SAMPLE PROJECTS

SAMPLE PROJECT 1: LINEAR PROGRAMMING

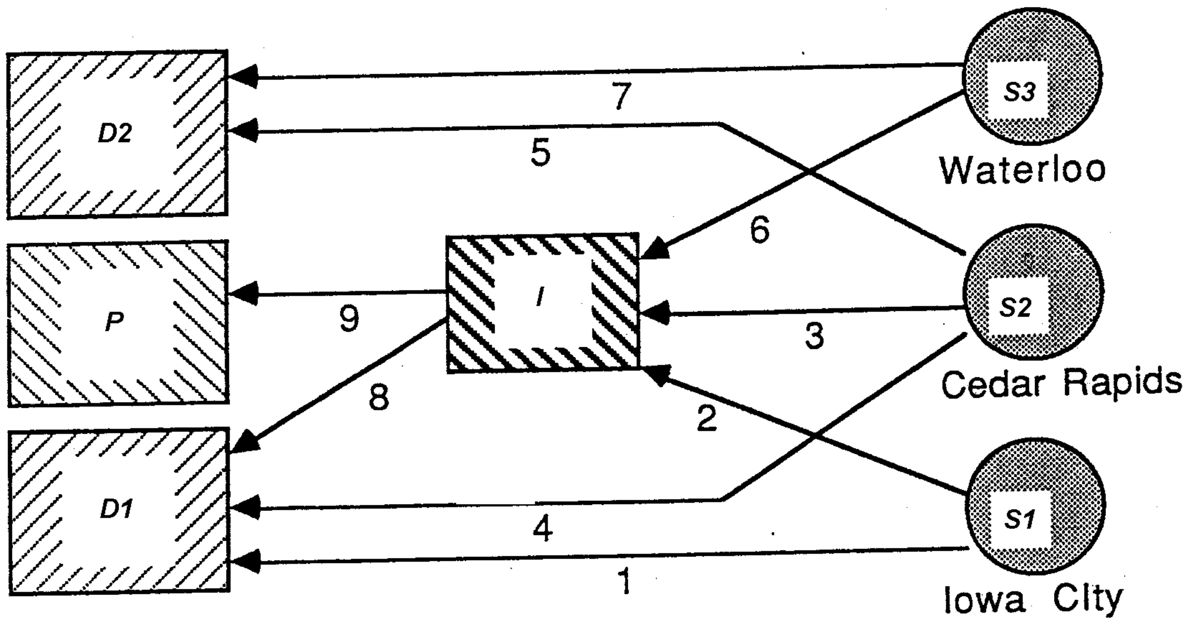
Solid Waste Management Plan

A residential solid waste management plan is required for the Iowa City-Cedar Rapids-Waterloo region. Each of these cities generates quantities of solid waste that must be disposed of in a landfill or processed at an intermediate facility. At the intermediate facility, useful material, such as dry fuel, may be recovered from the solid waste and then sold, thereby reducing the overall disposal cost. Residue of the intermediate facility must be shipped to a landfill.

A proposed system is shown in the figure. In addition to the three sources $S1$, $S2$ and $S3$, representing Iowa City, Cedar Rapids and Waterloo, respectively, there is one intermediate facility, I, that converts 60% of the residential waste into a dry-fuel product and can process 5000 tons per week, an electric generating station, P, that can use up to 4500 tons of dry fuel per week, and two landfills, $D1$ and $D2$, that can accept up to 10,000 tons of waste per week each. There are nine transportation links in the system. Data on quantities and costs are listed in Tables 1, 2 and 3. The disposal/processing costs in Table 3 apply to all incoming material at a facility and include capital and operating costs less revenues from the sale of useful material.

Answers to the following questions are needed for the evaluation of the proposed system. You must be able to answer all the questions without repeating the analysis performed to answer the first question. Do not repeat the analysis to answer the remaining questions. If you are unable to answer a question based on the results obtained in question 1, state that you are unable to answer the question and why.

1. What is the optimum operating plan?
2. What is the maximum operating cost at the intermediate processing facility that will allow its operation at capacity?
3. What is the consequence of enlarging the capacity of that facility to 9000 tons per week?
4. Which of the following increases in capacity will result in the most economical system and why or why not?
    a. The intermediate facility to 9000 tons per week.
    b. The landfill, $D1$, to 15,000 tons per week.
    c. The landfill, $D2$, to 18,000 tons per week.
    d. The power plant to 9000 tons per week.



Solid Waste Disposal System

Table 1. Solid Waste Quantities

Source		Amount, tons/week
S1	Iowa City	4800
S2	Cedar Rapids	7900
S3	Waterloo	9700

Table 2. Transportation Costs

Link	Cost, $/ton
1	2.00
2	2.50
3	2.70
4	3.50
5	3.50
6	3.00
7	2.10
8	1.00
9	0.20

Table 3. Disposal/Processing Costs

Facility		Cost, $/ton
I	Intermediate	1.90
P	Power plant	−0.80
*D*1	Dump I	
	Unprocessed waste	3.40
	Residue from I	1.10
*D*2	Dump 2	3.10

SAMPLE PROJECT 2: LP

Project No. 3 Air Pollution Control Model

Electric power generating plants have been criticized for contributing excessively to air pollution problems. The emissions from plant smokestacks contain pollutants of various kinds that are considered harmful to the environment. As a result, power plants are subject to regulations governing the amounts of specific pollutants that are allowed in the stack emissions.

A steam power plant has a choice of burning coal, oil and natural gas at the same time and in any combination to generate power. The fuels must provide a heat input to the plant of 40×10^{8} BTU/hr to generate the required power out.

Air pollution criteria must be met, and so choice of fuel is not based only on cost and efficiency. The following table gives the necessary data for cost in $per ton, heating value in BTU per pound (lb) and pollutants produced in lb per ton for each of the fuels. The maximum allowable limit for each of the pollutants in lb/hr is also given in the table. Note that standard ton is used; i.e., 1 ton = 2000 lb

	Coal	**Oil**	**Gas**	**Maximum Allowed**
Hydrocarbons	1 lb/ton	2 lb/ton	1 lb/ton	100 lb/hr
NO, NO_2, NO_3	1 lb/ton	1 lb/ton	1 lb/ton	100 lb/hr
SO_2	140 lb/ton	80 lb/ton	1 lb/ton	2,000 lb/hr
CO	100 lb/ton	90 lb/ton	90 lb/ton	20,000 lb/hr
Particulate	6 lb/ton	2 lb/ton	1 lb/ton	80 lb/hr
Cost	$60/ton	$100/ton	$160/ton	
Heating Value	14,000 BTU/lb	19,000 BTU/lb	22,000 BTU/lb	

An electrostatic precipitator can also be installed to remove particulate matter for $5 per pound of particulate removed. This could be a cost-effective way to remove particulate matter, and therefore needs to be considered in the formulation of the problem.

Using Excel Solver to generate your solution, address the following questions in your report:

a. Formulate the problem and determine the <u>fuel usage policy on per hour basis</u> that minimizes the cost of fuels and meets the air pollution requirements.
b. What is the consequence of an increase in the price of gas to $170/ton?
c. What is the consequence of reducing the permissible SO_2 output to 1500 lb/hr to minimize the acid rain problem?

For parts b and c, predict the results using the sensitivity information for the original results obtained in part a, and then verify your results by re-solving the problem with the modified data using the program (Hint: Use the allowable ranges in the Sensitivity Sheet of the Solver output).

Note: You must define the design variables very precisely and watch the units of various quantities carefully.

Project Reporting Requirements

Submit your report using the provided Microsoft Word .doc file. Remember to download the file and before working on it to rename the file including your first and last name. For example, “project1.doc” is the name of the file that you download. Assuming that “Aye Ten” is a student name, the renamed file would be “project1_AyeTen.doc”.

Your report must be formatted properly and carefully, and include the following 4 parts:

1) Complete problem formulation process: problem description, data/information, clear definition of design variables, cost function, and constraints.
2) Solution for the parts a, b and c, and verification of the predictions in parts b and c.
3) A brief discussion and conclusions for the project.
4) Submit the Excel file for the problem which should be **organized clearly** and in a **readable format.**

Project Grading Rubric: Graded based on 50 points

Report attributes ↓	**Meets all expectations**	**Partially meets expectations**	**Below expectations**
Formulation of the problem (20 points)	***(20 points)*** Complete and clear presentation that includes complete problem formulation process with proper formatting.	***(10 points)*** Complete, but sloppy presentation that either includes poorly defined design variables and/or cost function and/or constraints	***(5 points)*** Incomplete formulation that either lacks clear definition of design variables and/or cost function and/or constraints
Problem solution (10 points)	***(10 points)*** *Complete solution, active constraints*	***(6 points)*** Partial solution	***(2 points)*** Incorrect solution or no solution
Sensitivity analysis (10 points)	***(10 points)*** Solution based on sensitivity analysis and its verification.	***(6 points)*** Incomplete sensitivity solution and/or missing verification.	***(2 points)*** Incorrect sensitivity solution. Missing verification
Excel sheet for the problem (10 points)	***(10 points)*** Nicely organized Excel sheet in a clear readable and logical format	***(5 points)*** Excel sheet incomplete and/or unreadable and not formatted well	***(0 points)*** Not included

Overall Score: 1) Best effort: 45 or more**; 2) Acceptable effort:** 35 to 44**; 3); Needs improvement:** 20 to 34

