Aerodynamics - Sheet 2 - Solution

Fundamentals of Fluid Mechanics

1.

Let us use the continuity equation for a three-dimensional flow, i.e., equation (2.1):

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \overset{?}{=} 0$$

For constant density flow, this equation becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Thus,

$$\frac{\partial}{\partial x}\left\{-\frac{2xyz}{(x^2+y^2)^2}U_\infty L\right\} + \frac{\partial}{\partial y}\left\{\frac{(x^2-y^2)z}{(x^2+y^2)^2}U_\infty L\right\} + \frac{\partial}{\partial z}\left\{\frac{y}{x^2+y^2}U_\infty L\right\} = 0$$

Since U_∞ and L are constants and since they appear in every term, they can be divided out leaving:

$$-\frac{2yz}{(x^2+y^2)^2} - \frac{2xyz(-2)(2x)}{(x^2+y^2)^3} - \frac{2yz}{(x^2+y^2)^2} + \frac{(x^2-y^2)z(-2)(2y)}{(x^2+y^2)^3}$$

$$= -\frac{4yz}{(x^2+y^2)^2} - \frac{-8x^2yz + 4x^2yz - 4y^3z}{(x^2+y^2)^3}$$

$$= \frac{-4x^2yz - 4y^3z + 8x^2yz - 4x^2yz + 4y^3z}{(x^2+y^2)^3} = 0$$

Therefore, the continuity equation is satisfied.

2.

For incompressible flow this becomes:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Find the derivatives of the given velocity components:

$$\frac{\partial u}{\partial x}=3(x^2+z) \quad \frac{\partial u}{\partial y}=3(y^2+z)$$

Therefore:

$$\frac{\partial w}{\partial z}=-3(x^2+y^2+2z)$$

Integrating yields:

$$w=-3z\,(x^2+y^2+z)=f(x,y,z,t)$$

Where $f(x,y,z,t)$ is an arbitrary function (x,y,z,t). Since the first two velocity components are not a function of time, it may be possible to assume the flow is steady and drop the time function from the arbitrary constant.

3.

Given: Velocity components for a 2D incompressible flow:

$$u = \frac{C\left(y^2 - x^2\right)}{\left(x^2 + y^2\right)^2} \qquad v = -\frac{2Cxy}{\left(x^2 + y^2\right)^2}$$

Assume 2D incompressible flow and that C is a constant. For 2D incompressible flow the continuity equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Taking the required derivatives yields:

$$\frac{\partial u}{\partial x} = C\left(y^2 - x^2\right)\left(-2\right)\left(x^2 + y^2\right)^{-3}\left(2x\right) + C\left(-2x\right)\left(x^2 + y^2\right)^{-2}$$

$$\frac{\partial v}{\partial y} = -2Cxy\left(-2\right)\left(x^2 + y^2\right)^{-3}\left(2y\right) + C\left(-2x\right)\left(x^2 + y^2\right)^{-2}$$

$$\frac{-4Cx\left(y^2 - x^2\right)}{\left(x^2 + y^2\right)^3} - \frac{2Cx}{\left(x^2 + y^2\right)^2} + \frac{8Cxy^2}{\left(x^2 + y^2\right)^3} - \frac{2Cx}{\left(x^2 + y^2\right)^2} = 0$$

after some algebra and patience!

4.

Referring to the continuity equation for a two-dimensional, incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \frac{1}{2}\frac{a_1 y}{x^{1.5}} - \frac{3}{2}\frac{a_2 y^3}{x^{2.5}}$$

Integrating with respect to y

$$v = +\frac{1}{4}\frac{a_1 y^2}{x^{1.5}} - \frac{3}{8}\frac{a_2 y^4}{x^{2.5}} + C$$

To evaluate the constant of integration C, we note that $v = 0$ when $y = 0$. Thus, $C = 0$ and

