SPC 307 - Aerodynamics
Sheet 4
Dynamics of an incompressible, inviscid flow field

1. The velocity in a certain flow field is given by the equation
$$\vec{V} = x\,\hat{\imath} + x^2\,\hat{\jmath} + yz\,\hat{k}$$
Determine the expressions for the three rectangular components of acceleration.

2. The flow in the plane two-dimensional channel shown in Fif. 1. Gas x- and y-components of velocity given by
$$u = u_0\left(1 + \frac{x}{l}\right)\left[1 - \left(\frac{y}{Y}\right)^2\right]$$
$$v = u_0\left[\frac{y^3}{lY_0^2}\left(1 + \frac{x}{l}\right)^2 - \frac{y}{l}\right]$$
Calculate the linear acceleration, rotation, vorticity, rate of volumetric strain, and rate of shear deformation of the flow.

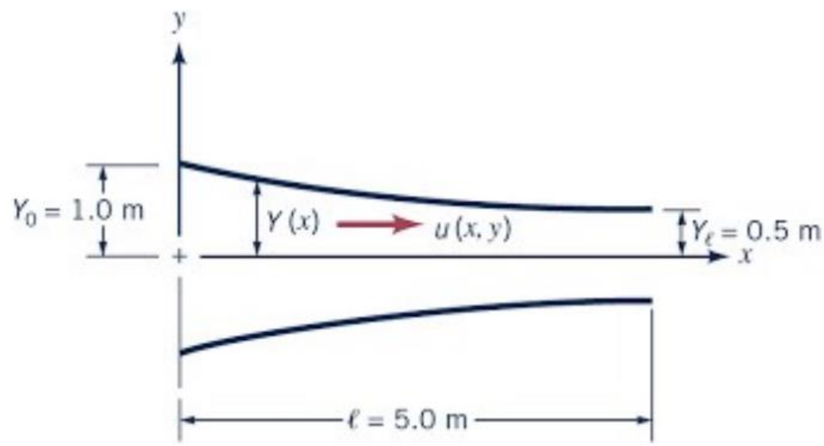

Fig. 1.

3. If Determine an expression for the vorticity of the flow field described by
$$\vec{V} = -xy^3\,\hat{\imath} + y^4\,\hat{\jmath}$$
Is the flow irrotational?

4. The Equation of the x-velocity in fully developed laminar flow between parallel plates is given by

$$u = \frac{1}{2\mu}\left(\frac{\delta p}{\delta x}\right)(y^2 - h^2)$$

   The y-velocity is v=0. Determine the volumetric strain rate, the vorticity and the rate of angular deformation. What is the shear stress at the plate surface?

5. A The stream function for a given two-dimensional flow field is

$$\Psi = 5x^2y - \left(\frac{5}{3}y^3\right)$$

   Determine the corresponding velocity potential.
6. The velocity potential for a certain inviscid flow field is

$$\phi = -(3x^2y - y^3)$$

   where ϕ has the units of ft^2/s when x and y are in feet. Determine the pressure difference (in psi) between the points (1,2) and (4,4), where the coordinates are in feet, if the fluid is water and elevation changes are negligible.

7. A The velocity potential for a certain inviscid, incompressible flow field is given by the equation

$$\phi = 2x^2y - \left(\frac{2}{3}\right)y^3$$

   where ϕ has the units of m^2/s when x and y are in meters. Determine the pressure at the point x = 2 m, y = 2 m if the pressure at x = 1 m, y = 1 m is 200 kPa. Elevation changes can be neglected, and the fluid is water.

8. Consider the two-dimensional flow air flow around the corner shown in in Fig. 2. The x- and y-direction velocities are

$$u = \frac{v_0}{l} \sin h \left(\frac{x}{l}\right) \cos h \left(\frac{y}{l}\right)$$

and

$$u = -\frac{v_0}{l} \cos h \left(\frac{x}{l}\right) \sin h \left(\frac{y}{l}\right)$$

respectively. Assume constant density, steady flow, negligible gravity and inviscid flow. Find $p(x,y)$.

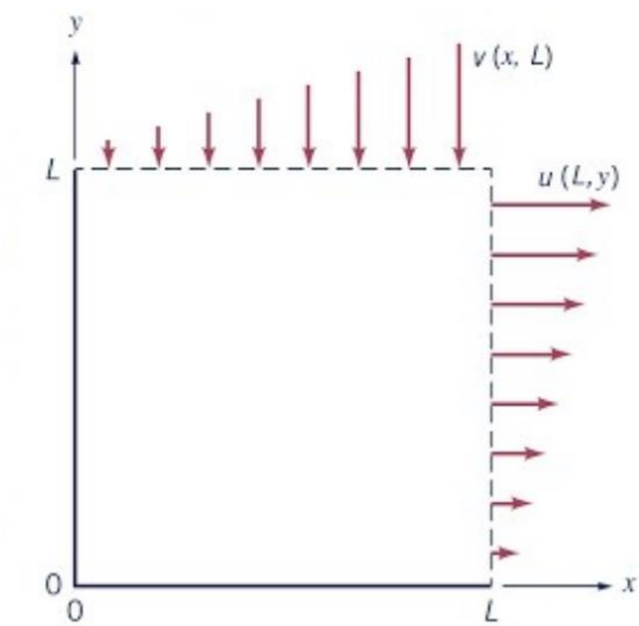

Fig. 2.

9. The velocity potential

$$\phi = -k(x^2 - y^2) \quad (k = \text{constant})$$

may be used to represent the flow against an infinite plane boundary, as illustrated in Fig. 3. For flow in the vicinity of a stagnation point, it is frequently assumed that the pressure gradient along the surface is of the form

$$\frac{\delta p}{\delta x} = Ax$$

where A is a constant. Use the given velocity potential to show that this is true.

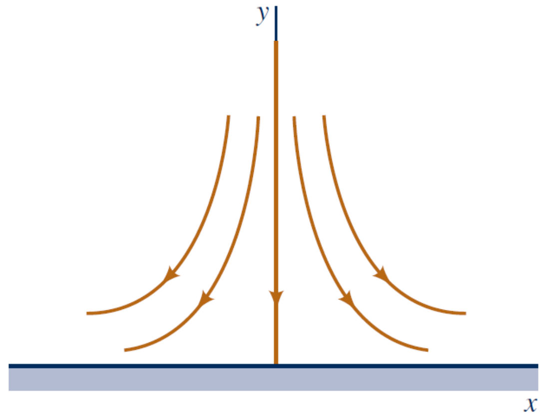

Fig. 3.

10. The velocity potential for a given two-dimensional flow field is

$$\phi = \left(\frac{5}{3}\right)x^3 - 2xy^2$$

Show that the continuity equation is satisfied and determine the corresponding stream function.