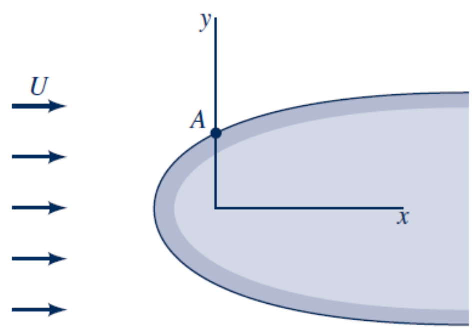
# SPC 307 Spring 2017

# Quiz 3

A body having the general shape of a half-body is placed in a stream of fluid. At a great distance upstream the velocity is U as shown in Fig. 1. Show how a measurement of the differential pressure between the stagnation point and point A can be used to predict the free-stream velocity, U. Express the pressure differential in terms of U and fluid density. Neglect body forces and assume that the fluid is nonviscous and incompressible.




| | | | | |
|---|---|---|---|---|
| Velocity potential | | $\mathbf{V} = \nabla\phi$ | | (6.65) |
| Laplace's equation | | $\nabla^2\phi = 0$ | | (6.66) |
| Uniform potential flow | $\phi = U(x\cos\alpha + y\sin\alpha)$ | $\psi = U(y\cos\alpha - x\sin\alpha)$ | $u = U\cos\alpha$<br>$v = U\sin\alpha$ | |
| Source and sink | $\phi = \dfrac{m}{2\pi}\ln r$ | $\psi = \dfrac{m}{2\pi}\theta$ | $v_r = \dfrac{m}{2\pi r}$<br>$v_\theta = 0$ | |
| Vortex | $\phi = \dfrac{\Gamma}{2\pi}\theta$ | $\psi = -\dfrac{\Gamma}{2\pi}\ln r$ | $v_r = 0$<br>$v_\theta = \dfrac{\Gamma}{2\pi r}$ | |
| Doublet | $\phi = \dfrac{K\cos\theta}{r}$ | $\psi = -\dfrac{K\sin\theta}{r}$ | $v_r = -\dfrac{K\cos\theta}{r^2}$<br>$v_\theta = -\dfrac{K\cos\theta}{r^2}$ | |

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \qquad v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \qquad v_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}$$

## Navier-Stokes Equation

($x$ direction)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

($y$ direction)

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

($z$ direction)

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$