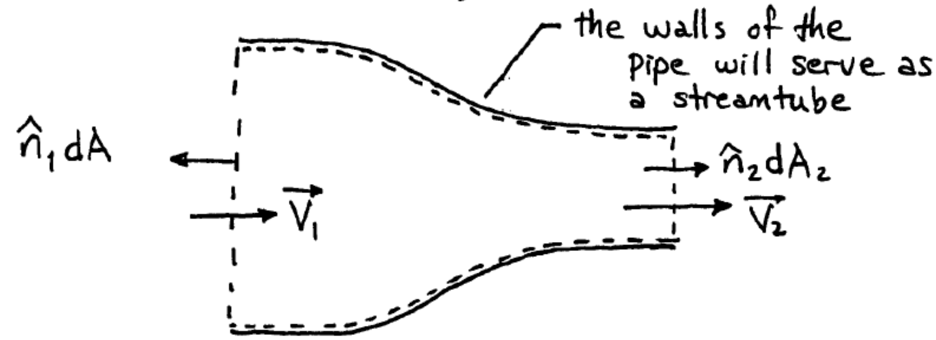
$$v = \frac{1}{4}\frac{a_1 y^2}{x^{1.5}} - \frac{3}{8}\frac{a_2 y^4}{x^{2.5}}$$

5.

Using the integral form of the continuity equation for steady flow, we can use equation (2.5)

$$\frac{\partial}{\partial t}\iiint_{Vol} \rho \, d(vol) + \oiint_{Area} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

to solve this problem. Let us draw a control volume between stations 1 and 2,



The vectors representing the areas ($\hat{n} dA$) are directed outward from the control volume, as shown in the sketch. The velocities represent an assumed flow from left to right. Using the vector dot products and noting that the flow properties do not vary across the cross-section, we obtain:

$$-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$$

where V_1 and V_2 are the magnitudes of the velocity vectors, $\vec{V}_1$ and $\vec{V}_2$, respectively. Thus,

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \text{ (and by extension)} = \rho_3 V_3 A_3$$

(Often we see the expression for steady, one-dimensional flow in a streamtube as:

$$\rho V A = \text{constant}$$

The duct need not be straight, providing the flow is approximately one-dimensional. Thus, the equation is often applied to flow in curved pipes and elbows.)

For this flow, water can be assumed to be of constant density. Thus,

$$\rho_1 = \rho_2 = \rho_3$$

As a result,

$$V_1 A_1 = V_2 A_2 = V_3 A_3 = 0.5 \frac{m^3}{s}$$

$$V_1 \left[\frac{\pi}{4}(0.4)^2\right] = V_2 \left[\frac{\pi}{4}(0.2)^2\right] = V_3 \left[\frac{\pi}{4}(0.6)^2\right] = 0.5$$

Solving,

$$V_1 = 3.979 \frac{m}{s} \; ; \; V_2 = 15.915 \frac{m}{s} \; ; \; V_3 = 1.768 \frac{m}{s}$$

6.

Let us use the integral form of the continuity equation. Note that the effects of viscosity are such that there is a significant reduction of the velocity in the wake of the airfoil (at station 2). Thus, for this rectangular control volume, a significant fraction of the mass influx at station 1 does not leave the control volume through station 2. Thus, some fluid must exit through planes 3 and 4. Thus, they are obviously not streamlines.

$$\frac{\partial}{\partial t} \iiint \rho \, d(vol) + \oiint \rho \vec{V} \cdot \hat{n} \, dA = 0$$

By continuity, we know that the is a v-component of velocity in the wake of the airfoil and that $v(x, y)$ in 2. Along surface 3

$$\vec{V}_3 = U_\infty \hat{i} + v_\infty(x)\hat{j}$$

and along surface (4)

$$\vec{V}_4 = U_\infty \hat{i} - v_\infty(x)\hat{j}$$

Since the flow is steady, the mass fluxes per unit depth in the continuity equation can be written:

$$\underbrace{\rho \int_{-H}^{+H} \left[U_\infty \hat{i}\right]\cdot\left[-\hat{i}\,dy\right]}_{(1)} + \underbrace{\rho \int_{-H}^{0} \left[-\frac{U_\infty y}{H}\hat{i} - v\hat{j}\right]\cdot\left[\hat{i}\,dy\right]}_{(2a)}$$

