# Midterm 1 - Solution

**1/11/2016**

1. Consider a flow impinging on the sharp-edged nose of an aircraft. As the Mach number increases from 0.1 to 5, can you explain what happens to the flow over the nose? (use drawings to enhance your answer).

   Solution:

   a) For a subsonic flow $M_\infty < 0.8$, no shock wave appears over the nose.

   b) For a transonic flow $0.8 < M_\infty < 1$, a shock wave appears over the nose.

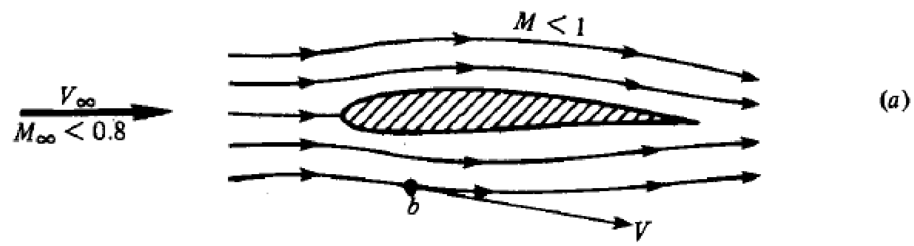
   c) For a transonic flow $1 < M_\infty < 1.2$, a bow shock wave appears before the nose.

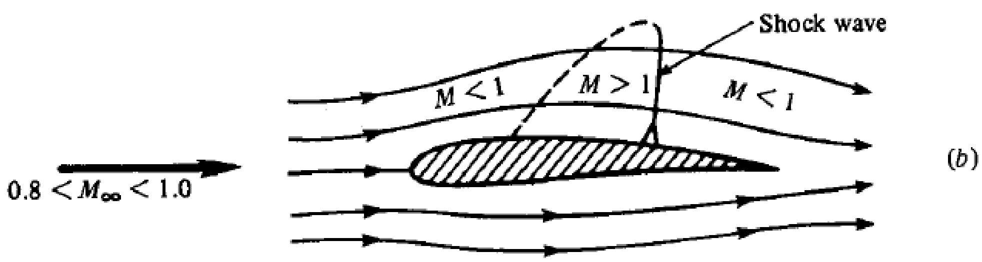
   d) For a transonic flow $M_\infty > 1.2$

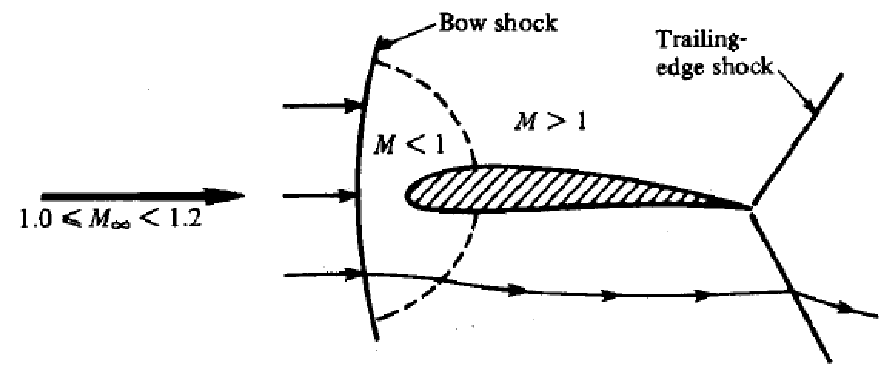
      When the wedge half-angle $\delta$ is greater than the maximum deflection angle $\theta_{max}$, the shock becomes curved and detaches from the nose of the wedge, forming what is called a detached oblique shock or a bow wave.

      When $\delta$ is less than $\theta_{max}$, the oblique shock is attached to the nose.

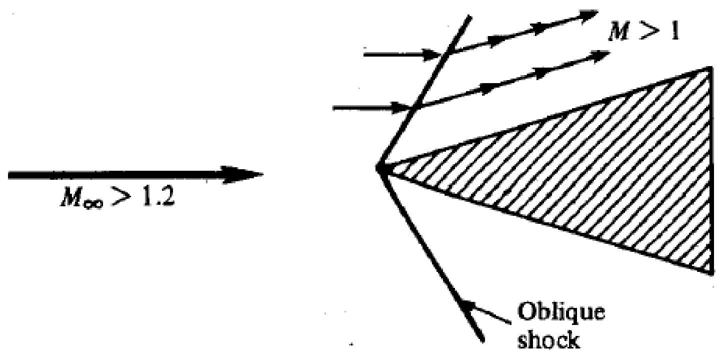
   e) As the Mach number increases in the case of $\delta$ is less than $\theta_{max}$, the shock wave angle $\beta$ decreases.



