SPC 307 - Aerodynamics
Sheet 1 - Solution
Introduction to Aerodynamics

1)

Given:
T-38A weighing 5,000 *lbf* and flying at 26,000 *ft*
From Fig. 1, the maximum Mach number with afterburner ("Max" thrust curve) is $M \approx 1.075$. From Table 1.2b, the standard day speed of sound at 26,000 *ft* is 1011.88 *ft/s*. Using the definition of the Mach number as $M = V/a$, the velocity is:

$$V = Ma = (1.075)(1011.88\ ft/s) = 1087.771\ ft/s$$

The maximum lift-to-drag ratio, $(L/D)_{max}$, occurs at the point of minimum drag, $D_{min} = 850\ lbf$. Assuming that the airplane is in steady, level, unaccelerated flight, $L = W = 5{,}000\ lbf$, and:

$$(L/D)_{max} = 5{,}000\ lbf / 850\ lbf = 5.88$$

The Mach number where this occurs is $M \approx 0.53$. The minimum velocity of the aircraft under the given conditions is $M \approx 0.34$ and is due to the buffet (or stall) limit.

2)
Given:
T-38A weighing 10,000 lbf and flying at 20,000 ft with "Mil" thrust at M = 0.65
From Eqn. 1.1, the total energy of the aircraft is:

$$E = 0.5mV^2 + mgh$$

The mass of the airplane is given by Eqn. 1.2:

$$m = W/g = 10{,}000\ lbf / 32.174\ ft/s^2 = 310.81\ slugs$$

As in Problem 1, the velocity of the airplane can be found as:

$$V = Ma = (0.65)(1036.94\ ft/s) = 674.01\ ft/s$$

which yields a total energy of:

$$E = 0.5mV^2 + mgh$$

$$E = 0.5(310.81\ slugs)(674.01 ft/s^2) + (310.81 slugs)(32.174\ ft/s)(20{,}000\ ft)$$

= $270.60\ 10^6$ *ft-lbf*

The energy height is given by Eqn. 1.3:

$$H_e = E / W = 270.60{*}10^6\,ft\text{-}lbf / 10{,}000\,lbf = 27{,}060\,ft$$

The specific excess power is given by Eqn. 1.7:

$$P_s = \frac{(T-D)V}{W}$$

At the given conditions, $T = 2{,}500$ *lbf* and $D = 1{,}000$ *lbf* from Fig. 1, and the specific excess power is:

$$P_s = \frac{(T-D)V}{W} = \frac{(2{,}500\,lbf - 1{,}000\,lbf)(674.01\,ft/s)}{10{,}000\,lbf} = 101.1\ ft/s$$

3)

Given:

T-38A weighing 10,000 *lbf* and flying at 20,000 *ft* with “Mil” thrust at $M = 0.65$

The acceleration possible is given by Eqn. 1.5 as:

$$\frac{(T-D)V}{W} = \frac{V}{g}\frac{dV}{dt}$$

For the conditions of Problem 1.2, the acceleration would be:

$$\frac{dV}{dt} = \frac{(T-D)V}{W}\frac{g}{V} = P_s\frac{g}{V} = (101.1\,ft/s)\frac{32.174\,ft/s^2}{674.01\,ft/s} = 4.826\,ft/s^2$$

The rate of climb is given by Eqn. 1.7 as:

$$\frac{dh}{dt} = P_s = 101.1\,ft/s = 6{,}066\,ft/m$$

4)

Given:

T-38A flying at 20,000 *ft* with weight of 8,000, 10,000, and 12,000 *lbf*

As in Problem 1.1, the maximum lift-to-drag ratio, $(L/D)_{\max}$, occurs at the minimum drag, $D_{\min}$. Also, assuming that the airplane is in steady, level, unaccelerated flight, $L = W$. For the three weights you can generate a table for finding $(L/D)_{\max}$ using values from Fig. P1.1 and Tab. 1.2b:

$W = L$ (lbf)	$D_{\min}$ (lbf)	$(L/D)_{\max}$	$M_{(L/D)_{\max}}$	$V_{(L/D)_{\max}}$ (ft/s)
8,000	650	12.31	0.47	487.3
10,000	850	11.76	0.53	549.6
12,000	1050	11.43	0.60	622.2

As can be seen, higher weight requires higher velocities to maintain aerodynamic efficiency, but also results in a reduction of that efficiency.