$$+ \underbrace{\rho \int_{0}^{H} \left[\frac{U_\infty y}{H}\hat{i} + v\hat{j}\right]\cdot\left[\hat{i}\,dy\right]}_{(2b)} + \underbrace{\rho \int_{0}^{L} \left[U_\infty \hat{i} + v_\infty \hat{j}\right]\cdot\left[\hat{j}\,dy\right]}_{(3)}$$

$$+ \rho \int_{0}^{L} \left[U_\infty \hat{i} - v_\infty \hat{j}\right]\cdot\left[-\hat{j}\,dx\right] = 0$$

Note that the vertical component of velocity does not transport fluid across the surface at station (2) and that the horizontal component of velocity does not transport fluid across the surface at stations (3) and (4). This is because these velocity components are perpendicular to the area "vectors" at the station. Thus,

$$-\rho U_\infty 2H + \rho \frac{U_\infty}{H}\left(-\frac{y^2}{2}\right|_{-H}^{0} + \rho \frac{U_\infty}{H}\left(+\frac{y^2}{2}\right|_{0}^{H}$$

$$+ \rho \int_0^L v_\infty\,dx + \rho \int_0^L v_\infty\,dx = 0$$

The last two terms represent the total mass flow across the surfaces (3) and (4). The density is common to every term. We can divide by the density to get the volumetric flow across (3) and (4) $\left[2\int_0^L v_\infty\,dx\right] = U_\infty H$

7.

Let us apply the integral form of the continuity equation. Note that, since surfaces 3 and 4 are streamlines, flow passes through only surfaces 1 and 2.

$$\frac{\partial}{\partial t}\cancel{\iiint \rho\, d(vol)}^{\text{steady}} + \oiint \rho \vec{V}\cdot\hat{n}\,dA = 0$$

Since the flow is incompressible and steady, we can write the continuity equation as

$$\underbrace{-\rho U_\infty \int_{-H_u}^{+H_u} dy}_{1} + \underbrace{\rho U_\infty \int_{-H_D}^{0}\left(-\frac{y}{H_D}\right)dy}_{2a}$$

$$+\underbrace{\rho U_\infty \int_{0}^{H_D}\left(\frac{y}{H_D}\right)dy}_{2b} = 0$$

(Refer to Problem 2.10 to see how to handle the u-component of velocity at station 2.)

$$-\rho U_\infty (2H_u) - \frac{\rho U_\infty}{H_D}\left(\frac{y^2}{2}\right|_{-H_D}^{0} + \frac{\rho U_\infty}{H_D}\left(\frac{y^2}{2}\right|_{0}^{H_D} = 0$$

Rearranging and dividing through by ρU_∞ (which is a common factor to every term), we obtain:

$$H_u = \frac{1}{2}H_D$$

8.

Let us apply the integral form of the continuity equation. Note that the effects of viscosity have caused a significant reduction of velocity in the wake of the airfoil (at station ②). As a result, there is a v-component of velocity which produces a mass flux across planes ③ and ④, because they are horizontal (perpendicular to the v-component).

The flow is steady and incompressible. As a result, the integral continuity equation becomes.

$$\oiint \vec{V} \cdot \hat{n}\, dA = 0$$

$$\underbrace{\int_{-H}^{+H} \left[U_\infty \hat{i}\right] \cdot \left[-\hat{i}\, dy\right]}_{1} + \underbrace{\int_{-H}^{+H} \left[U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2H}\right)\hat{i} + v\hat{j}\right] \cdot \left[\hat{i}\, dy\right]}_{2}$$

$$+ \underbrace{\int_0^L \left[U_\infty \hat{i} + v_\infty \hat{j}\right] \cdot \left[\hat{j}\, dx\right]}_{3} + \underbrace{\int_0^L \left[U_\infty \hat{i} - v_\infty \hat{j}\right] \cdot \left[-\hat{j}\, dx\right]}_{4} = 0$$