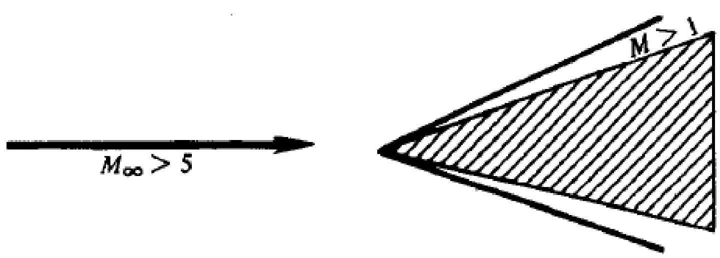




Bow shock
Trailing-edge shock
$M > 1$
$M < 1$
$1.0 \leqslant M_\infty < 1.2$

(*c*)


$M > 1$
$M_\infty > 1.2$
Oblique shock

(*d*)


$M > 1$
$M_\infty > 5$

(*e*)

2. Consider a Pitot static tube mounted on the nose of an experimental airplane. A Pitot tube measures the total pressure at the tip of the probe (hence sometimes called the Pitot pressure), and a Pitot static tube combines this with a simultaneous measurement of the free-stream static pressure. The Pitot and free-stream static measurements are given below for three different flight conditions. Calculate the free-stream Mach number at which the airplane is flying for each of the three different conditions:
    a. Pitot pressure = 1.22 x $10^5$ Pa, static pressure = 1 .01 x $10^5$ Pa
    b. Pitot pressure = 3.4579117 x $10^5$ Pa, static pressure = 1.0131461 x $10^5$ Pa
    c. Pitot pressure= 6.2756644 x $10^5$ Pa, static pressure= 4.883786 x $10^5$ Pa

Solution:

(a) $p/p_o = \dfrac{1.01 \ x \ 10^5}{1.22 \ x \ 10^5}$ = 0.8264. From Table A.13 by interpolation: $\boxed{M_\infty = 0.53}$

(b) $p/p_o = \dfrac{1.0131461 \ x \ 10^5}{3.4579117 \ x \ 10^5}$ = 0.293. From Table A.13, by interpolation: $M_\infty$ = 1.45.

However, since this is supersonic, a normal shock sits in front of the Pitot tube. Hence, $P_o$ is now the total pressure behind a normal shock wave. Thus, we have to use Table A.14.

$p_{o_2} / p_1 = \dfrac{3.4579117 \ x \ 10^5}{1.0131461 \ x \ 10^5}$ = 3.412. From Table A.14: $\boxed{M_\infty = 1.5}$

(c) $p_{o_2} / p_1 = \dfrac{6.2756644 \ x \ 10^5}{4.883786 \ x \ 10^4}$ = 12.85. From Table A.14 by interpolation: $\boxed{M_\infty = 3.1}$

3. Consider the flow through a rocket engine nozzle. Assume that the gas flow through the nozzle is an isentropic expansion of a calorically perfect gas. In the combustion chamber, the gas which results from the combustion of the rocket fuel and oxidizer is at a pressure and temperature of 15 atm and 2500 K, respectively; the molecular weight and specific heat at constant pressure of the combustion gas are 12 and 4157 J/kg.K, respectively. The gas constant of the mixture R = 692.8 J/kg.K and specific heat ratio of 1.2. The gas expands to supersonic speed through the nozzle, with a temperature of 1350 K at the nozzle exit. Calculate the pressure, Mach number and density at the exit of the rocket nozzle.

   Solution:

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 15\ atm\ \left(\frac{1350}{1500}\right)^{\frac{1.4}{0.4}} = 0.372\ atm = 0.372 \times 1.013 \times 10^5\ Pa$$

$$p_2 = \boxed{0.376 \text{ x } 10^5 \text{ Pa}}$$

   In the combustion chamber the flow velocity is very low; hence we can assume that the pressure and temperature in the combustion chamber are essentially $p_o$, and $T_o$, respectively. Moreover, since the flow expansion through the nozzle is isentropic, then $p_o$ , and $T_o$ are constant values throughout the nozzle flow.

$$\left(\frac{p_o}{p}\right)_2 = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

   Solving for $M_2$

$$M_2 = \left\{\frac{2}{\gamma - 1}\left[\left(\frac{p_o}{p_2}\right)^{(\gamma-1)/\gamma} - 1\right]\right\}^{1/2}$$

$$= \left\{\frac{2}{0.2}\left[\left(\frac{15}{0.372}\right)^{0.167} - 1\right]\right\}^{1/2} = \boxed{2.919}$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{(1.2)(692.8)(1350)} = 1059.4 \text{ m/s}$$

$$V_2 = M_2 a_2 = (2.919)(1059.4) = \boxed{3092 \text{ m/s}}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{0.376 \times 10^5}{(692.8)(1350)} = \boxed{0.04 \text{ kg/m}^3}$$