5)

Given:

10,000 *lbf* T-38A flying at 20,000 *ft* at $M = 0.35$, $M_{(L/D)_{\max}}$, and 0.70

As in Problem 1.2, the specific excess power is given by Eqn. 1.7:

$$P_s = \frac{(T - D)V}{W}$$

For the three velocities you can generate a table for specific excess power using values from Fig. P1.1:

M	T (lbf)	D (lbf)	V (ft/s)	P_s (ft/s)
0.35	2250	1600	362.93	23.6
0.53	2400	850	549.58	85.2
0.70	2600	1100	725.86	108.9

Notice the large increase in specific excess power from $M = 0.35$ to 0.53 as the airplane becomes more aerodynamically efficient. While the specific excess power continues to increase as the speed increases to $M = 0.70$, the increase in P_s is not as dramatic due to the increase in drag.

6)

Given:

Air and nitrogen with associate gas and Sutherland's law constants at $p = 586\ N/m^2$ and $T = 54.3K$.

For a thermally perfect gas, the equation of state is given by Eqn. 1.10:

$$p = \rho RT$$

The fluid density can be found as:

$$\rho = \frac{p}{RT}$$

where R, the gas constant, is given in the problem statement. The resulting densities are:

$$\rho_{Air} = \frac{p_{Air}}{R_{Air} T_{Air}} = \frac{58.6 N/m^2}{\left(287.05 \dfrac{N-m}{kg-K}\right)(54.3K)} = 0.03760\ kg/m^3$$

$$\rho_N = \frac{p_{N_2}}{R_{N_2} T_{N_2}} = \frac{(58.6 N/m^2)}{\left(297 \dfrac{N-m}{kg-K}\right)(54.3K)} = 0.03634\ kg/m^3$$

These values are very close to each other due to the very similar gas constants. The viscosity is given by Sutherland's law in English units, Eqn. 1.12a:

$$\mu = C_1 \frac{T^{1.5}}{(T + C_2)}$$

where the constants are also given in the problem statement. The resulting viscosities are:

$$\mu_{Air} = 1.458 \times 10^{-6} \frac{kg}{s-m-K^{0.5}} \frac{(54.3K)^{1.5}}{(54.3K + 110.4K)} = 5.18694 \times 10^{-7} \frac{lbf-s}{ft^2}$$

$$\mu_{N_2} = 1.458 \times 10^{-6} \frac{kg}{s-m-K^{0.5}} \frac{(54.3K)^{1.5}}{(54.3K + 102K)} = 5.01012 \times 10^{-7} \frac{lbf-s}{ft^2}$$

Finally, the kinematic viscosity is given by Eqn. 1.6 as:

$$\nu = \frac{\mu}{\rho}$$

and the resulting kinematic viscosities are:

$$\nu_{Air} = \frac{\mu_{Air}}{\rho_{Air}} = \frac{5.18694 \times 10^{-7} \dfrac{lbf - s}{ft^2}}{0.01553 \dfrac{slug}{ft^3}} = 3.33995 \times 10^{-5}\ ft^2/s$$

$$\nu_{N_2} = \frac{\mu_{N_2}}{\rho_{N_2}} = \frac{5.01012 \times 10^{-7} \dfrac{lbf - s}{ft^2}}{0.01502 \dfrac{slug}{ft^3}} = 3.33563 \times 10^{-5}\ ft^2/s$$

7)

Given:

Initial pressure	$p_1 = 1\ atm$
Final pressure	$p_2 = 4\ atm$
Initial Temperature	$T_1 = 300K$
Final Temperature	$T_2 = 400K$
Initial density	$\rho_1 = 1.176\ kg/m^3$

