathbf{x}) = 0$$

When g(x) = 0 the constraint is said to be ACTIVE or BINDING

Violated inequality constraint

Case C: $g(\mathbf{x}) > 0$

For example let $\mathbf{x}^{(3)} = [3, 3]^T$, then for

$$g(\mathbf{x}) = x_1 + x_2 - 5 \leq 0$$

$$g(\mathbf{x}) = 3 + 3 - 5 = 1$$

$$g(\mathbf{x}) = 1, \text{therefore}$$

$$g(\mathbf{x}) > 0$$

When g(x) > 0, the constraint is VIOLATED

Constraint Activity/Condition

Constraint Type	**Satisfied**	**Violated**
Equality	$h(\mathbf{x}) = 0$	$h(\mathbf{x}) \neq 0$
Inequality	$g(\mathbf{x}) < 0 \quad \text{inactive}$ $g(\mathbf{x}) = 0 \quad \text{active}$	$g(\mathbf{x}) > 0$

Feasible Design

The design of a system is a set of numerical values assigned to the design variables (ie, a particular design variable vector x). Even if this design is absurd (eg, negative radius) or inadequate in terms of its function, it can still be called a design. Clearly, some designs are useful and others are not. A design meeting all requirements is called a feasible design (acceptable or workable). An infeasible design (unacceptable) does not meet one or more of the requirements.

Feasible Designs

feasible design - A design candidate that meets design specifications and/or satisfies design constraints.

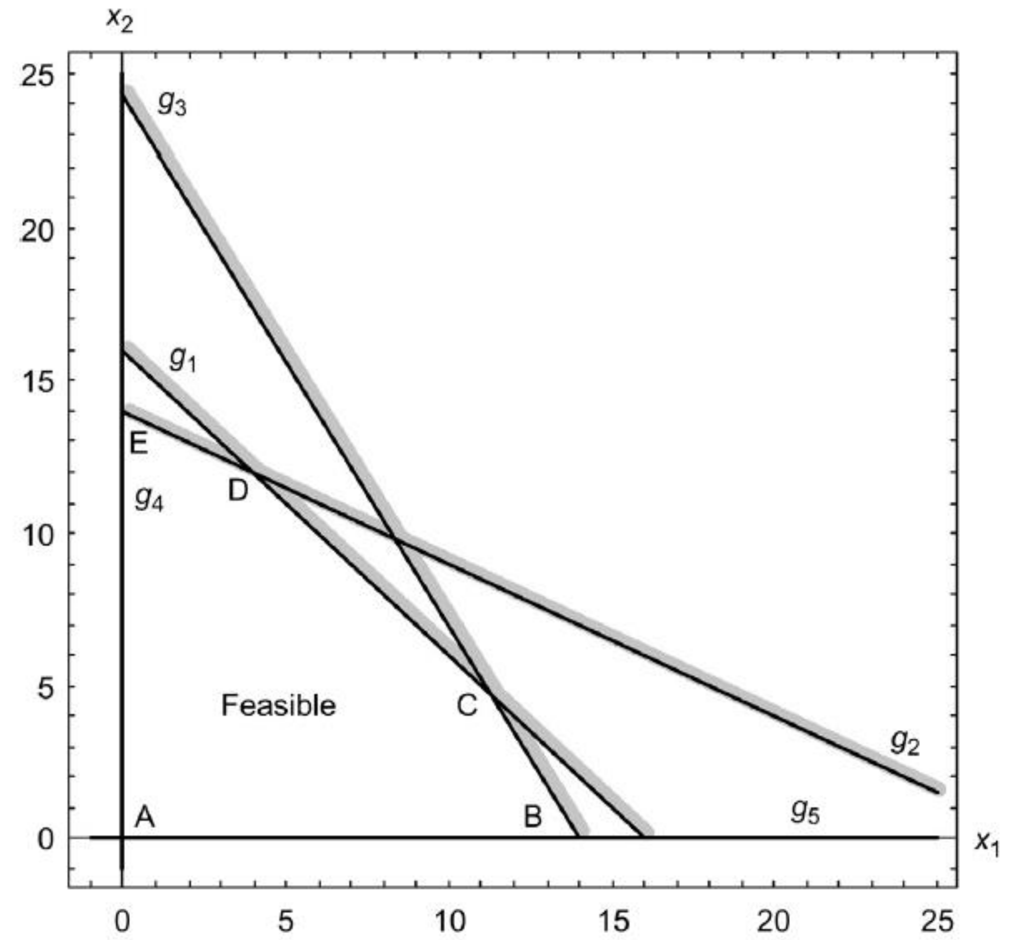
feasible design - a point in the design space that satisfies all constraints.

feasible design region (i.e. feasible design space) – the set of points in the design space that satisfies all constraints.

feasible region = feasible design space

When a point violates any constraint it is said to be **INFEASIBLE.**

Feasible Designs



Feasible region for the profit maximization problem ABCDE.