Note that the vertical component of velocity does not transport fluid across the surface at station ② and that the horizontal component of velocity does not transport fluid across stations ③ and ④. This is because these velocity components are perpendicular to the area "vectors". Thus,

$$-U_\infty(2H) + U_\infty \left[y - 0.5 \frac{2H}{\pi} \sin \frac{\pi y}{2H} \right] \Bigg|_{-H}^{+H}$$

$$+ \int_0^L v_\infty\, dx + \int_0^L v_\infty\, dx = 0$$

The last two terms represent the total volumetric flow across surfaces ③ and ④. Since the flow is planar symmetric at stations ① and ②, we'll assume that the volumetric flow rate across ③ is equal to that across ④.

$$\left[2 \int_0^L v_\infty\, dx\right] = 2HU_\infty - 2HU_\infty + \frac{HU_\infty}{\pi}\left[1 - (-1)\right]$$

$$\left[2 \int_0^L v_\infty\, dx\right] = \frac{2HU_\infty}{\pi}$$

9.

Let us apply the integral form of the continuity equation. Note that, since surfaces ③ and ④ are streamlines, fluid can cross only surfaces ① and ②. Since the flow is steady,

$$\underbrace{\cancel{\frac{\partial}{\partial t}\iiint}}_{\text{steady}=0} \rho\, d(\text{vol}) + \oiint \rho \vec{V}\cdot\hat{n}\, dA = 0$$

Because the flow is incompressible (i.e., ρ = constant),

$$-\rho U_\infty \int_{-H_u}^{+H_u} dy + \rho U_\infty \int_{-H_D}^{+H_D} \left(1 - 0.5\cos\frac{\pi y}{2H_D}\right) dy = 0$$

Note that we have eliminated the v-component of velocity at station ②, since it doesn't contribute to the mass flux. See the discussion of terms (2a) and (2b) in Problem 2.10.

We can divide through by ρU_∞ and obtain:

$$-2H_u + \left[y - 0.5\,\frac{2H_D}{\pi}\sin\frac{\pi y}{2H_D}\right]_{-H_D}^{+H_D} = 0$$

$$-2H_u + \left[2H_D - \frac{H_D}{\pi}(1+1)\right] = 0$$

$$H_u = H_D\left[1 - \frac{1}{\pi}\right] = 0.6817\, H_D$$

10.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + u\frac{\partial\vec{V}}{\partial x} + v\frac{\partial\vec{V}}{\partial y} + w\frac{\partial\vec{V}}{\partial z}$$

$$\vec{a} = 2t\hat{i} - 10\hat{j} + \left[6 + 2xy + t^2\right]\left[2y\hat{i} - y^2\hat{j}\right]$$

$$- \left[xy^2 + 10t\right]\left[2x\hat{i} - 2xy\hat{j}\right] + 25\,[0]$$

when (x, y, z) is $(3, 0, 2)$ and $t = 1$

$$\vec{a} = \hat{i}\,[2 - 60] + \hat{j}\,[-10] = -58\hat{i} - 10\hat{j}$$

11.

Given: A mass flow rate for the cabin air of:

$$\dot{m}_c = -0.040415 \frac{p_c}{\sqrt{T_c}} [A_{hole}]$$

Using the Ideal Gas Law and the definition of density:

$$p = \rho RT \qquad \rho = \frac{m}{\forall}$$

The pressure becomes:

$$p = \frac{m}{\forall} RT$$

And the mass flow rate equation can be rewritten as:

$$\dot{m}_c = -0.040415 \frac{m_c}{\forall} R_c \sqrt{T_c} [A_{hole}]$$

and:

$$\frac{\dot{m}_c}{m_c} = -\frac{0.040415}{\forall} R_c \sqrt{T_c} [A_{hole}]$$

But the mass flow rate is defined as $\dot{m} = dm/dt$ and the relationship can be integrated as:

$$\int_{m_{c_i}}^{m_{c_f}} \frac{dm}{m} = -\frac{0.040415}{\forall} R_c \sqrt{T_c} [A_{hole}] \int_0^{t_f} dt$$

where i represents an initial value and f represents a final value. Solving for the final time:

$$t_f = \frac{-\forall}{0.040415 R_c \sqrt{T_c} A_{hole}} \ln\left(\frac{m_{c_f}}{m_{c_i}}\right)$$