For perfect gas $p = \rho RT$

Or $p/\rho T = R$ (gas constant)

Therefore $p_1/\rho_1 T_1 = p_2/\rho_2 T_2$

$$\rho_2 = (p_2/T_2) \times (\rho_1 T_1/ p_1) = (4/400) \times (1.176 \times 300/1)$$

$$\rho_2 = 3.528\ kg/m^3$$

8)

Given:

Final pressure (p_2) = (1/3) × initial pressure (p_1)

Initial density (ρ_1) = 1.176 kg/m^3

For isentropic expansion of perfect air takes place such that p/ρ^γ is a constant. Where $\gamma = 1.4$ for air.

Therefore $\quad p_1/\rho_1^\gamma = p_2/\rho_2^\gamma$

Or $\quad \rho_2 = \rho_1 \times (p_2/p_1)^{1/\gamma} = 1.176 \times (1/3)^{1/1.4}$

$\rho_2 = 0.536\ kg/m^3$

9)

From Table 1.2, at 15*km* altitude

$T = 216.650\ K$, $R = 287.05\ N\text{-}m/kg\text{-}K$ for air

$p/p_{sl} = 0.11953$ where $p_{sl} = 101.325\ kN/m^2$.

Therefore $\quad p = 12.11\ kN/m^2$

From equation 1.10 $\quad \rho = \dfrac{p}{RT}$

$$\rho = 0.195\ kg/m^3$$

From equation 1.12a

$$\mu = C_1 \frac{T^{1.5}}{T + C_2}.$$

For SI units where temperature, T, is in units of K and μ is in units of $kg/m\text{-}s$
Where C_1=1.458 × 10^{-6} and $C_2 = 110.4$

Therefore $\quad \mu = 1.4216 \times 10^{-5}\ kg/m\text{-}s$

10)

From Problem 9

$\mu = 1.4216 \times 10^{-5}$ *kg/m-s*

$\rho = 0.195$ *kg/m³*

$T = 216.650$ K

$\gamma = 1.4$ and $R = 287.05$ *N-m/kg-K* for air

Kinematic viscosity

$$\vartheta = \frac{\mu}{\rho} \frac{1.4216 \times 10^{-5} \frac{\text{kg}}{\text{m}-\text{s}}}{0.195 \frac{\text{kg}}{\text{m}^3}} = 7.29 \times 10^{-5} \text{ m}^2/\text{s}$$

And sound speed

$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.05 \times 216.65} = 295.07 \text{ m/s}$$

11)

From Table 1.2, at 10 *km* altitude

Temperature (T) = 223.252 K

And given stagnation temperature $T_o = 625\ K$

$\gamma = 1.4$ for air

From stagnation temperature T_o and Mach number M relation:

$T_o / T = 1 + (\gamma - 1)\, M^2/2$

$625/223.252 = 1 + (1.4 - 1)\, M^2/2$

Therefore $M = 3$

From Table 1.2, at 10 *km* altitude sound speed (a) = 299.53 m/s

$M = u/a$

$3 = u/299.53$

Therefore $u = 898.59\ m/s$

12)

$$P_\infty = 586\ {}^{N}/_{mm^2}\ ;\ T_\infty = 54.3\ K\ ;\ M_\infty = 7$$

Using the perfect gas law: $\rho_\infty = \dfrac{586\ {}^{N}/_{mm^2}}{\left(287.05\ \frac{N.m}{Kg\ K}\right)(54.3K)} = 0.03760\ \frac{Kg}{m^3}$

$$\mu_\infty = 1.458 \times 10^{-6}\ \frac{54.3^{1.5}}{54.3+110.4} = 3.542 \times 10^{-6}\ \frac{kg}{s.m}$$

$$U_\infty = M_\infty\, a_\infty = 7.0\left[20.047\sqrt{54.3}\right] = 7.0(147.72) = 1034\ \frac{m}{s}$$