Solution: Air Pollution Control Model

Design Variables

x_1 = tons of coal used/hr

x_2 = tons of oil used/hr

x_3 = tons of gas used/hr

x_4 = pounds of particulate material removed/hr by the electrostatic precipitator

Cost function

Minimize the cost of fuel use/hr

$$f = 60x_1 + 100x_2 + 160x_3 + 5x_4$$

Constraints

Hydrocarbons:	$x_1 + 2x_2 + x_3 \le 100$
NO, NO_2, NO_3:	$x_1 + x_2 + x_3 \le 100$
SO_2:	$140x_1 + 80x_2 + x_3 \le 2000$
CO:	$100x_1 + 90x_2 + 90x_3 \le 20000$
Particulate:	$6x_1 + 2x_2 + x_3 - x_4 \le 80$
Heat Input:	$(14000x_1 + 19000x_2 + 22000x_3)2000 = 40 \times 10^8$
Nonnegative variables:	$x_i \ge 0; i = 1, 2, 3, 4$

Problem set up for Solver						
Variables	x1	x2	x3	x4	Sum of LHS	RHS Limit
Variable value	11.189056	4.419501	79.97193	75.94528		
Objective function: min	60	100	160	5	=B15*x_1+C15*x_2+D15*x_3+E15*x_4	
Hydrocarbons	1	2	1	0	=B16*x_1+C16*x_2+D16*x_3+E16*x_4	100
NO	1	1	1	0	=B17*x_1+C17*x_2+D17*x_3+E17*x_4	100
SO2	140	80	1	0	=B18*x_1+C18*x_2+D18*x_3+E18*x_4	2000
CO	100	90	90	0	=B19*x_1+C19*x_2+D19*x_3+E19*x_4	20000
Particulate	6	2	1	-1	=B20*x_1+C20*x_2+D20*x_3+E20*x_4	80
Heat input	14	19	22	0	=B21*x_1+C21*x_2+D21*x_3+E21*x_4	2000

Microsoft Excel 15.0 Answer Report
Worksheet: [Project 3_LP-S2016.xlsx]Project 3-S2016
Report Created: 3/14/2016 1:57:42 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0 Seconds.
Iterations: 4 Subproblems: 0
Solver Options
Max Time 100 sec, Iterations 100, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, Solve Without Integer Constraints, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$F\$15	Objective function: min Sum of LHS	5	14288.53034

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$14	x_1	0	11.18905647	Contin
\$C\$14	x_2	0	4.419501929	Contin
\$D\$14	x_3	0	79.97193967	Contin
\$E\$14	x_4	1	75.94528236	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$F\$16	Hydrocarbons Sum of LHS	100	\$F\$16<=\$G\$16	Binding	0
\$F\$17	NO Sum of LHS	95.58049807	\$F\$17<=\$G\$17	Not Binding	4.419501929
\$F\$18	SO2 Sum of LHS	2000	\$F\$18<=\$G\$18	Binding	0
\$F\$19	CO Sum of LHS	8714.135391	\$F\$19<=\$G\$19	Not Binding	11285.86461
\$F\$20	Particulate Sum of LHS	80	\$F\$20<=\$G\$20	Binding	0
\$F\$21	Heat input Sum of LHS	2000	\$F\$21=\$G\$21	Binding	0

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [Project 3_LP-S2016.xlsx]Project 3-S2016
Report Created: 3/14/2016 1:57:42 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$14	x_1	11.18905647	0	60	4.6	42.23434808
$C$14	x_2	4.419501929	0	100	23.98238748	14.375
$D$14	x_3	79.97193967	0	160	1E+30	12.77777778
$E$14	x_4	75.94528236	0	5	0.92	5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$16	Hydrocarbons Sum of LHS	100	-25.79095054	100	18.32280299	4.109589041
$F$17	NO Sum of LHS	95.58049807	0	100	1E+30	4.419501929
$F$18	SO2 Sum of LHS	2000	-0.040336724	2000	1575	1276
$F$19	CO Sum of LHS	8714.135391	0	20000	1E+30	11285.86461
$F$20	Particulate Sum of LHS	80	-5	80	75.94528236	1E+30
$F$21	Heat input Sum of LHS	2000	8.674149421	2000	90.64748201	408.974359

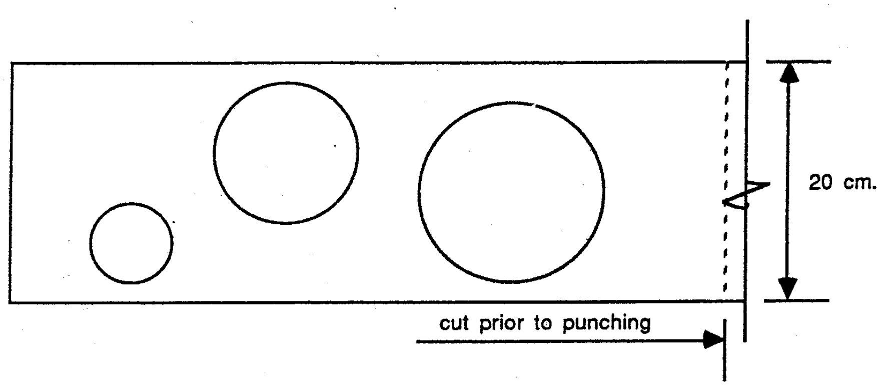
SAMPLE PROJECT 3: NONLINEAR PROGRAMMING

Optimum Use of Sheet Metal

A manufacturer is to produce circular steel end plates for cans for the food industry. Three diameters of cans are involved. The plates will be punched using a set of three circular punches, one punch for each size can, in some optimal configuration on the manufacturer's sheet metal stock. The three plate diameters are 15 cm., 11 cm., and 7 cm. The sheet metal stock is 20 cm. wide and will be cut to the appropriate length prior to punching. The problem becomes one of locating the punches so as to use as little metal stock as possible for each set of three end plates. Some physical limitations exist in order to produce a quality end plate. The punches will not produce a clean punch if the edge of the punch is within 1 cm. of the edge of the stock nor will it produce a clean punch if the edge of the punches are within 0.25 cm. of each other.

Formulate this as an optimal design problem to determine the locations of the three punches and the length of stock required. The formulation must be normalized and in standard form. The program IDESIGN will be used to solve the resulting problem.

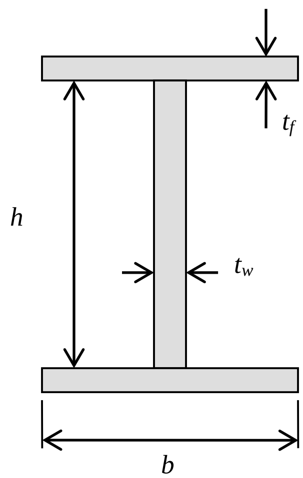
The manufacturer has the option of using a different metal stock which has a slightly greater resistance to damage from the punching. Because of this tolerance, the distance between the edges of the punches may be reduced to 0.12 cm. From the results of the problem solution above compute the savings in metal stock due to the additional tolerance.



SAMPLE PROJECT: PROBLEM FORMULATION

Optimum Design of Compression Members to Satisfy AISC LRFD Manual Requirements

Step 1. Project Statement: Columns made of wide flange sections are used in numerous applications. They need to be designed for minimum cost which is related directly to the mass of the column. Assume a length of the column as L ft, service dead load of P_D kips and live load of P_L kips. The optimum design must satisfy all the requirements of the AISC Load and Resistance Factor Design Manual. Formulate the design problem as an optimization problem using the five step procedure. State all the assumptions made in the formulation. Submit a word-processed report.



Cross-section of wide flange sections or plate girders

Step 2. Data and Information Collection:

Data: L, P_D, P_L, Steel grade (E, F_y, etc)

Expressions:

$$A_g = 2bt_f + ht_w$$

$$I_x = \frac{t_w h^3}{12} + 2\left[\frac{bt_f^3}{12} + bt_f\left(\frac{h}{2} + \frac{t_f}{2}\right)^2\right]$$

$$I_y = \frac{b^3 t_f}{6} + \frac{t_w^3 h}{12}$$

$$r_x = \sqrt{\frac{I_x}{A_g}}$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$

AISC LRFD Specifications:

Design load P_u: $P_u = \max\{1.2P_D + 1.6P_L, 1.4P_D\}$

Column design:

If $\frac{h}{t_w} \le 1.49\sqrt{\frac{E}{F_y}}$ and $\frac{b}{2t_f} \le 0.56\sqrt{\frac{E}{F_y}}$, local instability is prevented.

$$\lambda_c = \max\left\{\frac{K_x L_x}{r_x \pi}\sqrt{\frac{F_y}{E}}, \frac{K_y L_y}{r_y \pi}\sqrt{\frac{F_y}{E}}\right\}$$

$$F_{cr} = \begin{cases} 0.658^{\lambda_c^2} F_y & \text{for } \lambda_c < 1.5 \\ \frac{0.877}{\lambda_c^2} F_y & \text{for } \lambda_c \ge 1.5 \end{cases}$$

Design strength: $\phi_c P_n = \phi_c F_{cr} A_g = 0.85 F_{cr} A_g$

$$P_u \le \phi_c P_n \text{ or } 1.0 - \frac{\phi_c P_n}{P_u} \le 0$$

For wide flange sections, torsional buckling and flexural-torsional buckling need not be considered.

