SPC 407
Time allowed: 1 hr

Midterm 2

13/12/2016

Answer The following two questios

1. Air flows out of a pipe with a diameter of 0.3 m at a rate of 1000 m^3/min at a pressure and temperature of 150 kPa and 293 K, respectively. If the pipe is 50 m long, assuming that the friction factor, f = 0.005, find the Mach number at the exit, the inlet pressure, and the inlet temperature.

2. Air flows through a constant area duct. The pressure and temperature of the air at the inlet to the duct are 100 kPa and 10°C, respectively, and the inlet Mach number is 2.8. Heat is transferred to the air as it flows through the duct, and as a result, the Mach number at the exit is 1.3. Find the pressure and temperature at the exit. If no shock waves occur in the flow, find the maximum amount of heat that can be transferred to the air per unit mass of air. Also, find the exit pressure and temperature that would exist with this maximum heat transfer rate. Assume that the flow is steady, that the effects of wall friction can be neglected and that the air behaves as a perfect gas.

Equation Sheet

Isentropic Flow Equations

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\mathrm{Ma}^* = \mathrm{Ma}\sqrt{\frac{k+1}{2+(k-1)\mathrm{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)\right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left(1+\frac{k-1}{2}\mathrm{Ma}^2\right)^{-1}$$

Rayleigh Flow Equations

$$q + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$$

$$\frac{T_0}{T_0^*} = \frac{(k+1)\mathrm{Ma}^2[2+(k-1)\mathrm{Ma}^2]}{(1+k\mathrm{Ma}^2)^2}$$

$$\frac{P_0}{P_0^*} = \frac{k+1}{1+k\mathrm{Ma}^2}\left(\frac{2+(k-1)\mathrm{Ma}^2}{k+1}\right)^{k/(k-1)}$$

$$\frac{T}{T^*} = \left(\frac{\mathrm{Ma}(1+k)}{1+k\mathrm{Ma}^2}\right)^2$$

$$\frac{P}{P^*} = \frac{1+k}{1+k\mathrm{Ma}^2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1+k)\mathrm{Ma}^2}{1+k\mathrm{Ma}^2}$$

Fanno Flow Equations

$$T_0 = T_0^*$$

$$\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{\mathrm{Ma}}\left(\frac{2 + (k - 1)\mathrm{Ma}^2}{k + 1}\right)^{(k+1)/2(k-1)}$$

$$\frac{T}{T^*} = \frac{k + 1}{2 + (k - 1)\mathrm{Ma}^2}$$

$$\frac{P}{P^*} = \frac{1}{\mathrm{Ma}}\left(\frac{k + 1}{2 + (k - 1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \mathrm{Ma}\left(\frac{k + 1}{2 + (k - 1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{fL^*}{D} = \frac{1 - \mathrm{Ma}^2}{k\mathrm{Ma}^2} + \frac{k + 1}{2k}\ln\frac{(k + 1)\mathrm{Ma}^2}{2 + (k - 1)\mathrm{Ma}^2}$$

TABLE A–13

One-dimensional isentropic compressible flow functions for an ideal gas with $k = 1.4$

Ma	Ma*	A/A^*	P/P_0	ρ/ρ_0	T/T_0
0	0	∞	1.0000	1.0000	1.0000
0.1	0.1094	5.8218	0.9930	0.9950	0.9980
0.2	0.2182	2.9635	0.9725	0.9803	0.9921
0.3	0.3257	2.0351	0.9395	0.9564	0.9823
0.4	0.4313	1.5901	0.8956	0.9243	0.9690
0.5	0.5345	1.3398	0.8430	0.8852	0.9524
0.6	0.6348	1.1882	0.7840	0.8405	0.9328
0.7	0.7318	1.0944	0.7209	0.7916	0.9107
0.8	0.8251	1.0382	0.6560	0.7400	0.8865
0.9	0.9146	1.0089	0.5913	0.6870	0.8606
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.2	1.1583	1.0304	0.4124	0.5311	0.7764
1.4	1.2999	1.1149	0.3142	0.4374	0.7184
1.6	1.4254	1.2502	0.2353	0.3557	0.6614
1.8	1.5360	1.4390	0.1740	0.2868	0.6068
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.2	1.7179	2.0050	0.0935	0.1841	0.5081
2.4	1.7922	2.4031	0.0684	0.1472	0.4647
2.6	1.8571	2.8960	0.0501	0.1179	0.4252
2.8	1.9140	3.5001	0.0368	0.0946	0.3894
3.0	1.9640	4.2346	0.0272	0.0760	0.3571
5.0	2.2361	25.000	0.0019	0.0113	0.1667
∞	2.2495	∞	0	0	0