Note that (in a conventional hypersonic wind tunnel) the speed of the sound is very low (only 147.72 ${}^{m}/_{s}$) in this problem. Thus hypersonic flows are achieved at relatively low velocities See Chapter 12. Although the information was not requested, the unit Reynolds number is:

$$Re_\infty/_{length} = \frac{\rho_\infty U_\infty}{\mu_\infty} = 12.543\ \times 10^6/_{m}$$

13)

For a thermally perfect gas, the equation of state is

$$\rho = \frac{p}{RT}$$

The gas constant R has a particular value for each substance. The gas constant R = 287.05 *N-m/kg-K* for air in SI units.

$$\rho = \frac{170 \times 10^3\ \mathrm{N/m^2}}{(287.05\ \mathrm{N.\,m/kg.\,K})\times(650+273.15\ \mathrm{K})}$$

$$= 0.6415\ kg/m^3$$

The viscosity of air is independent of pressure for temperatures below 3000 *K*. In this temperature range, we could use Sutherland's equation to calculate the coefficient of viscosity:

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$$\mu = C_1 \frac{T^{1.5}}{T + C_2}.$$

For SI units where temperature, *T*, is in units of *K* and μ is in units of *kg/m-s* where C_1=1.458 × 10^{-6} and C_2 = 110.4

$$\mu = 1.458 \times 10^{-6} \frac{(650 + 273.15)^{1.5}}{(650 + 273.15) + 110.4} = 3.9567 \times 10^{-5} \text{ kg/m - s}$$

14)

Given:

$M_\infty = 4$, $p_\infty = 1100\ N/m^2$, $T_\infty = 219.15\ K$

The speed of sound for a perfect gas is $a_\infty = \sqrt{\gamma RT_\infty}$ where γ is the ratio of specific heats and *R* is the gas constant. For the range of temperature over which air behaves as a perfect gas, $\gamma = 1.4$ and $R = 287.05\ N.\ m/kg.\ K$

$$a_\infty = \sqrt{1.4 \times 287.05 \times 219.15} = 296.77\ m/s$$

We know that free-stream velocity $U_\infty = M_\infty a_\infty$

Therefore $U_\infty = 4 \times 296.77 = 1187.06\ m/s$

The free-stream density for a thermally perfect gas can be calculate using the equation of state is

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{1100\ N/m^2}{(287.05\ N.\ m/kg.\ K) \times (219.15\ K)} = 0.0175 \text{ kg/m}^3$$

The viscosity of air is independent of pressure for temperatures below 3000 *K*. In this temperature range, we could use Sutherland's equation to calculate the coefficient of viscosity:

$$\mu_\infty = C_1 \frac{T^{1.5}}{T + C_2}.$$

For SI units where temperature, *T*, is in units of *K* and μ is in units of *kg/m-s* where C_1=1.458 × 10^{-6} and C_2 = 110.4

$$\mu_\infty = 1.458 \times 10^{-6} \frac{(219.15)^{1.5}}{219.15 + 110.4} = 1.435 \times 10^{-5} \text{ kg/m - s}$$

Using the values for the static pressure given in Table 1.2, the pressure altitude simulated in the wind tunnel by this test condition is 35.67 *km*.

15)

Given Mach number $M = u/a = 0.8$

From Table 1.2, at 10 *km* altitude sound speed (a) = 299.53 m/s

$$M = u/a$$

$$0.8 = u/299.53$$

Therefore $u = 239.53\ m/s$

$$= 239.53 \times 3.28168 = 786.06\ ft/s$$

$$= 239.53 \times 1.943 = 465.42 \text{ knots}$$

16)

The values $T_0 = 288.15\ K\ and\ B = 0.0065\ K/m$ are standard for altitudes 0 to 11,000 m.