Slenderness ratio (recommendation) (SPEC B7):

$$\frac{KL}{r} \le 200$$

Step 3. Design Variables:

b = flange width, in
t_f = flange thickness
h = web height, in
t_w = web thickness, in

Intermediate Variables: $A_g, I_x, I_y, r_x, r_y, P_n, F_{cr}, \lambda_c$

Step 4. Definition of Objective (Cost) Function

The objective is to minimize the total cost of the wide flange column which can be directly related to the total volume of the column, as

$$f = A_g L, in^3$$

Or, just the area of the cross-section (since length of the column L is fixed), as

$$f = A_g$$

Step 5. Design Constraints:

Design Strength: $1.0 - \dfrac{\phi_c P_n}{P_u} \le 0$

Slenderness Ratio:

$$\frac{K_x L_x}{r_x} \le 200$$

$$\frac{K_y L_y}{r_y} \le 200$$

K_x, L_x, K_y, L_y are determined from the practical situations.

Local Buckling:

$$\frac{h}{t_w} \le 1.49\sqrt{\frac{E}{F_y}}$$

$$\frac{b}{2t_f} \le 0.56\sqrt{\frac{E}{F_y}}$$

Design Variable Bounds:

$$b_{min} \le b \le b_{max}$$

$$t_{f\,min} \le t_f \le t_{f\,max}$$

$$h_{min} \leq h \leq h_{max}$$
$$t_{wmin} \leq t_w \leq t_{wmax}$$

Note that it is permissible to use a cross-sectional shape that does not satisfy the width-thickness ratio requirements (slender sections), but such a member may not be permitted to carry as large a load as the one that does satisfy the requirements. In other words, the design strength could be reduced because of local buckling. In the above formulation, local bucking constraints are imposed and so local bucking is not allowed.

SAMPLE PROJECT: NONLINEAR PROGRAMMING

Optimum Design of Compression Members to Satisfy AISC LRFD Manual Requirements

Consider the design optimization problem that you have formulated earlier. Solve the problem for the data given below. Use a numerical method to solve the problem, such as in Excel, Matlab, Mathematica, etc. Submit a brief word-processed report containing your problem formulation, solution, computer files and a discussion of the solution.

DATA FOR PROJECT:

$$L = 30\ ft;\quad P_D = 2\times10^5\ lbs;\quad P_L = 2\times10^5\ lbs;\quad E = 29\times10^6\ psi;\quad F_y = 5\times10^4\ psi$$

Lower and upper bounds of the design variables:

$$1 \le h \le 20\ in;\quad 1 \le b \le 20\ in;\quad 0.1 \le t_f \le 1\ in;\quad 0.1 \le t_w \le 1\ in;$$

Solution for the problem is given in the below Excel sheets.

Project #1 (A)				
1. Design variable name	**Lower limit**	**Symbol**	**Value**	
web height	1	h	8.5718583504302	20
flange width	1	b	16.1374523605781	20
flange thickness	0.1	tf	0.598277890596105	1
web thickness	0.1	tw	0.238877211066465	1
2. Parameter name	**Symbol**	**Value**	**Units**	
Span length	L	=360	in	
Modulus of elasticity	E	=29000000	psi	
Yield stress	sigma_y	=50000	psi	
Dead load	PD	=200000	lb	
Live load	PL	=200000	lb	
3. Dependent variable name	**Symbol**	**Equation**		
Cross sectional area	Ag	=2*b*tf+h*tw		
Moment of inertia	Ix	=tw*h^3/12+2*(b*tf^3/12+b*tf*(0.5*h+0.5*tf)^2)		
Bending Moment	Iy	=b^3*tf/6+tw^3*h/12		
Radius of gyration w.r.t x axis	rx	=SQRT(Ix/Ag)		
Radius of gyration w.r.t y axis	ry	=SQRT(Iy/Ag)		
K	K	=1		
ramda_c	ramda_c	=MAX(K*L/(rx*PI())*SQRT(sigma_y/E),K*L/(ry*PI())*SQRT(sigma_y/E))		
Fcr	Fcr	=IF(ramda_c<1.5,(0.658^(ramda_c^2))*sigma_y,0.877*sigma_y/ramda_c^2)		
Pu	Pu	=MAX(1.2*PD+1.6*PL,1.4*PD)		
Pn	Pn	=Fcr*Ag		
4. Objective fuction name	**Symbol**	**Equation**	**Units**	
Volume of material	Vol	=Ag*L	in^3	
5. Constraints	**Value/Eq.**	**</>/=**	**Value/Eq.**	
Design strength	=1-0.85*Pn/Pu	<	=0	Allowable bending s
Slenderness ratio	=K*L/rx	<	=200	Allowable shear str
Slenderness ratio	=K*L/ry	<	=200	Allowable deflection
Local web buckling	=h/tw	<	=1.49*SQRT(E/sigma	Local buckling
Local flange buckling	=b/(2*tf)	<	=0.56*SQRT(E/sigma	Local buckling
Height to Flange width	=b-h	<	=0	

Microsoft Excel 15.0 Answer Report
Worksheet: [Project-Optimization of W Shape.xlsx]Problem
Report Created: 3/31/2016 4:31:20 PM
Result: Solver has converged to the current solution. All Constraints are satisfied.
Solver Engine
Engine: GRG Nonlinear
Solution Time: 0.062 Seconds.
Iterations: 17 Subproblems: 0
Solver Options
Max Time 100 sec, Iterations 10000, Precision 0.000001
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 5%, Solve Without Integer

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$28	Vol	21600	7688.514072

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$3	h	20	8.57185835	Contin
$D$4	b	20	16.13745236	Contin
$D$5	tf	1	0.598277891	Contin
$D$6	tw	1	0.238877211	Contin

Constraints

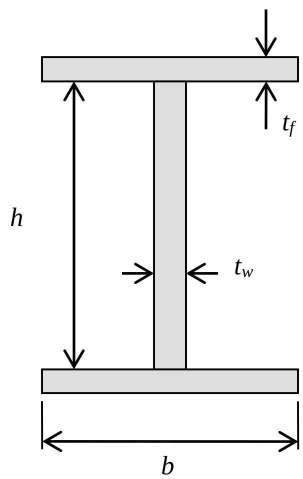
Cell	Name	Cell Value	Formula	Status	Slack
$B$31	Design strength Value/Eq.	-5.26407E-07	$B$31<=$D$31	Binding	0
$B$32	Slenderness ratio Value/Eq.	81.27159123	$B$32<=$D$32	Not Binding	118.7284088
$B$33	Slenderness ratio Value/Eq.	81.27165432	$B$33<=$D$33	Not Binding	118.7283457
$B$34	Local web buckling Value/Eq.	35.88395189	$B$34<=$D$34	Binding	0
$B$35	Local flange buckling Value/Eq.	13.48658593	$B$35<=$D$35	Binding	0
$B$3	web height Lower limit	1	$B$3<=$D$3	Not Binding	7.57185835
$B$4	flange width Lower limit	1	$B$4<=$D$4	Not Binding	15.13745236
$B$5	flange thickness Lower limit	0.1	$B$5<=$D$5	Not Binding	0.498277891
$B$6	web thickness Lower limit	0.1	$B$6<=$D$6	Not Binding	0.138877211
$D$3	h	8.57185835	$D$3<=$E$3	Not Binding	11.42814165
$D$4	b	16.13745236	$D$4<=$E$4	Not Binding	3.862547639
$D$5	tf	0.598277891	$D$5<=$E$5	Not Binding	0.401722109
$D$6	tw	0.238877211	$D$6<=$E$6	Not Binding	0.761122789

SAMPLE PROJECT: PROBLEM FORMULATION

Optimum Design of Crane Girder

Step 1. Problem Statement

Plate girders are used as cranes in mechanical shops to transfer loads from one place to another. A crane girder needs to be designed with the objective of minimizing its total mass which is directly related to the total cost. The maximum wheel load is P and the span of girder is L. The hook center is moving between the ends of the girder. The design must satisfy the deflection, bending stress, shear stress, lateral-torsional buckling and local bucking constraints. Formulate the design problem as an optimization problem using the five step procedure. State all the assumptions made in your formulation. Submit a word-processed report.



Step 2. Data and Information

Data: L, P, δ_a, σ_a, τ_a, Steel grade (E, F_y, etc)

Expressions:

$$I = \frac{1}{12} t_w h^3 + 2\left[\frac{1}{12} b t_f^3 + b t_f \left(\frac{1}{2}h + \frac{1}{2}t_f\right)^2\right]$$

$$\delta = \frac{PL^3}{48EI}$$

$$\sigma = \frac{Mc}{I}, \quad c = \left(t_f + \frac{1}{2}h\right), \quad M = \frac{PL}{4}$$

$$\tau = \frac{VQ}{It_w}, \quad Q = \frac{1}{8}t_w h^2 + \frac{1}{2}bt_f(h + t_f), \quad V = P$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{1}{12}ht_w^3 + 2\left(\frac{1}{12}t_f b^3\right)}{ht_w + 2t_f b}}$$

Step 3. Design Variables

b : width of flange
t_f : thickness of flange
h : height of web
t_w: thickness of web

Step 4. Cost Function

Minimization total mass $f = \rho L(2bt_f + ht_w)$

Step 5. Constraints

Deflection: $\delta \le \delta_a$

Bending stress: $\sigma \le \sigma_a$

Shear stress: $\tau \le \tau_a$

Buckling:

Lateral torsional buckling: $L \le 1.76 r_y \sqrt{\frac{E}{F_y}}$

Local flange buckling: $\frac{b}{2t_f} \le 0.38\sqrt{\frac{E}{F_y}}$

Local web buckling: $\frac{h}{t_w} \le 3.76\sqrt{\frac{E}{F_y}}$

Design Variable bounds:

$$b_{min} \le b \le b_{max}$$
$$t_{f\,min} \le t_f \le t_{f\,max}$$
$$h_{min} \le h \le h_{max}$$
$$t_{w\,min} \le t_w \le t_{w\,max}$$

SAMPLE PROJECT: NONLINEAR PROGRAMMING

Optimum Design of Crane Girder

Consider the design optimization problem that you have formulated earlier. Solve the problem for the data given below. Use a numerical method to solve the problem, such as in Excel, Matlab, Mathematica, etc. Submit a brief word-processed report containing your problem formulation, solution, computer files and a discussion of the solution.

