

Optimum Design - Sheet 1
Mathematical Review

1. Determine the rank of the following matrices:

a.

$$\begin{bmatrix} 3 & 0 & 1 & 3 \\ 2 & 0 & 3 & 2 \\ 0 & 2 & -8 & 1 \\ -2 & -1 & 2 & -1 \end{bmatrix}$$

b.

$$\begin{bmatrix} 1 & 2 & 2 & 2 & 4 \\ 1 & 6 & 3 & 0 & 3 \\ 2 & 2 & 3 & 3 & 2 \\ 1 & 3 & 2 & 5 & 1 \end{bmatrix}$$

2. Obtain the solution of the following equations using the Gaussian elimination procedure.

a.

$$\begin{aligned} 2x_1 + 2x_2 + x_3 &= 5 \\ x_1 - 2x_2 + 2x_3 &= 1 \\ x_2 + 2x_3 &= 3 \end{aligned}$$

b.

$$\begin{aligned} x_2 - x_3 &= 0 \\ x_1 + x_2 + x_3 &= 3 \\ x_1 - 3x_2 &= -2 \end{aligned}$$

c.

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 7 \\ 4x_2 - 5x_3 &= -7 \\ x_1 - 2x_2 + 4x_3 &= 9 \end{aligned}$$

d.

$$\begin{aligned} 2x_1 + x_2 - 3x_3 + x_4 &= 1 \\ x_1 + 2x_2 + 5x_3 - x_4 &= 7 \\ -x_1 + x_2 + x_3 + 4x_4 &= 5 \\ 2x_1 - 3x_2 + 2x_3 - 5x_4 &= -4 \end{aligned}$$

3. Check the linear independence of the following set of vectors

a.

$$\mathbf{a}^{(1)} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \mathbf{a}^{(2)} = \begin{bmatrix} -3 \\ -4 \\ 1 \end{bmatrix}, \mathbf{a}^{(3)} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \mathbf{a}^{(4)} = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

b.

$$\mathbf{a}^{(1)} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \mathbf{a}^{(2)} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \mathbf{a}^{(3)} = \begin{bmatrix} 4 \\ 0 \\ -3 \\ 2 \\ 1 \end{bmatrix}$$

4. Find eigenvalues for the following matrices:

a.

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$$

b.

$$\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$$

c.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

d.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

e.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 5 \end{bmatrix}$$