The exponent g/RB, which is dimensionless, is equal to 5.26 for air.

$$T = T_0 - BZ$$

$$= 288.15 - 0.0065(9000)$$
$$= 229.65\ k$$

As compared with 229.733 k in Table 1.2.

$$P = P_0 \left[1 - \frac{BZ}{T_0}\right]^{g/RB}$$

$$P = P_0 \left[1 - \frac{(0.0065)(9000)}{288.15}\right]^{5.26} = 0.30312\ P_0$$

Since $P_0 = P_{SL}$, this value compares favorably with the value of $P = 0.30397 P_{SL}$ for 9 *km* in Table 1.2.

17)

$$p_2 = p_1 \exp\left[\frac{g(z_1 - z_2)}{RT}\right]$$

to 20,000 m is given by:

$$p = p_{11,000} \exp\left\{\frac{g\,(11,000 - z)}{RT}\right\}$$

$$\frac{g}{RT} = \frac{9.8066\ \text{m/s}^2}{\left(287.05\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(216.650\ \text{K})} = 0.0001577/\text{m}$$

$$p = \left\{0.22310\ p_{SL}\right\} \exp\left\{1.7347 - 0.0001577\,z\right\}$$

18)

Recall that the temperature is constant at 216.650K from 11,000 m to 20,000 m. Using the equation developed in Problem 1.9, the pressure at 18,000 m is:

$$p = \left\{0.22310\ p_{SL}\right\} \exp\left\{1.7347 - 2.8386\right\} = 0.07397\,p_{SL}$$

This compares favorably with the tabulated value of $0.074663\ p_{SL}$.

$$\rho = \frac{p}{RT} = \frac{7495.5\ \text{N/m}^2}{\left(287.05\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(216.65\text{K})} = 0.1205\ \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 1.458\times10^{-6}\ \frac{T^{1.5}}{T + 110.4} = 1.422\times10^{-5}\ \frac{\text{kg}}{\text{s}\cdot\text{m}}$$

which is equal to the value at 18,000 m in Table 1.2.

$$a = 20.047\sqrt{T} = 295.073\ \frac{\text{m}}{\text{s}}$$

19)

At 10,000 ft

$T = 518.67 - 0.003565\,z = 484.75°R$

$$\frac{p}{p_0} = \left[1 - 6.873\times10^{-6}\,z\right]^{5.26} = \left[0.93127\right]^{5.26} = 0.68759$$

$$\frac{\rho}{\rho_0} = \left[1 - 6.873\times10^{-6}\,z\right]^{4.26} = \left[0.93127\right]^{4.26} = 0.73834$$

The corresponding values from Table 1.2 are

$T = 483.03°R$; $\frac{p}{p_0} = 0.68783$; $\frac{\rho}{\rho_0} = 0.7386$

At 30,000 ft:

$T = 518.67 - 0.003565\,z = 411.72°R$

$$\frac{p}{p_0} = \left[1 - 6.873\times10^{-6}\,z\right]^{5.26} = \left[0.79380\right]^{5.26} = 0.29681$$

$$\frac{\rho}{\rho_0} = \left[1 - 6.873\times10^{-6}\,z\right]^{4.26} = \left[0.79380\right]^{4.26} = 0.37391$$

The corresponding values from Table 1.2 are

$T = 411.84°R$; $\frac{p}{p_0} = 0.29754$; $\frac{\rho}{\rho_0} = 0.3747$

At 65,000 ft:

$T = 389.97°R$

$$\frac{p}{p_0} = 0.2231\,\exp\left(1.7355 - 4.8075\times10^{-5}\,z\right)$$

$$\frac{p}{p_0} = 0.2231\,(0.24923) = 0.05560$$

$$\frac{\rho}{\rho_0} = 0.2967\,\exp\left(1.7355 - 4.8075\times10^{-5}\,z\right)$$

$$\frac{\rho}{\rho_0} = 0.2967\,(0.24923) = 0.07395$$

The corresponding values in Table 1.2 are:

$T = 389.97°R$; $\frac{p}{p_0} = 0.05620$; $\frac{\rho}{\rho_0} = 0.0747$

20)

Given:

An aircraft required to perform a 9g turn at 20,000 *ft* MSL.