DATA FOR PROJECT:

$L = 30\ ft;\quad P = 2\times10^5\ lbs;\quad E = 29\times10^6\ psi;\quad F_y = 5\times10^4\ psi;$

$\delta_a = \frac{L}{240};\quad \sigma_a = 3\times10^4\ psi;\quad \tau_a = 18\times10^3\ psi$

Lower and upper bounds of the design variables:

$1 \le h \le 40\ in;\quad 1 \le b \le 40\ in;\quad 0.1 \le t_f \le 3\ in;\quad 0.1 \le t_w \le 3\ in;$

The following Excel Answer Report gives the optimal solution for the problem. It is seen that the flange width is much larger than the height of the web which may not be practical. Therefore a constraint $b \le h$ is imposed. The Excel sheets on the following pages give the solution for this case.

Microsoft Excel 15.0 Answer Report
Worksheet: [Project-Opt of Crane Girder.xlsx]Crane Girder
Report Created: 4/1/2016 1:13:18 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: GRG Nonlinear
Solution Time: 0.062 Seconds.
Iterations: 19 Subproblems: 0
Solver Options
Max Time 100 sec, Iterations 10000, Precision 0.000001
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Central, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 3%, Solve Without Integer Constraints

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$25	Vol	16200	40785.27746

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$3	h	15	11.27013505	Contin
$D$4	b	15	30.77866011	Contin
$D$5	tf	1	1.681597762	Contin
$D$6	tw	1	0.867583703	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$28	Bending stress Value/Eq.	29469.90185	$B$28<=$D$28	Not Binding	530.09815
$B$29	Shear stress Value/Eq.	18000	$B$29<=$D$29	Binding	0
$B$30	Deflection Value/Eq.	1.5	$B$30<=$D$30	Binding	0
$B$31	Lateral torsional buckling Value/Eq.	360	$B$31<=$D$31	Binding	0
$B$32	Local flange buckling Value/Eq.	9.15161188	$B$32<=$D$32	Binding	0
$B$33	Local web buckling Value/Eq.	12.99025674	$B$33<=$D$33	Not Binding	77.56253449
$B$3	web height Lower limit	1	$B$3<=$D$3	Not Binding	10.27013505
$B$4	flange width Lower limit	1	$B$4<=$D$4	Not Binding	29.77866011
$B$5	flange thickness Lower limit	0.1	$B$5<=$D$5	Not Binding	1.581597762
$B$6	web thickness Lower limit	0.1	$B$6<=$D$6	Not Binding	0.767583703
$D$3	h	11.27013505	$D$3<=$E$3	Not Binding	28.72986495
$D$4	b	30.77866011	$D$4<=$E$4	Not Binding	9.221339893
$D$5	tf	1.681597762	$D$5<=$E$5	Not Binding	1.318402238
$D$6	tw	0.867583703	$D$6<=$E$6	Not Binding	2.132416297

The Excel sheet showing the additional constraint $b \le h$.

Project - Optimum Design of Crane					
1. Design variable name	**Lower limit**	**Symbol**	**Value**	**Upper limit**	**Units**
web height	1	h	30.8949952738024	40	in
flange width	1	b	30.8949952738024	40	in
flange thickness	0.1	tf	1.6879537549947	3	in
web thickness	0.1	tw	0.346633586010688	3	in
2. Parameter name	**Symbol**	**Value**	**Units**		
Span length	L	360	in		
Modulus of elasticity	E	29000000	psi		
Yield stress	sigma_y	50000	psi		
Concentrated load	P	200000	lb		
3. Dependent variable name	**Symbol**	**Equation**			
Cross sectional area	A	=2*b*tf+h*tw			
Moment of inertia	I	=1/12*tw*h^3+2*(1/12*b*tf^3+b*tf*(1/2*h+1/2*tf)^2)			
Bending Moment	M	=P*L/4			
Bending stress	sigma	=M*(tf+h/2)/I			
Shear force	V	=P			
Deflection	D	=P*L^3/(48*E*I)			
Shear stress	tau	=V*(1/8*tw*h^2+1/2*b*tf*(h+tf))/(I*tw)			
Radius of gyration w.r.t y axis	ry	=SQRT((1/12*h*tw^3+2*(1/12*tf*b^3))/A)			
4. Objective fuction name	**Symbol**	**Equation**	**Units**		
Volume of material	Vol	=A*L	in^3		
5. Constraints	**Value/Eq.**	**</>/=**	**Value/Eq.**	**Name**	
Bending stress	=sigma	<	30000	Allowable bending stress	
Shear stress	=tau	<	18000	Allowable shear stress	
Deflection	=D	<	=L/240	Allowable deflection	
Lateral torsional buckling	=L	<	=1.76*ry*SQRT(E/sigma_y)	Lateral torsional buckling	
Local flange buckling	=b/(2*tf)	<	=0.38*SQRT(E/sigma_y)	Local flange buckling	
Local web buckling	=h/tw	<	=3.76*SQRT(E/sigma_y)	Local web buckling	
b<=h	=b-h	<	0		

Microsoft Excel 15.0 Answer Report
Worksheet: [Project-Opt of Crane Girder.xlsx]Crane Girder
Report Created: 4/1/2016 1:24:04 PM
Result: Solver converged in probability to a global solution.
Solver Engine
Engine: GRG Nonlinear
Solution Time: 0.343 Seconds.
Iterations: 0 Subproblems: 31
Solver Options
Max Time 100 sec, Iterations 10000, Precision 0.000001
Convergence 0.00001, Population Size 100, Random Seed 0, Derivatives Central, Multistart, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 3%, Solve Without Integer Constraints, Ass

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$25	Vol	16200	41402.84024

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$3	h	15	30.89499527	Contin
$D$4	b	15	30.89499527	Contin
$D$5	tf	1	1.687953755	Contin
$D$6	tw	1	0.346633586	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$28	Bending stress Value/Eq.	10800.13724	$B$28<=$D$28	Not Binding	19199.86276
$B$29	Shear stress Value/Eq.	18000	$B$29<=$D$29	Binding	0
$B$30	Deflection Value/Eq.	0.234725073	$B$30<=$D$30	Not Binding	1.265274927
$B$31	Lateral torsional buckling Value/Eq.	360	$B$31<=$D$31	Binding	0
$B$32	Local flange buckling Value/Eq.	9.15161188	$B$32<=$D$32	Binding	0
$B$33	Local web buckling Value/Eq.	89.12868378	$B$33<=$D$33	Not Binding	1.424107456
$B$34	b<=h Value/Eq.	0	$B$34<=$D$34	Binding	0
$B$3	web height Lower limit	1	$B$3<=$D$3	Not Binding	29.89499527
$B$4	flange width Lower limit	1	$B$4<=$D$4	Not Binding	29.89499527
$B$5	flange thickness Lower limit	0.1	$B$5<=$D$5	Not Binding	1.587953755
$B$6	web thickness Lower limit	0.1	$B$6<=$D$6	Not Binding	0.246633586
$D$3	h	30.89499527	$D$3<=$E$3	Not Binding	9.105004726
$D$4	b	30.89499527	$D$4<=$E$4	Not Binding	9.105004726
$D$5	tf	1.687953755	$D$5<=$E$5	Not Binding	1.312046245
$D$6	tw	0.346633586	$D$6<=$E$6	Not Binding	2.653366414

It is seen that the height of the web has increased substantially, and the optimum material has also increased accordingly.

SAMPLE PROJECT: GRAPHICAL OPTIMIZATION
Optimum Design of Torsion Rod

Design a hollow torsion rod shown in Fig.E3.34 to satisfy the following:

1. The calculated shear stress, τ , shall not exceed the allowable shear stress $\tau_a$under the normal operation torque T_o (N·m).
2. The calculated angle of twist, θ, shall not exceed the allowable twist, θ_a (radians).
3. The member shall not buckle under a short duration torque of T_{max} (N·m).

Requirements for the rod and material properties are given in Table E3.34(A) and E3.34(B) (use length = 0.75m and Titanium material). Use the following design variables: x_1 = outside diameter of the shaft; x_2 = ratio of inside/outside diameter, d_i/d_o.