TABLE A–15

Rayleigh flow functions for an ideal gas with $k = 1.4$

Ma	T_0/T_0^*	P_0/P_0^*	T/T^*	P/P^*	V/V^*
0.0	0.0000	1.2679	0.0000	2.4000	0.0000
0.1	0.0468	1.2591	0.0560	2.3669	0.0237
0.2	0.1736	1.2346	0.2066	2.2727	0.0909
0.3	0.3469	1.1985	0.4089	2.1314	0.1918
0.4	0.5290	1.1566	0.6151	1.9608	0.3137
0.5	0.6914	1.1141	0.7901	1.7778	0.4444
0.6	0.8189	1.0753	0.9167	1.5957	0.5745
0.7	0.9085	1.0431	0.9929	1.4235	0.6975
0.8	0.9639	1.0193	1.0255	1.2658	0.8101
0.9	0.9921	1.0049	1.0245	1.1246	0.9110
1.0	1.0000	1.0000	1.0000	1.0000	1.0000
1.2	0.9787	1.0194	0.9118	0.7958	1.1459
1.4	0.9343	1.0777	0.8054	0.6410	1.2564
1.6	0.8842	1.1756	0.7017	0.5236	1.3403
1.8	0.8363	1.3159	0.6089	0.4335	1.4046
2.0	0.7934	1.5031	0.5289	0.3636	1.4545
2.2	0.7561	1.7434	0.4611	0.3086	1.4938
2.4	0.7242	2.0451	0.4038	0.2648	1.5252
2.6	0.6970	2.4177	0.3556	0.2294	1.5505
2.8	0.6738	2.8731	0.3149	0.2004	1.5711
3.0	0.6540	3.4245	0.2803	0.1765	1.5882

TABLE A–16

Fanno flow functions for an ideal gas with $k = 1.4$

Ma	P_0/P_0^*	T/T^*	P/P^*	V/V^*	fL^*/D
0.0	∞	1.2000	∞	0.0000	∞
0.1	5.8218	1.1976	10.9435	0.1094	66.9216
0.2	2.9635	1.1905	5.4554	0.2182	14.5333
0.3	2.0351	1.1788	3.6191	0.3257	5.2993
0.4	1.5901	1.1628	2.6958	0.4313	2.3085
0.5	1.3398	1.1429	2.1381	0.5345	1.0691
0.6	1.1882	1.1194	1.7634	0.6348	0.4908
0.7	1.0944	1.0929	1.4935	0.7318	0.2081
0.8	1.0382	1.0638	1.2893	0.8251	0.0723
0.9	1.0089	1.0327	1.1291	0.9146	0.0145
1.0	1.0000	1.0000	1.0000	1.0000	0.0000
1.2	1.0304	0.9317	0.8044	1.1583	0.0336
1.4	1.1149	0.8621	0.6632	1.2999	0.0997
1.6	1.2502	0.7937	0.5568	1.4254	0.1724
1.8	1.4390	0.7282	0.4741	1.5360	0.2419
2.0	1.6875	0.6667	0.4082	1.6330	0.3050
2.2	2.0050	0.6098	0.3549	1.7179	0.3609
2.4	2.4031	0.5576	0.3111	1.7922	0.4099
2.6	2.8960	0.5102	0.2747	1.8571	0.4526
2.8	3.5001	0.4673	0.2441	1.9140	0.4898
3.0	4.2346	0.4286	0.2182	1.9640	0.5222

End of Exam.