In order to evaluate these performance parameters you might need to know Standard Day values for pressure, density, temperature, viscosity, and speed of sound. These values would be used as free-stream values for various calculations and would be found in Table 1.2b. At 20,000 the values are:

$$p_\infty = \frac{p}{p_{SL}}\, p_{SL} = (0.45991)\left(2116.22\frac{lbf}{ft^2}\right) = 973.27\ lbf/ft^2$$

$$\rho_\infty = \frac{\rho}{\rho_{SL}}\, \rho_{SL} = (0.53316)\left(0.002377\frac{slug}{ft^3}\right) = 0.001267\ slug/ft^3$$

$$\mu_\infty = \frac{\mu}{\mu_{SL}}\, \mu_{SL} = (0.88953)\left(3.740 \times 10^{-7}\frac{lbf-s}{ft^2}\right) = 3.3268 \times 10^{-7}\ \frac{lbf-s}{ft^2}$$

$$T_\infty = 447.42\ °R$$

$$a_\infty = 1036.94\ ft/s$$

21)

Given:

Aircraft flying at an altitude of 20,000 *ft* with $p = 1195.57\ \frac{lbf}{ft^2}$ and $T = 475.90\ °R$

a) The density altitude will require calculating the density using the perfect gas law:
b)

$$\rho = \frac{p}{RT} = \frac{1195.57\ \frac{lbf}{ft^2}}{\left[53.34\frac{ft-lbf}{lbm-°R}\right](32.174 lbm/slug)(475.90\ °R)} = 0.0014638\ slug/ft^3$$

Using Table 1.12b and interpolating to the nearest 500 *ft*:

$$h_p = 15kft$$
$$h_T = 12kft$$
$$h_\rho = 15.63kft \approx 15.5kft$$

In a standard atmosphere an aircraft flying at 20,000 *ft* would experience pressure, temperature, and density altitudes of 20,000 *ft*.

22)

At a section XX

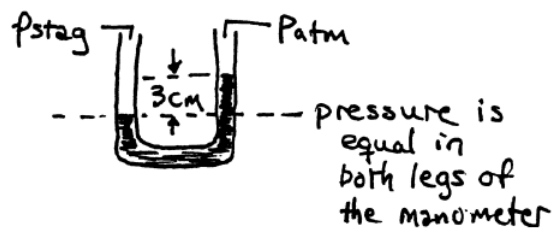
$$p_A + (\rho g h)_{Water} = p_{atm} + (\rho g h)_{Mercury}$$

$$p_A + 1000 \times 9.8 \times 20\times10^{-2} = 101325 + 13.6\times1000\times9.8\times50\times10^{-2}$$

Therefore $\quad p_A = 165.747 \times 10^3\ N/m^2$

23)

The pressure at the surface of the mercury in the left leg is p_{stag}.



Since the pressure at a given level in the mercury is equal in both legs,

$$p_{stag} = p_{atm} + \rho_{Hg}\, g(\Delta h)$$

$$p_{stag} - p_{atm} = \left[13595.1\ \frac{Kg}{m^3}\right]\left[9.8066\ \frac{m}{s^2}\right]\left[0.03\ m\right]$$

$$= 3999.65\ \frac{N}{m^2}$$

Since the difference $p_{stag} - p_{atm}$ is the gage pressure

$$p_{stag} = 3999.65\ \frac{N}{m^2},\ \text{gage}$$

24)

Refer to Figure 12.5, "Thermodynamic properties of air in chemical equilibrium"

When dissociation occurs, the equation of state can be written

$$p = z \rho R T$$

If $z = 1$, $m_o = m$ and the equation of state matches that for part (a). There is a line in Fig. 12.5 for which $z = 1.01$

(b) Referring to Fig. 12.5, the line designated 1000K (or 1800°R) is "horizontal", independent of pressure. Thus, within our ability to read these lines, a calorically perfect gas is one where the temperature is "1000K", or less.