Formulate the design optimization problem, and using graphical optimization, determine the inside and outside diameters for a minimum mass rod to meet the above design requirements. Compare the hollow rod with an equivalent solid rod ($d_i/d_o = 0$). Use consistent set of units (e.g. Newtons and millimeters) and let the minimum and maximum values for design variables be given as

$$0.02 \le d_o \le 0.5 \text{ m}, \;\; 0.60 \le \frac{d_i}{d_o} \le 0.999$$

Useful expressions for the rod are:

Mass of rod: $$M = \frac{\pi}{4}\rho l(d_o^2 - d_i^2), kg$$

Calculated shear stress: $$\tau = \frac{c}{J}T_o, Pa$$

Calculated angle of twist: $$\theta = \frac{l}{GJ}T_o, radians$$

Critical buckling torque: $$T_{cr} = \frac{\pi d_o^3 E}{12\sqrt{2}(1-\nu^2)^{0.75}}\left(1 - \frac{d_i}{d_o}\right)^{2.5}, \text{N.m}$$

Notation

M = mass of the rod (kg)
d_o= outside diameter of the rod (m)
d_i= inside diameter of the rod (m)
ρ = mass density of material (kg/m^3)
l = length of the rod (m)
T_0 = Normal operation torque (N · m)
c = Distance from rod axis to extreme fiber (m)
J = Polar moment of inertia (m^4)
θ= Angle of twist (radians)
G = Modulus of rigidity (Pa)
T_{cr} = Critical buckling torque (N · m)
E = Modulus of elasticity (Pa)
ν = Poisson's ratio.

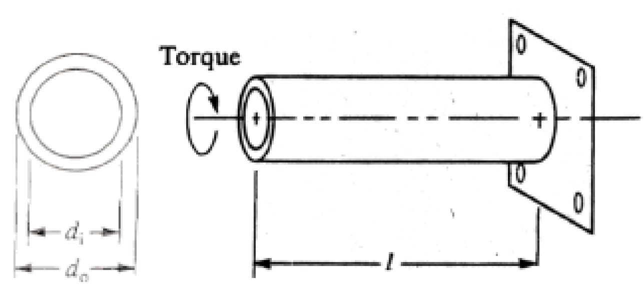


FIGURE E3-34 Hollow torsion rod.

TABLE E3-34(A) Rod Requirements

Torsion rod number	*Length, l* (m)	*Normal torque,* T_0 (kN · m)	*Max. torque,* T_{max} (kN · m)	*Allowable twist,* θ_a *(degrees)*
1	0.50	10.0	20.0	2
2	0.75	15.0	25.0	2
3	1.00	20.0	30.0	2

TABLE E3-34(B) Materials and Properties for the Torsion Rod

Material	*Density,* ρ (kg/m^3),	*Allowable Shear stress,* τ_a (MPa)	*Elastic modulus, E* (GPa)	*Shear modulus, G* (GPa)	*Poisson's ratio* (ν)
1. 4140 alloy steel	7850	275	210	80	0.30
2. Aluminum alloy 24 ST4	2750	165	75	28	0.32
3. Magnesium alloy A261	1800	90	45	16	0.35
4. Berylium	1850	110	300	147	0.02
5. Titanium	4500	165	110	42	0.30

Project 1 Reporting Requirements

Submit your report using the provided Microsoft Word .doc file. Remember to download the file and before working on it to rename the file including your first and last name. For example, "project1.doc" is the name of the file that you download. Assuming that "Aye Ten" is a student name, the renamed file would be "project1_AyeTen.doc".

Your report should include the following 4 parts:

5) *Formulation of the problem: clear definition of design variables, cost function, and constraints.*
6) *Graphical representation of the problem using MATLAB; copy the MATLAB code in the end of the Word report in an Appendix.*
7) *Final solution for the problem and discussion about the solution, which should include:*
   a) *Final design variables and cost function values*
   b) *Active/inactive constraints at optimum*
   c) *Discussion of the problem formulation and the final solution.*
8) *Submit the MATLAB ".m" file for the problem (in addition to including it in the body of the report); the code should be organized clearly and in a readable format.*

Project 1 Grading Rubric: Graded based on 50 points

Report attributes↓	**Meets all expectations**	**Partially meets expectations**	**Below expectations**
Formulation of the problem (15 points)	***(15 points)*** Complete and clear presentation that includes clearly defined design variables, accurate cost function and constraints	***(8 points)*** Complete, but sloppy presentation that either includes poorly defined design variables and/or cost function and/or constraints	***(3 points)*** Incomplete formulation that either lacks clear definition of design variables and/or cost function and/or constraints
Graphical representation of the problem (15 points)	***(15 points)*** *Complete legible graph with correct labels and correct identification of the optimum point*	***(8 points)*** Partially complete graph with either missing or incorrect labels and/or missing or incorrect optimum point	***(3 points)*** Graph illegible and/or labels missing and/or optimum point missing
Final solution for the problem and its discussion (10 points)	***(10 points)*** Complete solution that includes correct: Design variables Cost function value Active/inactive constraints at optimum. Complete discussion of the formulation and the solution	***(5 points)*** Partial solution that contains all of the components with either incorrect design variables and/or incorrect cost function and/or incorrect active/inactive constraints at optimum; incomplete or missing discussion of problem formulation or solution	***(2 points)*** Incomplete solution with either missing or incorrect design variables; missing or incorrect cost function; incorrect active/inactive constraints at optimum; missing and or incomplete discussion of problem formulation and/or solution.
MATLAB code for the problem (10 points)	***(10 points)*** Nicely organized MATLAB code in a clear readable and logical format	***(5 points)*** MATLAB code incomplete and/or unreadable and not formatted well	***(0 points)*** Not included
Overall Score	**Best effort 45 or more**	**Acceptable effort 35 to 44**	**Needs improvement 20 to 34**

Solution

Design Variables: x_1 = outside diameter of the shaft; x_2 = ratio of inside/outside diameter, d_i/d_o
Units of mass, force and length are kg, N and mm respectively.

Cost Function: minimize mass of hollow shaft

$$f = \rho(\pi/4)\left(d_o^2 - d_i^2\right)l = \rho(\pi/4)d_o^2\left(1-\left(d_i/d_o\right)^2\right)l = (\pi\rho l/4)x_1^2\left(1-x_2^2\right)$$

Constraints:

$$g_1 = \tau - \tau_a = T_o c/J - \tau_a = T_o\,(0.5x_1)\Big/\left(\pi x_1^4(1-x_2)/32\right) - \tau_a = (16T_o/\pi)\Big/x_1^3\left(1-x_2^4\right) - \tau_a \le 0$$

$$g_2 = \theta - \theta_a = T_0 l/GJ - \theta_a = T_0 l\Big/\left(G\pi\left(d_o^4 - d_i^4\right)/32\right) - \theta_a = (32T_0 l/G\pi)\Big/x_1^4\left(1-x_2^4\right) - \theta_a \le 0$$

$$g_3 = -T_{cr} + T_{max} = -\left(\frac{\pi E}{12\sqrt{2}\left(1-\upsilon^2\right)^{0.75}}\right)\left(x_1^3(1-x_2)2.5\right) + T_{max} \le 0$$

Transform the parameters used in the foregoing equations to have consistent units. Note the first case of requirements and first case of material properties in Tables E3.34(A) and E3.34(B) are used.

l = 0.5 m = 500 mm;
T_o = 10.0 kN.m = 10.0(10^3)(10^3) = 10^7 N.mm;
T_{max} = 20.0 kN.m = 2.0×10^7 N.mm;
$\theta_a = 2° = 2(\pi/180) = \pi/90$ rad;
ρ = 7850 kg/m^3 = 7850(10^{-9}) = (7.85×10^{-6}) kg/mm^3;
τ_a = 275 MPa = 275 N/mm^2
E = 210 GPa = (2.1×10^5) N/mm2;
G = 80 GPa = (8.0×10^4) N/mm^2;
υ = 0.3

$$f = \left(7.85\times10^{-6}\right)(\pi/4)(500)x_1^2\left(1-x_2^2\right) = \left(3.08269\times10^{-3}\right)x_1^2\left(1-x_2^2\right)$$

$$g_1 = \left(16\times10^7/\pi\right)\Big/x_1^3\left(1-x_2^4\right) - 275 = 5.093\times10^7\Big/x_1^3\left(1-x_2^4\right) - 275 \le 0$$

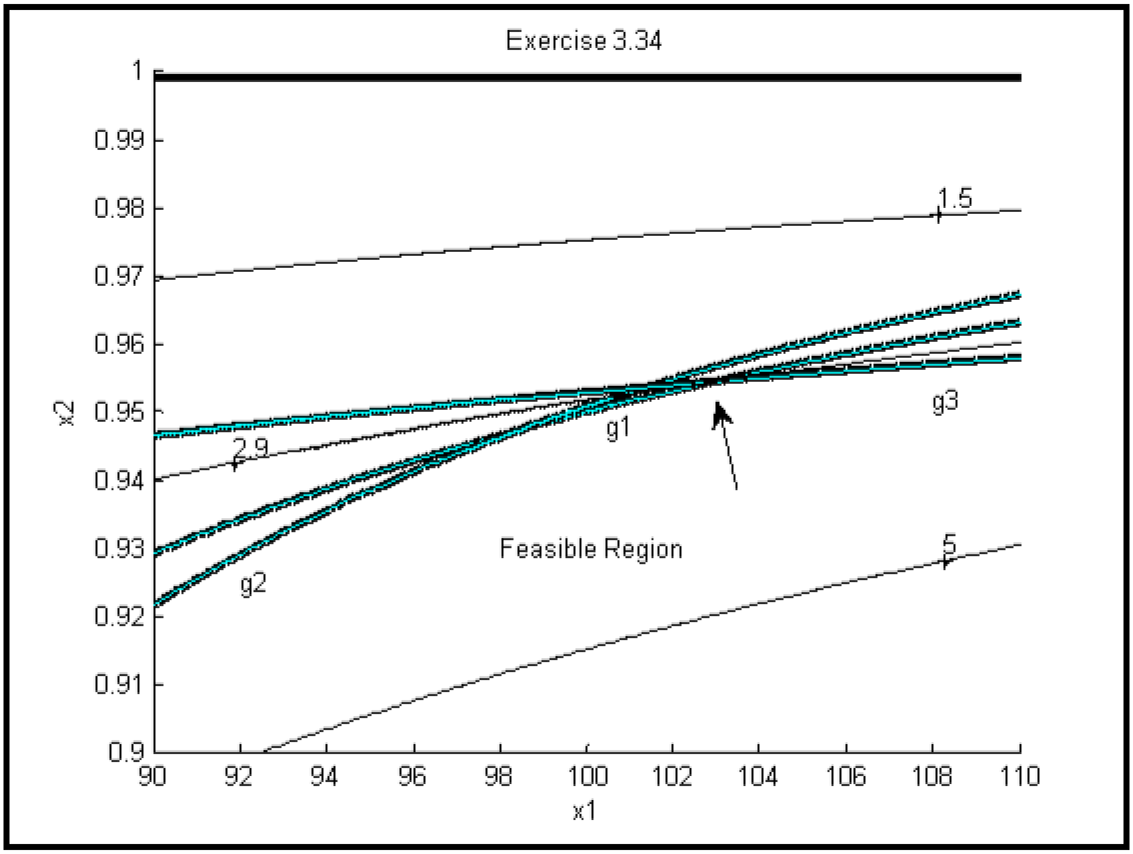
$$g_2 = \frac{32\left(10^7\right)(500)}{\left(8.0\times10^4\right)\pi}\Bigg/x_1^4\left(1-x_2^4\right) - \pi/90 = 6.36619\times10^5\big/x_1^4\left(1-x_2^4\right) - 3.49066\times10^{-2} \le 0$$

