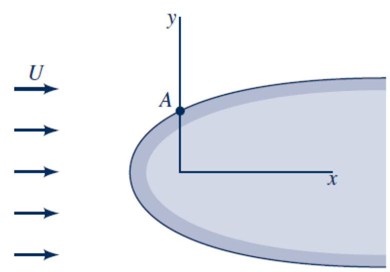
SPC 307 Spring 2017

Quiz 3 - Solution

A body having the general shape of a half-body is placed in a stream of fluid. At a great distance upstream the velocity is U as shown in Fig. 1. Show how a measurement of the differential pressure between the stagnation point and point A can be used to predict the free-stream velocity, U. Express the pressure differential in terms of U and fluid density. Neglect body forces and assume that the fluid is nonviscous and incompressible.



Write Bernoulli equation between stagnation point and point A to obtain

$$p_{stag} = p_A + \frac{1}{2}\rho V_A^2 \qquad (1)$$

the stagnation point will occur at $x = -b$ where

$$U = \frac{m}{2\pi b}$$

$$b = \frac{m}{2\pi U}$$

The value of the stream function at the stagnation point can be obtained by evaluating ψ at $r = b$ and $\theta = \pi$, which yields from Eq. 6.97

$$\psi_{\text{stagnation}} = \frac{m}{2}$$

Since $m/2 = \pi b U$ (from Eq. 6.99) it follows that the equation of the streamline passing through the stagnation point is

$$\pi b U = Ur \sin\theta + bU\theta$$

or

$$r = \frac{b(\pi - \theta)}{\sin\theta} \qquad \textbf{(6.100)}$$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U\cos\theta + \frac{m}{2\pi r}$$

and

$$v_\theta = -\frac{\partial\psi}{\partial r} = -U\sin\theta$$

Thus, the square of the magnitude of the velocity, V, at any point is

$$V^2 = v_r^2 + v_\theta^2 = U^2 + \frac{Um\cos\theta}{\pi r} + \left(\frac{m}{2\pi r}\right)^2$$

and since $b = m/2\pi U$

$$V^2 = U^2\left(1 + 2\frac{b}{r}\cos\theta + \frac{b^2}{r^2}\right) \qquad \textbf{(6.101)}$$

At point A $\theta = \frac{\pi}{2}$ so that

$$r_A = \frac{b\left(\pi - \frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = \frac{\pi b}{2}$$

or

$$\frac{b}{r_A} = \frac{2}{\pi} \qquad (2)$$

Substitution of Eq. (2) into Eq. 6.101 yields

$$V_A^{\,2} = U^2\left(1 + 0 + \frac{4}{\pi^2}\right)$$

and therefore from Eq. (1)

$$p_{stag} = p_A + \frac{1}{2}\rho U^2\left(1 + \frac{4}{\pi^2}\right) = p_A + 0.703\,\rho U^2$$

Thus,

$$\underline{\underline{p_{stag} - p_A = 0.703\,\rho U^2}}$$