Midterm 2 - Solution

13/12/2016

1. Air flows out of a pipe with a diameter of 0.3 m at a rate of 1000 m^3/min at a pressure and temperature of 150 kPa and 293 K, respectively. If the pipe is 50 m long, assuming that the friction factor, f = 0.005, find the Mach number at the exit, the inlet pressure, and the inlet temperature.

Solution

In this case, the flow situation being considered is shown in Figure 1.

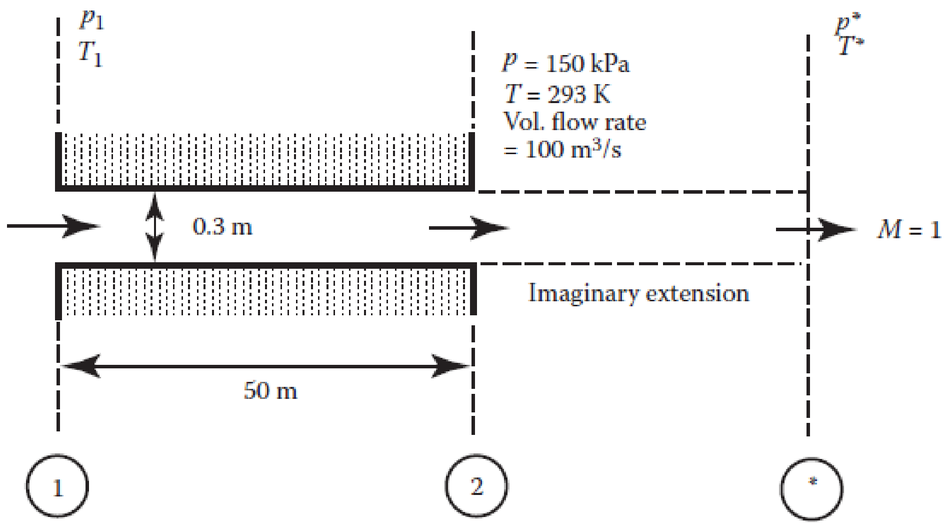

Figure 1.

At the exit, i.e., at point 2

$$V_2 = \frac{Q}{A_2} = \frac{(1000/60)}{(\pi/4)(0.3)^2} = 236 \text{ m/s}$$

However,

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{1.4 \times 287 \times 293} = 343 \text{ m/s}$$

Hence,

$$M_2 = \frac{V_2}{a_2} = \frac{236}{343} = 0.688$$

At point 2 therefore, the relations derived above or tables or the software give

$$\frac{\bar{f}l_2^*}{D} = 0.228$$

$$\frac{p_2}{p^*} = 1.54, \quad \frac{T_2}{T^*} = 1.10$$

Now,

$$l_{1-2} = l_1^* - l_2^*$$

Hence,

$$\frac{fl_{1-2}}{D} = \frac{fl_1^*}{D} - \frac{fl_2^*}{D}$$

From which it follows that

$$\frac{(0.005)(50)}{0.3} = \frac{fl_1^*}{D} - 0.228$$

which gives

$$\frac{fl_1^*}{D} = 1.12$$

From the relations derived above or from tables or using the software for fl_1^*/D= 1.12, the following is obtained:

$$M_1 = 0.4958, \quad p_1/p^* = 2.161, \quad T_1/T^* = 1.1437$$

Hence,

$$p_1 = \frac{p_1/p^*}{p_2/p^*} \times p_1 = \frac{2.161}{1.54} \times 150 = 210.48 \text{ kPa}$$

and

$$T_1 = \frac{T_1/T^*}{T_2/T^*} \times p_1 = \frac{1.1437}{1.1} \times 293 = 304.64 \text{ K} = 31..64°\text{C}$$

Therefore, the Mach number, pressure, and temperature at the inlet are 0.4958, 210.48 kPa, and 31.64°C, respectively.

2. Air flows through a constant area duct. The pressure and temperature of the air at the inlet to the duct are 100 kPa and 10°C, respectively, and the inlet Mach number is 2.8. Heat is transferred to the air as it flows through the duct, and as a result, the Mach number at the exit is 1.3. Find the pressure and temperature at the exit. If no shock waves occur in the flow, find the maximum amount of heat that can be transferred to the air per unit mass of air. Also, find the exit pressure and temperature that would exist with this maximum heat transfer rate. Assume that the flow is steady, that the effects of wall friction can be neglected and that the air behaves as a perfect gas.