$$g_3 = \frac{-\pi\left(2.1\times10^5\right)}{12\sqrt{2}\left(1-0.3^2\right)^{0.75}}\left(x_1^3(1-x_2)2.5\right) + \left(2.0\times10^7\right)$$

$$= 2.0\times10^7 - \left(4.17246\times10^4\right)\left(x_1^3\right)(1-x_2)^{2.5} \le 0$$

$$g_4 = 20 - x_1 \le 0;$$
$$g_5 = x_1 - 500 \le 0;$$
$$g_6 = 0.6 - x_2 \le 0;$$
$$g_7 = x_2 - 0.999 \le 0$$

Optimum solution: $x_1^* \doteq 103.0$ mm, $x_2^* \doteq 0.955$, $f^* \doteq 2.9$ kg; g_1 (shear stress constraint) and g_3 (buckling constraint) are active.



MATLAB Code

```
[x1,x2]=meshgrid(90:0.1:110, 0.9:0.001:1);
                              %Enter functions for the minimization problem
f=(3.08269*10^-3)*(x1.^2).*(1-x2.^2);
g1=(5.093*10^7)-(275)*(x1.^3).*(1-x2.^4);
g2=(6.36619*10^5)-(3.49066*10^-2)*(x1.^4).*(1-x2.^4);
g3=(2*10^7)-(4.17246*10^4)*(x1.^3).*(1-x2).^2.5;
g4=20-x1;
g5=x1-500;
g6=0.6-x2;
g7=x2-0.999;
cla reset
axis auto                     %Minimum and maximum values for axes are determined
automatically
xlabel('x1'),ylabel('x2')     %Specifies labels for x- and y-axes
hold on                       %retains the current plot and axes properties for all subsequent
plots
cv1=[0 0];
const1=contour(x1,x2,g1,cv1,'k','LineWidth',3);
text(100.5,0.948,'g1')
cv11=[0.01:0.001:0.1];
const1=contour(x1,x2,g1,cv11,'c');
const2=contour(x1,x2,g2,cv1,'k','Linewidth',3);
const2=contour(x1,x2,g2,cv11,'c');
text(92,0.925,'g2')
const3=contour(x1,x2,g3,cv1,'k','Linewidth',3);
const3=contour(x1,x2,g3,cv11,'c');
text(108,0.952,'g3')
const4=contour(x1,x2,g4,cv1,'k','Linewidth',4);
const4=contour(x1,x2,g4,cv11,'c');
text(0.1,0.06,'g4')
const5=contour(x1,x2,g5,cv1,'k','Linewidth',3);
const5=contour(x1,x2,g5,cv11,'c');
text(0.1,0.02,'g5')
const6=contour(x1,x2,g6,cv1,'k','Linewidth',3);
const6=contour(x1,x2,g6,cv11,'c');
text(2.5,0.005,'g6')
const7=contour(x1,x2,g7,cv1,'k','Linewidth',3);
const7=contour(x1,x2,g7,cv11,'c');
text(6,200,'g7')
text(98,0.93,'Feasible Region')
fv=[1.5 2.9 5];               %Defines contours for the minimization function
fs=contour(x1,x2,f,fv,'k');   %'k' specifies black dashed lines for function contours
clabel(fs)                    %Automatically puts the contour value on the graph
hold off                      %Indicates end of this plotting sequence
```

SAMPLE PROJECT: VERIFICATION OF KKT CONDITIONS

Optimum Design of a Tripod

Tripods are used in many military and commercial applications. The objective of this project is to design a minimum mass tripod of height H to support a vertical load W = 60 kN. The tripod base is an equilateral triangle with sides B = 1200 mm. The struts have a solid circular cross section of diameter D (Fig. E3.54).

The axial stress in the struts must not exceed the allowable stress in compression, and axial load in the strut P must not exceed the critical buckling load P_{cr} divided by a safety factor FS = 2. Use consistent units of **Newtons** and **centimeters**. The minimum and maximum values for design variables are $0.5 \le H \le 1$m and $1 \le D \le 10$ cm. Material properties and other relationship are given below:

Material: aluminum alloy 2014-T6

Allowable compressive stress,	σ_a = 150MPa
Young's modulus,	E = 75 GPa
Mass density,	ρ = 2800 kg/m^3
Strut length,	$l = (H^2 + \frac{1}{3}B^2)^{0.5}$
Critical buckling load,	$P_{cr} = \dfrac{\pi^2 EI}{l^2}$
Moment of inertia,	$I = \dfrac{\pi}{64}D^4$
Strut load,	$P = \dfrac{Wl}{3H}$

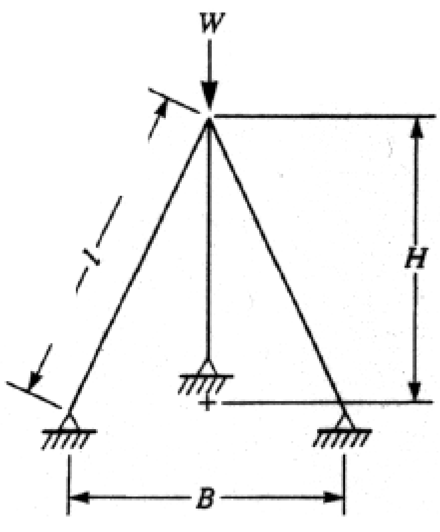


FIGURE E3.54 A tripod.

Project 2 Reporting Requirements

Submit *<u>your report</u>* *using the provided Microsoft Word .doc file. Remember to download the file and before working on it to rename the file including your first and last name. For example, "project1.doc" is the name of the file that you download. Assuming that "Aye Ten" is a student name, the renamed file would be "project1_AyeTen.doc".*

Your report should include the following 4 parts:

9) *Complete problem formulation process: problem description, data/information, clear definition of design variables, cost function, and constraints.*
10) *Graphical representation and optimum solution of the problem using MATLAB (design variable values, cost function, active constraints; zoomed-in figure for the optimum point); copy the MATLAB code in the end of the Word report in an Appendix.*
11) *Verify the KKT necessary conditions for the graphical solution:*
    a) *Write KKT conditions for the solution case.*
    b) *Calculate the Lagrange multipliers for the active constraints and verify their sign.*
    c) *A brief discussion of the solution process, final solution, KKT Conditions, and conclusions.* ***Note:*** *This discussion should only contain information pertinent to the scope of this project.*
12) *Submit the Matlab/Mathematica ".m/.nb" file for the problem (in addition to including it in the body of the report); the code should be organized clearly and in a readable format.*

Project 2 Grading Rubric: Graded based on 50 points

Report attributes	**Meets all expectations**	**Partially meets expectations**	**Below expectations**
Formulation of the problem (10 points)	***(10 points)*** Complete and clear presentation that includes complete problem formulation process with proper formatting.	***(6 points)*** Complete, but sloppy presentation that either includes poorly defined design variables and/or cost function and/or constraints	***(2 points)*** Incomplete formulation that either lacks clear definition of design variables and/or cost function and/or constraints
Graphical representation of the problem (10 points)	***(10 points)*** *Complete legible graph with correct labels and correct identification of the optimum point*	***(6 points)*** Incomplete graph with either missing or incorrect labels and/or missing or incorrect optimum point	***(2 points)*** Graph illegible and/or labels missing and/or optimum point missing
Final solution, verification of KKT conditions, discussion (20 points)	***(20 points)*** Complete solution; complete and correct KKT conditions. Correct solution of Lagrange multipliers. Proper discussion of results/KKT conditions.	***(10 points)*** Incomplete solution. Incomplete or incorrect verification of KKT conditions. Lack of discussion of results/KKT conditions.	***(4 points)*** Incorrect solution. Incorrect verification of KKT conditions; no discussion of results/KKT conditions.
MATLAB code for the problem (10 points)	***(10 points)*** Nicely organized MATLAB code in a clear readable and logical format	***(5 points)*** MATLAB code incomplete and/or unreadable and not formatted well	***(0 points)*** Not included