Since $T_c = 22^\circ C$ we see that $m_{c_f} / m_{c_i} = p_{c_f} / p_{c_i}$ and:

$$t_f = \frac{-\forall}{0.040415 R_c \sqrt{T_c} A_{hole}} \ln\left(\frac{p_{c_f}}{p_{c_i}}\right)$$

Using $\forall = 71\ m^3$ and consistent units, we get:

$$t_f = 5589 s = 1.55 hours$$

12.

Let us apply the integral form of the momentum equation. Since we are interested in the drag, we only need to consider the x-component of this vector equation. Refer to the solution for Problem 2.7 for the discussion of the continuity equation of this flow.

$$\sum F_x = \frac{\partial}{\partial t}\iiint \rho V_x \, d(vol) + \oiint (\rho \vec{V}\cdot \hat{n}\, dA) V_x$$

Since the pressure is constant over the external surface of the control volume, the only force for the left-hand side is the force of the airfoil on the fluid within the control volume, which is the negative of the drag per unit span.

$$-d = \underbrace{\rho \int_{-H}^{+H} (U_\infty \hat{i})(-\hat{i}\, dy) U_\infty}_{1}$$

$$+ \underbrace{\rho \int_{-H}^{0} \left[\left(-U_\infty \frac{y}{h}\hat{i} - v\hat{j}\right)\cdot(\hat{i}\, dy)\right] \frac{-U_\infty y}{h}}_{2}$$

$$+ \underbrace{\rho \int_{0}^{H} \left[\left(U_\infty \frac{y}{h}\hat{i} + v\hat{j}\right)\cdot(\hat{i}\, dy)\right] \frac{U_\infty y}{h}}_{2}$$

$$+ \underbrace{\rho \int_{0}^{L} \left[(U_\infty \hat{i} + v_\infty \hat{j})\cdot(\hat{j}\, dx)\right] U_\infty}_{3}$$

$$+ \underbrace{\rho \int_{0}^{L} \left[(U_\infty \hat{i} - v_\infty \hat{j})\cdot(-\hat{j}\, dx)\right] U_\infty}_{4}$$

Note that because of the approximations that we have employed, the velocity at the boundaries 3 and 4 actually exceeds U_∞, while the static pressure remains unchanged. These are "second-order inconsistencies" introduced by our flow model approximations.

Note also that v_∞ is some unspecified function of x. The exact functional relationship is not important. Using the result from the application of the continuity equation in Problem 2.10:

$$2\int_0^L v_\infty \, dx = U_\infty H$$

Thus, $$-d = -\rho U_\infty^2(2H) + \rho \frac{U_\infty^2}{H^2}\left(\frac{y^3}{3}\right)\Bigg|_{-H}^{0} + \rho \frac{U_\infty^2}{H^2}\left(\frac{y^3}{3}\right)\Bigg|_{0}^{H} + \rho U_\infty \left[2\int_0^L v_\infty \, dx\right]$$

can be written:

$$-d = -\rho U_\infty^2(2H) + \rho U_\infty^2 \frac{H}{3} + \rho U_\infty^2 \frac{H}{3} + \rho U_\infty^2 H$$

$$d = \frac{1}{3}\rho U_\infty^2 H$$

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\frac{1}{3}\rho U_\infty^2 H}{\frac{1}{2}\rho U_\infty^2 c} = \frac{1}{60} = 0.0167$$

13.

This is very similar to Problem 2.27, except that the side boundaries of the control volume are streamlines. Thus, instead of using the continuity equation to determine the flow through sides 3 and 4 as was done for Problem 2.27, the continuity equation must be used to determine the relation between H_U and H_D.

