Midterm 2

1. An atmospheric boundary layer is formed when the wind blows over the Earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. 1, typical values are n = 0.40 for urban areas, n = 0.28 for woodland or suburban areas, and n = 0.16 for flat open country.

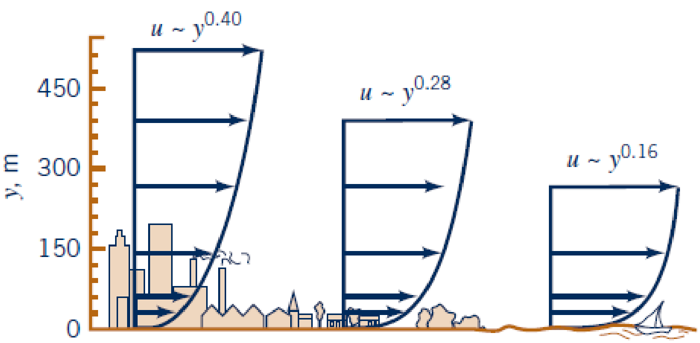
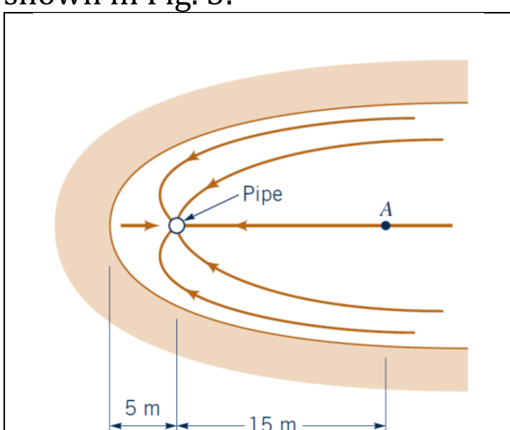


Fig.1

A 30-story office building (each story is 3.66 m tall) is built in a suburban industrial park. Plot the dynamic pressure, $\rho u^2/2$, as a function of elevation if the wind blows at 120.7 km/hr (in case of a hurricane) at the top of the building Plot four points only in the graph.

2. One end of a pond has a shoreline that resembles a half-body as shown in Fig. 2. A vertical porous pipe is located near the end of the pond so that water can be pumped out. When water is pumped at the rate of 0.02 m²/s (flow rate per unit length), what will be the velocity at point A in Fig. 2? Hint: Consider the flow inside a half-body. This flow is similar to the flow shown in Fig. 3.



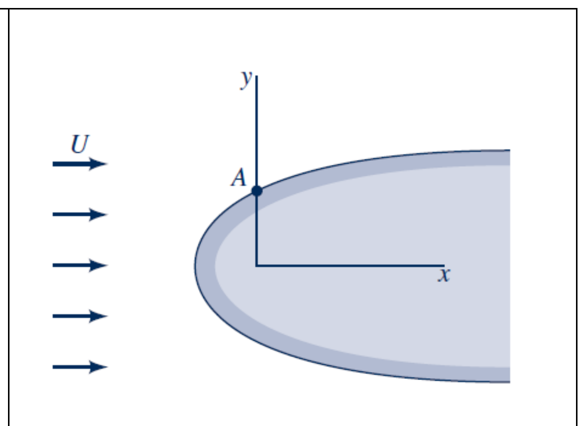


Fig.2	**Fig.3**

Equations and Graphs

Velocity potential		$\mathbf{V} = \nabla\phi$		(6.65)
Laplace's equation		$\nabla^2\phi = 0$		(6.66)
Uniform potential flow	$\phi = U(x\cos\alpha + y\sin\alpha)$	$\psi = U(y\cos\alpha - x\sin\alpha)$	$u = U\cos\alpha$ $v = U\sin\alpha$	
Source and sink	$\phi = \frac{m}{2\pi}\ln r$	$\psi = \frac{m}{2\pi}\theta$	$v_r = \frac{m}{2\pi r}$ $v_\theta = 0$	
Vortex	$\phi = \frac{\Gamma}{2\pi}\theta$	$\psi = -\frac{\Gamma}{2\pi}\ln r$	$v_r = 0$ $v_\theta = \frac{\Gamma}{2\pi r}$	
Doublet	$\phi = \frac{K\cos\theta}{r}$	$\psi = -\frac{K\sin\theta}{r}$	$v_r = -\frac{K\cos\theta}{r^2}$ $v_\theta = -\frac{K\cos\theta}{r^2}$	

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \qquad v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \qquad v_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}$$

Navier-Stokes Equation

(x direction)

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

(y direction)

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

(z direction)

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$