Overall Score: 1) Best effort: 45 or more**; 2) Acceptable effort:** 35 to 44**; 3); Needs improvement:** 20 to 34___

Solution: Design of a tripod-

Design Variables: H = height of the tripod; D = diameter of cross-section of the struts

Data and Information: Using units of Newtons and centimeters, the data are calculated as:
Area, $A = \frac{\pi D^2}{4}$; Moment of inertia, $I = \frac{\pi D^4}{64}$
$\sigma_a = 150 \text{ MPa} = 1.5\times10^4 \text{ N/cm}^2$; $E = 75 \text{ GPa} = 7.5\times10^6 \text{ N/cm}^2$;
$\rho = 2800 \text{ kg/m}^3 = 2.8\times10^{-3} \text{kg/cm}^3$; $B = 1200 \text{ mm} = 120 \text{ cm}$; $W = 60 \text{ kN} = 6.0\times10^4 \text{ N}$
Various expressions are defined in the problem statement.
Cost Function: minimize mass; $f = 3(\rho A l)$, kg

Constraints:

$$\mathrm{g}_1 = \frac{P}{A} - \sigma_a \le 0$$

$$\mathrm{g}_2 = P - \frac{P_{\mathrm{cr}}}{\mathrm{FS}} \le 0$$

$$\mathrm{g}_3 = H - 100 \le 0;$$

$$\mathrm{g}_4 = -H + 50 \le 0;$$

$$\mathrm{g}_5 = D - 10 \le 0;$$

$$\mathrm{g}_6 = -D + 1 \le 0$$

The optimization problem is to find D and H to minimize mass f subject to inequality constraints g_1 to g_7.
In the computer program, the above formulation may be used as it is. Or, it may be reduced to be only in terms of the design variables by substituting various constants and expressions into the cost and constraints:

$$f = 3\left(2.8\times10^{-3}\right)\left(\pi D^2/4\right)\left(H^2 + 120^2/3\right)^{\frac{1}{2}} = \left(6.59734\times10^{-3}\right)D^2\left(H^2 + 4800\right)^{\frac{1}{2}}$$

$$\mathrm{g}_1 = \left(2.546475\times10^4\right)\left(H^2 + 4800\right)^{\frac{1}{2}} \Big/ D^2 H - 1.5\times10^4 \le 0;$$

$$\mathrm{g}_2 = \frac{\left(2.0\times10^4\right)\left(H^2 + 4800\right)^{\frac{1}{2}}}{H} - \left(1.816774\times10^6\right)D^4 \Big/ \left(H^2 + 4800\right) \le 0;$$

$$\mathrm{g}_3 = H - 500 \le 0;$$

$$\mathrm{g}_4 = 50 - H \le 0;$$

$$\mathrm{g}_5 = D - 50 \le 0;$$

$$\mathrm{g}_6 = 0.5 - D \le 0$$

Optimum solution from the graph: $H^* = 50.0$ cm, $D^* \doteq 3.42$ cm, $f^* \doteq 6.6$ kg; g_2 (buckling load constraint) and g_4 (maximum height constraint) are active.

Verification of KKT conditions:

```
In[1]:= ρ = 0.0028; B = 120; A = (Pi / 4) * Drod^2; W = 60 000;

In[2]:= l = Sqrt[H^2 + (1/3) * B^2]
```

Out[2]= $\sqrt{H^2 + 4800}$

```
In[3]:= f = 3 * ρ * l * A // Simplify
```

Out[3]= $0.00659734\,\text{Drod}^2 \sqrt{H^2 + 4800}$

```
In[4]:= P = (W * l) / (3 * H) // Simplify
```

Out[4]= $\frac{20\,000 \sqrt{H^2 + 4800}}{H}$

```
In[5]:= Ir = (Pi * Drod^4) / 64
```

Out[5]= $\frac{\pi\,\text{Drod}^4}{64}$

```
In[7]:= Es = 7 500 000
```

Out[7]= 7 500 000

```
In[8]:= Pcr = ((Pi^2) * Es * Ir) / (l^2) // Simplify
```

Out[8]= $\frac{234\,375\,\pi^3\,\text{Drod}^4}{2\,(H^2 + 4800)}$

```
In[9]:= P - Pcr / 2
```

Out[9]= $\frac{20\,000 \sqrt{H^2 + 4800}}{H} - \frac{234\,375\,\pi^3\,\text{Drod}^4}{4\,(H^2 + 4800)}$

```
In[10]:= L = f + u2 * (P - Pcr / 2 + s1^2) + u4 * (50 - H + s4^2) // Simplify
```

Out[10]= $\text{u2}\left(-\frac{234\,375\,\pi^3\,\text{Drod}^4}{4\,(H^2 + 4800)} + \frac{20\,000 \sqrt{H^2 + 4800}}{H} + \text{s1}^2\right) + 0.00659734\,\text{Drod}^2 \sqrt{H^2 + 4800} + \text{u4}\,(-H + \text{s4}^2 + 50)$

```
In[11]:= PartialLH = D[L, H]
```

Out[11]= $\text{u2}\left(\frac{234\,375\,\pi^3\,\text{Drod}^4\,H}{2\,(H^2 + 4800)^2} - \frac{20\,000 \sqrt{H^2 + 4800}}{H^2} + \frac{20\,000}{\sqrt{H^2 + 4800}}\right) + \frac{0.00659734\,\text{Drod}^2\,H}{\sqrt{H^2 + 4800}} - \text{u4}$

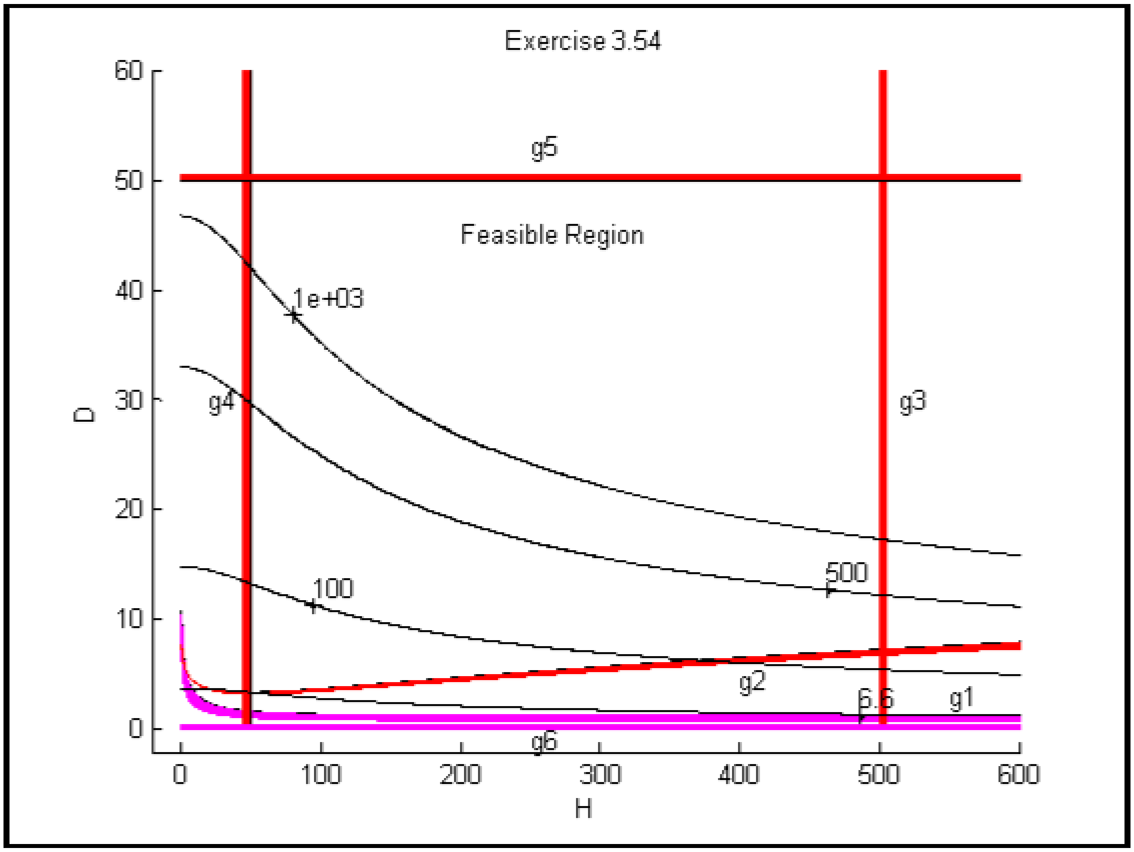
```
In[12]:= PartialLD = D[L, Drod]
```

Out[12]= $0.0131947\,\text{Drod} \sqrt{H^2 + 4800} - \frac{234\,375\,\pi^3\,\text{Drod}^3\,\text{u2}}{H^2 + 4800}$