Again, the pressure is constant over the external surface of the control volume for this steady, incompressible flow. Thus, the only force acting on the system of the fluid particles within the control volume is the negative of the drag.

$$-d = \rho \underbrace{\int_{-H_U}^{+H_U} \left[(U_\infty \hat{i}) \cdot (-\hat{i} dy) \right] U_\infty}_{1}$$

$$+ \rho \underbrace{\int_{-H_D}^{0} \left[\left(-U_\infty \frac{y}{H_D} \hat{i} - v\hat{j}\right) \cdot (\hat{i} dy) \right] \left(-U_\infty \frac{y}{H_D}\right)}_{2}$$

$$+ \rho \underbrace{\int_{0}^{H_D} \left[\left(U_\infty \frac{y}{H_D} \hat{i} + v\hat{j}\right) \cdot (\hat{i} dy) \right] \left(U_\infty \frac{y}{H_D}\right)}_{2}$$

There is no momentum transport across boundaries 3 and 4, since they are streamlines.

$$-d = \rho U_\infty^2 \left[2H_U\right] + \rho U_\infty^2 \left[\frac{y^3}{3H_D^2}\right]_{-H_D}^{0} + \rho U_\infty^2 \left[\frac{y^3}{3H_D^2}\right]_{0}^{+H_D}$$

$$d = \rho U_\infty^2 \left[2H_U - \frac{2}{3} H_D\right]$$

We can use the integral continuity equation to determine the relation between H_U and H_D for this steady, incompressible flow:

$$+\underbrace{\int_{-H_U}^{+H_U}\left[(U_\infty\hat{i})\cdot(-\hat{i}\,dy)\right]}_{①} + \underbrace{\int_{-H_D}^{0}\left[\left(-\frac{U_\infty y}{H_D}\hat{i} - v\hat{j}\right)\cdot(\hat{i}\,dy)\right] + \int_{0}^{H_D}\left[\left(+\frac{U_\infty y}{H_D}\hat{i} + v\hat{j}\right)\cdot(\hat{i}\,dy)\right]}_{②} = 0$$

Thus, $$U_\infty 2H_U - \frac{U_\infty y^2}{2H_D}\Bigg|_{-H_D}^{0} + \frac{U_\infty y^2}{2H_D}\Bigg|_{0}^{H_D} = 0$$

Therefore, $U_\infty 2H_U = U_\infty H_D$

as was shown in Problem 2.11, $H_U = \frac{1}{2}H_D$

$$d = \rho U_\infty^2\left[H_D - \frac{2}{3}H_D\right] = \frac{1}{3}\rho U_\infty^2 H_D$$

$$C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c} = \frac{\frac{1}{3}\rho U_\infty^2\left(\frac{1}{40}c\right)}{\frac{1}{2}\rho_\infty U_\infty^2 c} = \frac{1}{60} = 0.0167$$

As one would expect, we have gotten the same result as was obtained in Problem 2.28. Therefore, the result is not dependent on the control volume.

14.

Let us apply the integral form of the momentum equation. Since we are interested in the drag acting on the airfoil, which is aligned with the x-axis, we need only consider the x-component of this vector equation.

$$\sum F_x = \frac{\partial}{\partial t}\iiint \rho V_x \, d(\text{vol}) + \oiint \rho(\vec{V}\cdot\hat{n}\,dA)V_x$$

Since atmospheric pressure acts over the entire external surface of the control volume, the only force acting on the fluid particles within the control volume (i.e., the left-hand side of this equation) is the negative of the drag. Furthermore, the flow is steady and the first term on the right-hand side is zero. The flow is incompressible (or the density is constant). Thus,

$$-d = +\rho\int_{-H}^{+H}\left[(U_\infty\hat{i})\cdot(-\hat{i}\,dy)\right]U_\infty$$