Solution:

The flow situation under consideration is shown in Figure 2.

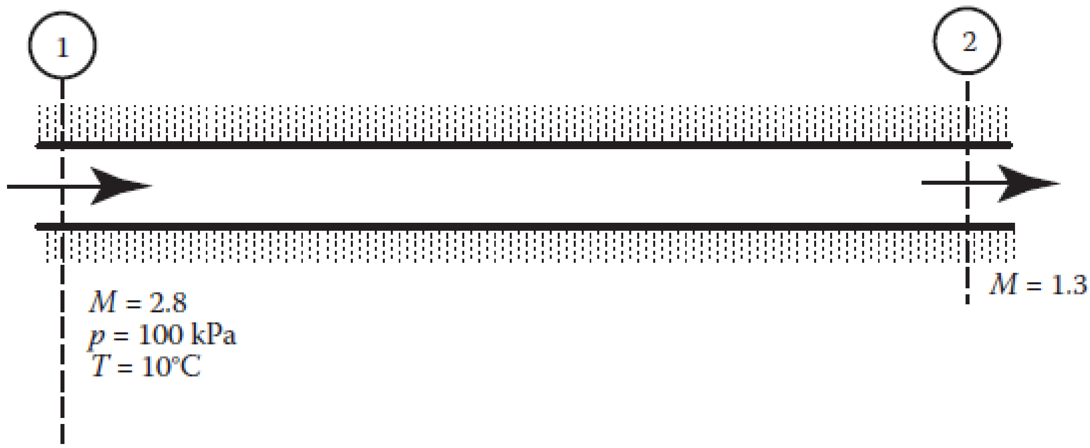

Figure 2.

Now for Mach 2.8, using

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + 0.2M^2$$

or using the isentropic tables gives $T_1/T_{01} = 0.3894$ or $T_{01}/T_1 = 2.568$. Hence,

$$T_{01} = 2.568 \times (273 + 10) = 726.7 \text{ K}$$

Next, using the tables for frictionless flow in constant area duct with heat exchange gives

For M = 2.8,

$$\frac{p}{p^*} = 0.2004, \quad \frac{T}{T^*} = 0.3149, \quad \frac{T_0}{T_0^*} = 0.6738$$

For M = 1.3,

$$\frac{p}{p^*} = 0.7130, \quad \frac{T}{T^*} = 0.8592, \quad \frac{T_0}{T_0^*} = 0.9580$$

Using these values gives

$$p_2 = \frac{p_2/p^*}{p_1/p^*} p_1 = \frac{0.7130}{0.2004} \times 100 = 355.8 \text{ kPa}$$

and

$$T_2 = \frac{T_2/T^*}{T_1/T^*} T_1 = \frac{0.8592}{0.3149} \times 283 = 772.2 \text{ K} = 499.2°\text{C}$$

Therefore, the pressure and temperature of the air at the outlet to the duct are 355.8 kPa and 499.2°C, respectively.
When the maximum amount of heat has been transferred, the Mach number at the exit is 1 and $T_{02} = T_0^*$. Hence, in this case,

$$T_{02} = \frac{T_{01}}{T_{01}/T_0^*} = \frac{726.7}{0.6738} = 1078.5 \text{ K}$$

However, Assuming that c_p = 1.007 kJ/kg.

$$q = c_p(T_{02} - T_{01}) = 1.007 \times (1078.5 - 726.7) = 354.3 \text{ kJ/kg}$$

Also, when the maximum amount of heat has been transferred,

$$p_2 = p^* = \frac{p_1}{p_1/p^*} = \frac{100.7}{0.2004} = 499.0 \text{ kPa}$$

and

$$T_2 = T^* = \frac{T_1}{T_1/T^*} = \frac{283}{0.3149} = 898.7 \text{ K} = 625.7°\text{C}$$

Therefore, when the maximum amount of heat is transferred, the heat added is 354.3 kJ for every kilogram of air flowing through the duct, and under these circumstances, the pressure and temperature of the air at the outlet to the duct are 499.0 kPa and 625.7°C, respectively.