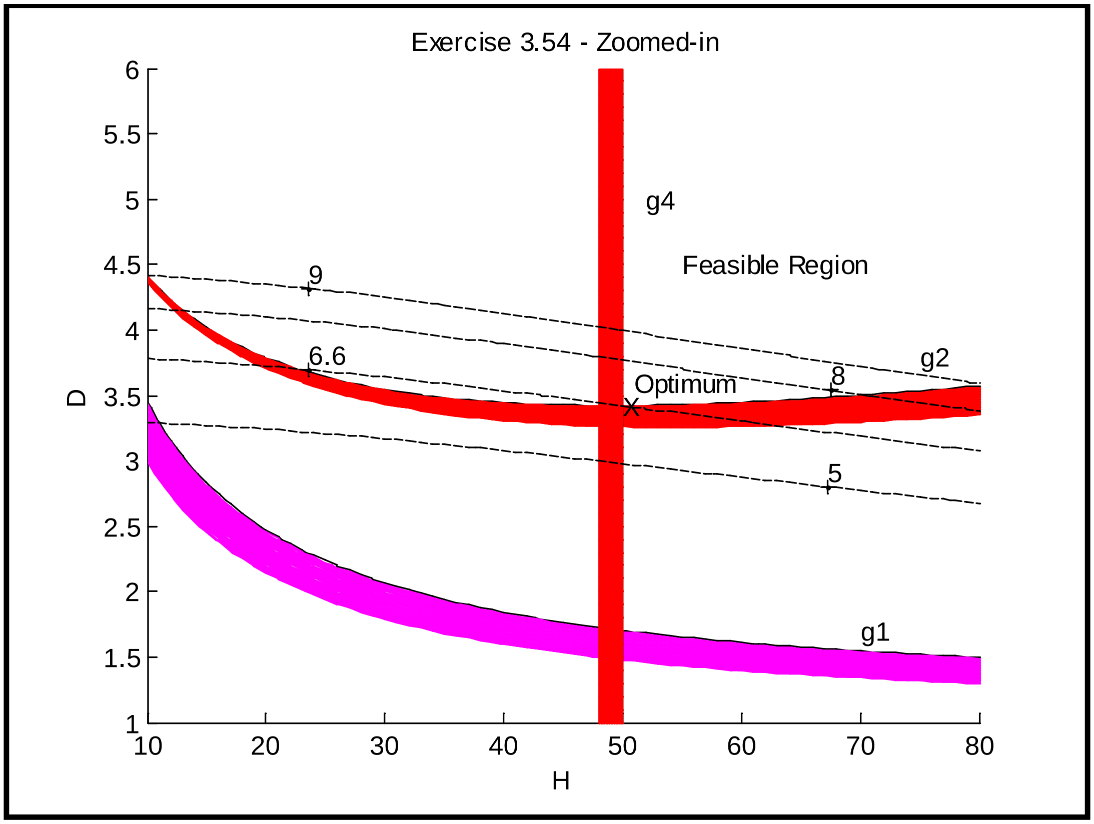
```
In[13]:= Drod = 3.42; H = 50;
```

```
In[14]:= PartialLH
Out[14]= 16.9631 u2 - u4 + 0.0451575

In[15]:= PartialLD
Out[15]= 3.85555 - 39821.4 u2

In[17]:= sol = NSolve[{PartialLH == 0, PartialLD == 0}, {u2, u4}]
Out[17]= {{u2 → 0.0000968212, u4 → 0.0467999}}

In[18]:= u2 = sol[[1, 1, 2]]
Out[18]= 0.0000968212

In[19]:= u4 = sol[[1, 2, 2]]
Out[19]= 0.0467999
```

Optimum solution: $H^* = 50.0$ cm, $D^* \doteq 3.42$ cm, $f^* \doteq 6.6$ kg; g_2 (buckling load constraint) and g_4 (maximum height constraint) are active.



Zoomed-in graph



MATLAB Code: 3.54

```
 [H,D]=meshgrid(1.0:1.0:600.0, 0.1:0.1:60.0);
f=6.59734e-3*(D.^2).*(sqrt(H.^2+4800));
g1=2.546479e4*sqrt(H.^2+4800)./((D.^2).*H)-1.5e4;
g2=2.0e4*sqrt(H.^2+4800)./H-1.816774e6*(D.^4)./(H.^2+4800);
g3=H-500;
g4=50-H;
g5=D-50;
g6=0.5-D;
cla reset
axis([-20,600,-2,60])
xlabel('H'),ylabel('D')
title('Exercise 3.54')
hold on
cv=[0 0];
const1=contour(H,D,g1,cv,'k');
text(550,3,'g1')
cv1=[1000.0:1000.0:20000.0];
const11=contour(H,D,g1,cv1,'m');
const2=contour(H,D,g2,cv,'k');
text(400,4.5,'g2')
cv2=[1000.0:1000.0:5000.0];
const21=contour(H,D,g2,cv2,'r');
const3=contour(H,D,g3,cv,'k');
cv3=[0.5:0.5:5.0];
text(515,30,'g3')
const31=contour(H,D,g3,cv3,'r');
const4=contour(H,D,g4,cv,'k');
text(20,30,'g4')
const41=contour(H,D,g4,cv3,'r');
const5=contour(H,D,g5,cv,'k');
cv5=[0.05:0.05:0.5];
text(250,53,'g5')
const51=contour(H,D,g5,cv5,'r');
const6=contour(H,D,g6,cv,'k');
text(250,-1,'g6')
const61=contour(H,D,g6,cv5,'m');
text(200,45,'Feasible Region')
fv=[6.6, 100, 500, 1000];
fs=contour(H,D,f,fv,'k--');
clabel(fs)
hold off
```

Code for zoomed-in figure

```
[H,D]=meshgrid(10.0:1.0:80.0, 1:0.1:6.0);
f=6.59734e-3*(D.^2).*(sqrt(H.^2+4800));
g1=2.546479e4*sqrt(H.^2+4800)./((D.^2).*H)-1.5e4;
g2=2.0e4*sqrt(H.^2+4800)./H-1.816774e6*(D.^4)./(H.^2+4800);
g3=H-500;
g4=50-H;
g5=D-50;
g6=0.5-D;
cla reset
axis([10,80,1,6])
xlabel('H'),ylabel('D')
title('Exercise 3.54 - Zoomed-in')
hold on
cv=[0 0];
const1=contour(H,D,g1,cv,'k');
text(70,1.7,'g1')
cv1=[200.0:100.0:5000.0];
const11=contour(H,D,g1,cv1,'m');
const2=contour(H,D,g2,cv,'k');
text(75,3.8,'g2')
cv2=[300.0:100.0:6000.0];
const21=contour(H,D,g2,cv2,'r');
const3=contour(H,D,g3,cv,'k');
cv3=[0.04:0.02:2.0];
text(515,30,'g3')
const31=contour(H,D,g3,cv3,'r');
const4=contour(H,D,g4,cv,'k');
text(52,5,'g4')
const41=contour(H,D,g4,cv3,'r');
const5=contour(H,D,g5,cv,'k');
cv5=[0.02:0.01:0.4];
text(250,53,'g5')
const51=contour(H,D,g5,cv5,'r');
const6=contour(H,D,g6,cv,'k');
text(250,-1,'g6')
const61=contour(H,D,g6,cv5,'m');
text(55,4.5,'Feasible Region')
text (50,3.42, 'X')
text (51,3.6,'Optimum')
fv=[5, 6.6, 8, 9];
fs=contour(H,D,f,fv,'k--');
clabel(fs)
hold off
```

```
%Exercise 3.54 - Alternate Formulation
[H,D]=meshgrid(1.0:1.0:120.0, 0.1:0.1:12.0);

%Data for the problem
W=60000;  B=120;   ro=2.8/1000;  E=7.5e6;  sigma_a=15000;
D_min=1; D_max=10; H_min=50; H_max=100; FS=2;

%Analysis Expressions
L=(H.*H+B.*B./3).^0.5;
I=pi.*D.^4./64;
A=pi.*D.*D./4;
P_cr=pi.*pi.*E.*I./(L.*L);
P=W.*L./(3.*H);
Mass=3.*ro.*A.*L;
sigma=P./A;

%Formulation
f=Mass;
g1=sigma - sigma_a;
g2=P-P_cr./FS;

%f=6.59734e-3*(D.^2).*(sqrt(H.^2+4800));
%g1=2.546479e4*sqrt(H.^2+4800)./((D.^2).*H)-1.5e4;
%g2=2.0e4*sqrt(H.^2+4800)./H-1.816774e6*(D.^4)./(H.^2+4800);
g3=H - H_max;
g4=H_min - H;
g5=D - D_max;
g6=D_min - D;
cla reset
axis([1,120,.1,12])
xlabel('Height, H'),ylabel('Diameter, D')
title('Exercise 3.54')
hold on
cv=[0 0];
const1=contour(H,D,g1,cv,'k');
text(80,2,'g1')
cv1=[1000.0:100.0:20000.0];
const11=contour(H,D,g1,cv1,'m');
const2=contour(H,D,g2,cv,'k');
text(110,4.5,'g2')
cv2=[1000.0:100.0:10000.0];
const21=contour(H,D,g2,cv2,'r');
const3=contour(H,D,g3,cv,'k');
cv3=[0.5:0.1:5.0];
text(95,9,'g3')
const31=contour(H,D,g3,cv3,'r');
const4=contour(H,D,g4,cv,'k');
text(52,9,'g4')
const41=contour(H,D,g4,cv3,'r');
const5=contour(H,D,g5,cv,'k');
cv5=[0.05:0.01:0.5];
text(75, 9.5,'g5')
const51=contour(H,D,g5,cv5,'r');
const6=contour(H,D,g6,cv,'k');
text(15,1.3,'g6')
```

```
const61=contour(H,D,g6,cv5,'m');
text(60,7,'Feasible Region')
fv=[4, 6.6, 8, 10, 12];
fs=contour(H,D,f,fv,'k--');
clabel(fs)
hold off
```