$$+\rho\int_{-H}^{+H}\left\{\left[U_\infty\left(1-0.5\cos\frac{\pi y}{2H}\right)\hat{i}\pm v\hat{j}\right]\cdot(\hat{i}\,dy)\right\}\left[U_\infty\left(1-0.5\cos\frac{\pi y}{2H}\right)\right] + 2\rho\int_0^L\left[U_\infty\hat{i}+v_\infty\hat{j}\right]\cdot(\hat{j}\,dx)\,U_\infty$$

$$-d = \rho U_\infty^2 y\Big|_{-H}^{+H} + \rho U_\infty^2\int_{-H}^{+H}\left[1-\cos\frac{\pi y}{2H}+0.25\cos^2\frac{\pi y}{2H}\right]dy + 2\rho U_\infty\int_0^L v_\infty\,dx$$

We can use the integral form of the continuity equation to find $\int_0^L v_\infty\,dx$

$$\rho\int_{-H}^{+H}(U_\infty\hat{i})\cdot(-\hat{i}\,dy) + \rho\int_{-H}^{+H}\left[U_\infty\left(1-0.5\cos\frac{\pi y}{2H}\right)\hat{i} + v\hat{j}\right]\cdot(\hat{i}\,dy) + 2\rho\int_0^L\left[U_\infty\hat{i}+v_\infty\hat{j}\right]\cdot\hat{j}\,dx = 0$$

$$-\rho U_\infty y\Big|_{-H}^{+H} + \rho U_\infty\int_{-H}^{+H}\left(1-0.5\cos\frac{\pi y}{2H}\right)dy + 2\rho\int_0^L v_\infty\,dx = 0$$

$$-\rho U_\infty(2H) + \rho U_\infty(2H) - \rho U_\infty\frac{1}{2}\frac{2H}{\pi}\left[1-(-1)\right] + 2\rho\int_0^L v_\infty\,dx = 0$$

Thus, $2\rho\int_0^L v_\infty\,dx = \rho U_\infty\frac{2H}{\pi}$

Substituting this result into the momentum equation

$$-d = -\rho U_\infty^2(2H) + \rho U_\infty^2\left(y\Big|_{-H}^{+H} - \frac{2H}{\pi}\sin\frac{\pi y}{2H}\Big|_{-H}^{+H}\right.$$

15.

let us apply the integral equations to solve this problem. Again, since atmospheric pressure acts over the external surface of the control volume, the only force acting on the fluid particles within the control volume, i.e., the left-hand side of the integral momentum equation, is the negative of the drag. Furthermore, the flow is steady, so that the first term on the right-hand side of the momentum equation is zero; incompressible, so that the density is constant; and surfaces ③ and ④ are streamlines. Thus, there is no flux of momentum across these streamlines.

$$-d = -\rho U_\infty^2 \int_{-H_U}^{+H_U} dy + \rho \int_{-H_D}^{+H_D} \left[U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2 H_D}\right) \right.$$

$$\left. \text{times } dy \right] U_\infty \left(1 - 0.5 \cos \frac{\pi y}{2 H_D}\right)$$

$$-d = -\rho U_\infty^2 (2 H_U) + \rho U_\infty^2 \int_{-H_D}^{+H_D} \left(1 - 1.0 \cos \frac{\pi y}{2 H_D} + 0.25 \cos^2 \frac{\pi y}{2 H_D}\right) dy$$

$$-d = -\rho U_\infty^2 (2 H_U) + \rho U_\infty^2 \left[y - \frac{2 H_D}{\pi} \sin \frac{\pi y}{2 H_D} + \frac{1}{4}\left(\frac{y}{2} + \frac{2 H_D}{4\pi} \sin \frac{\pi y}{2 H_D}\right)\right]\Bigg|_{-H_D}^{+H_D}$$

$$-d = -\rho U_\infty^2 (2 H_U) + \rho U_\infty^2 (2 H_D) - \rho U_\infty^2 \frac{2}{\pi} H_D (1 + 1) + \rho U_\infty^2 \frac{1}{4}\left(\frac{H_D}{2} + \frac{H_D}{2}\right)$$

