SPC 307 - Aerodynamics
Sheet 6 - Solution
Boundary Layer

1. Water flows past a flat plate that is oriented parallel to the flow with an upstream velocity of 0.5 m/s. Determine the approximate location downstream from the leading edge where the boundary layer becomes turbulent. What is the boundary layer thickness at this location?

$$Re_{cr} = 5\times10^5 = \frac{U x_{cr}}{\nu}$$

$$x_{cr} = \frac{5\times10^5 \nu}{U} = \frac{5\times10^5 (1.12\times10^{-6}\ m^2/s)}{0.5\ m/s} = \underline{\underline{1.12\ m}}$$

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{(1.12\times10^{-6} m^2/s)\,1.12 m}{0.5 m/s}} = \underline{\underline{7.92\times10^{-3} m}}$$

2. A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.

Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12\times10^{-3} m}{\sqrt{1.3 m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x \sim m, \delta \sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

3. Approximately how fast can the wind blow past a 0.25- in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re = 1)? Explain. Repeat for a 0.004 in. diameter hair and a 6-ft-diameter smokestack.

$Re = \frac{UD}{\nu} < 1$ or $U < \frac{\nu}{D}$ if viscous effects are to be important throughout the flow.

For standard air $\nu = 1.57 \times 10^{-4} \frac{ft^2}{s}$

Thus,

$U < \frac{1.57 \times 10^{-4}}{D}$, where D is the diameter in feet.

object	D, ft	U, $\frac{ft}{s}$
twig	2.08×10^{-2}	7.54×10^{-3}
hair	3.33×10^{-4}	0.471
smokestack	6	2.62×10^{-5}

4. Air enters a square duct through a 1-ft opening as is shown in Fig. 1. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant U = 2 ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, d, as a function of x for $0 \leq x \leq 10$ ft if U is to remain constant. Assume laminar flow.

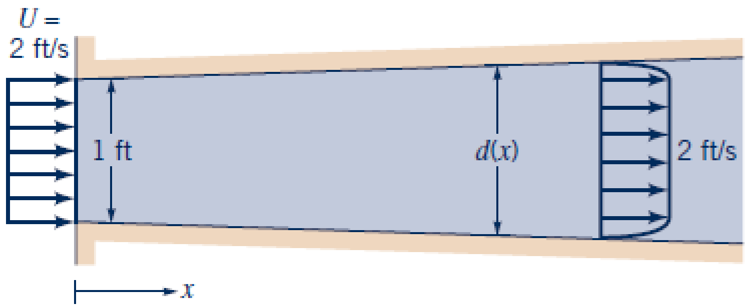


Fig.1

For incompressible flow $Q_o = Q(x)$ where Q_o = flowrate into the duct $= UA_o = (2\,\frac{ft}{s})(1\,ft^2) = 2\,\frac{ft^3}{s}$
and
$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_o = U(d - 2\delta^*)^2 \quad \text{or} \quad d = 1\,ft + 2\delta^*, \qquad (1)$$

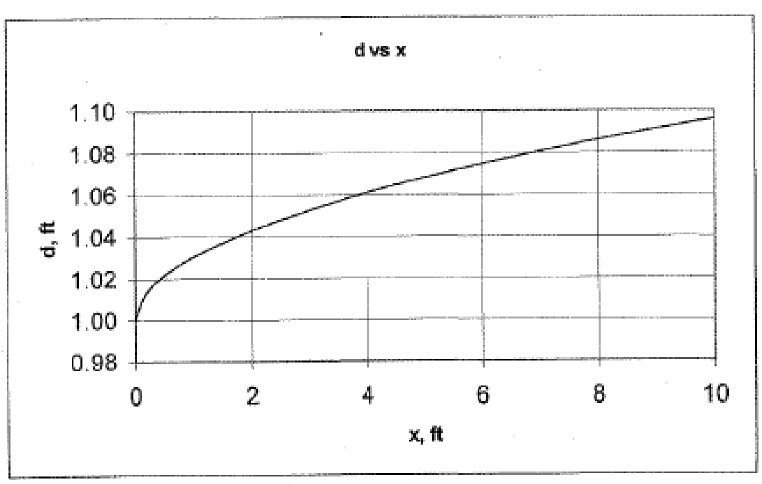
where

$$\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\left[\frac{(1.57\times10^{-4}\,\frac{ft^2}{s})\,x}{2\,\frac{ft}{s}}\right]^{1/2} = 0.0152\sqrt{x}\ ft, \text{ where } x \sim ft$$

Hence, from Eq.(1)

$$\underline{\underline{d = 1 + 0.0304\sqrt{x}\ ft}}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.



5. An atmospheric boundary layer is formed when the wind blows over the Earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. 2, typical values are n = 0.40 for urban areas, n = 0.28 for woodland or suburban areas, and n = 0.16 for flat open country.
(a) If the velocity is 20 ft/s at the bottom of the sail on your boat (y = 4 ft), what is the velocity at the top of the mast (y = 30 ft)?
(b) If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

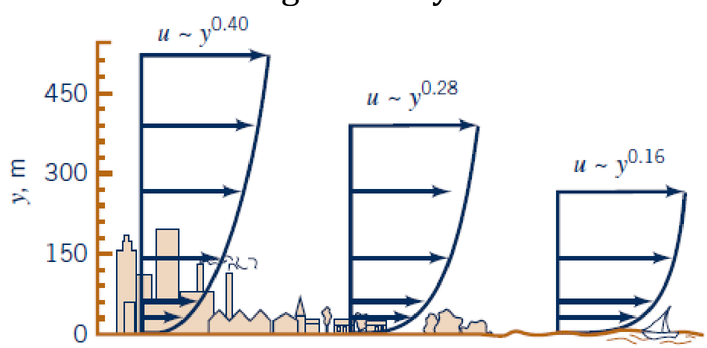


Fig.2

(a) $u = C\, y^{0.16}$, where C is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.16}$ or $u_2 = 20\,\frac{ft}{s}\left(\dfrac{30\,ft}{4\,ft}\right)^{0.16} = \underline{\underline{27.6\,\frac{ft}{s}}}$

(b) $u = \tilde{C}\, y^{0.4}$, where $\tilde{C}$ is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.40}$ or $u_2 = 10\,mph\left(\dfrac{60}{10}\right)^{0.4} = \underline{\underline{20.5\,mph}}$