Using the results from Problem 2.13 that

$$H_U = \left(1 - \frac{1}{\pi}\right) H_D$$

$$-d = -\rho U_\infty^2 H_D \left[2 - \frac{2}{\pi} - 2 + \frac{4}{\pi} - \frac{1}{4}\right]$$

$$d = \rho U_\infty^2 H_D (0.3866)$$

The drag coefficient is:

$$C_d = \frac{d}{\frac{1}{2} \rho U_\infty^2 c} = \frac{\rho U_\infty^2 (0.025 c)(0.3866)}{\frac{1}{2} \rho U_\infty^2 c} = 0.01933$$

Again, comparison of the results from Problems 2.30 and 2.31 shows that the drag coefficient does not depend on the control volume chosen.

16.

a) Given:

Diameter of golf ball (d) is = 4.5 cm

Velocity of golf ball (u_∞) is = 60 m/s.

From Table 1.2, at standard sea level atmosphere

$$\rho_\infty = 1.225\ kg/m^3 \quad \mu_\infty = 1.7894 \times 10^{-5}\ kg/m\text{-}s$$

And the sound speed a_∞= 340.29 m/s

So $$Re_{\infty,\ d} = \frac{\rho_\infty u_\infty d}{\mu_\infty} = \frac{1.225 \times 60 \times 4.5 \times 10^{-2}}{1.7894 \times 10^{-5}}$$

$$Re_{\infty,\ d} = 1.848 \times 10^5$$

$$M_\infty = \frac{u_\infty}{a_\infty} = \frac{60}{340.29} = 0.176$$

b) Given:

Characteristic length (L) is = 70.6 m

Speed of Boeing 747 (u_∞) is = 250 m/s

From Table 1.2, at an altitude of 10 km from sea level

$$\rho_\infty = 0.4135\ kg/m^3 \quad \mu_\infty = 1.4576 \times 10^{-5}\ kg/m\text{-}s$$

And the sound speed a_∞= 299.53 m/s

So $$Re_{\infty,\ d} = \frac{\rho_\infty u_\infty d}{\mu_\infty} = \frac{0.4135 \times 250 \times 70.6}{1.4576 \times 10^{-5}}$$

$$Re_{\infty,\ d} = 5 \times 10^8$$

$$M_\infty = \frac{u_\infty}{a_\infty} = \frac{250}{299.53} = 0.834$$

17.

(a) $M_\infty = 3.0$ at an altitude of 20km. Using Table 1.2

$$a_\infty = 295.069 \frac{m}{s} \; ; \mu_\infty = 1.4216 \times 10^{-5} \frac{kg}{s.m} \; ; \rho_\infty = 0.0889 \frac{kg}{m^3}$$

$$Re_{\infty,L} = \frac{(0.0889)\left[(3.0)(295.069)\right](10.4)}{1.4216 \times 10^{-5}} = 5.757 \times 10^7$$

Again, the Reynolds number for a high-speed airplane is in excess of 10^7

(b) Referring to the previous problem, we found the English unit values for the density and for the viscosity at sea level. Thus,

$$Re_{\infty,L} = \frac{\left(2.376 \times 10^{-3} \frac{lbf.s^2}{ft^4}\right)\left[\left(160 \frac{mi}{h}\right)\frac{5280 \frac{ft}{mi}}{3600 \frac{s}{h}}\right](4.0\,ft)}{3.740 \times 10^{-7} \frac{lbf.s}{ft^2}}$$

$$Re_{\infty,L} = 5.963 \times 10^6$$

18.

Given:

Mach number of SR-71 (M_∞) is = 2.0.

From Table 1.2, at an altitude of 25 km from sea level

Temperature (T_∞) = 221.555 K

From Mach number and total temperature relationship

$$\frac{T_t}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \qquad \text{For air } \gamma = 1.4$$

$$T_t = 221.555\left(1 + \frac{1.4 - 1}{2}\,[2.0]^2\right) = 398.8\ K$$

The total temperature is much higher as compared to the static temperature. Hence the kinetic energy of this flow is very high. Therefore the convective heating can be a problem for air craft flying at this speed.