SPC 407

Supersonic & Hypersonic Fluid Dynamics

Lecture 2

September 25, 2016

What is Shock Wave?

To watch the videos, click on the links below

[Understanding Shock Waves in Aerospace Applications]
[The sonic boom problem]
[What is a Shockwave]

Flow Regimes

To watch the video, click on the links below

[Flow Regime]

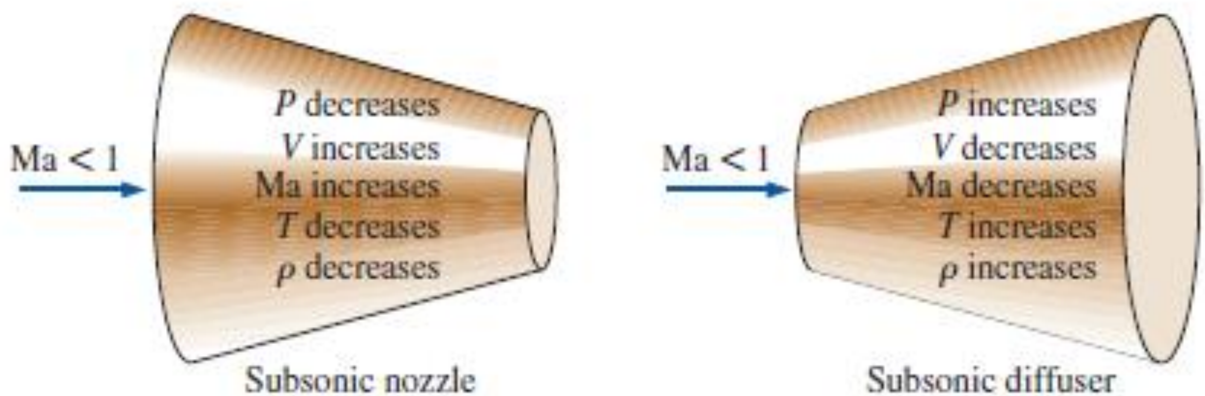
Hypersonic Flow Applications

To watch the videos, click on the links below

[How fast is that land air]
[How fast is that spacecraft 1]
[How fast is that spacecraft 2]

Governing Equations

- Flow inside Nozzle and Diffuser



(a) Subsonic flow

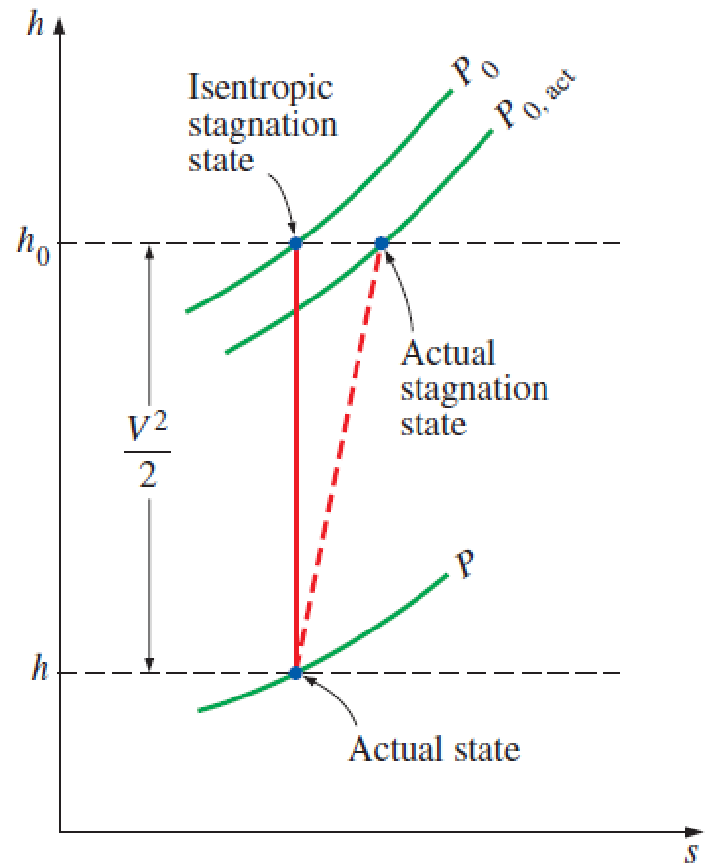
Governing Equations

- Enthalpy

$$h = u + P/\rho$$

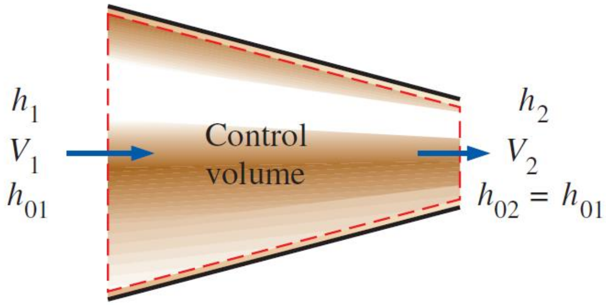
- Stagnation Properties
  - Stagnation Enthalpy

$$h_0 = h + \frac{V^2}{2} \qquad \text{(kJ/kg)}$$



Governing Equations

Consider the steady flow of a fluid through a duct such as a nozzle, diffuser, or some other flow passage where the flow takes place adiabatically and with no shaft or electrical work, as shown in Fig



Assuming the fluid experiences little or no change in its elevation

$$\dot{E}_{in} = \dot{E}_{out}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

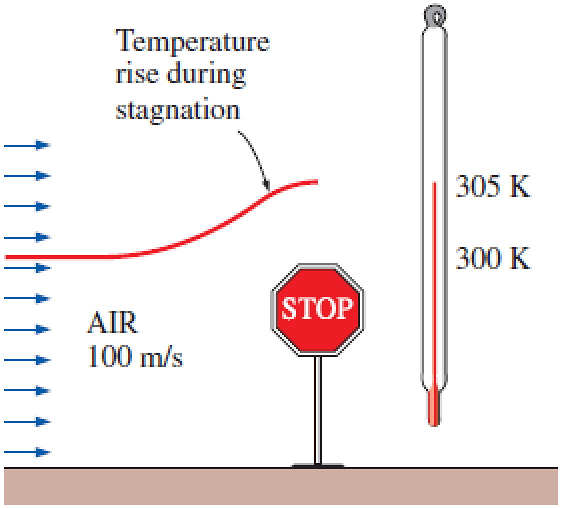
$$h_{01} = h_{02}$$

Governing Equations

When the fluid is approximated as an *ideal gas* with constant specific heats, its enthalpy can be replaced by

$$c_p T_0 = c_p T + \frac{V^2}{2}$$

$$T_0 = T + \frac{V^2}{2c_p}$$



For ideal gases with constant specific heats

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)}$$

Governing Equations

Then the energy balance for a single-stream, steady-flow device can be expressed as

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$q_{\text{in}} + w_{\text{in}} + (h_{01} + gz_1) = q_{\text{out}} + w_{\text{out}} + (h_{02} + gz_2)$$

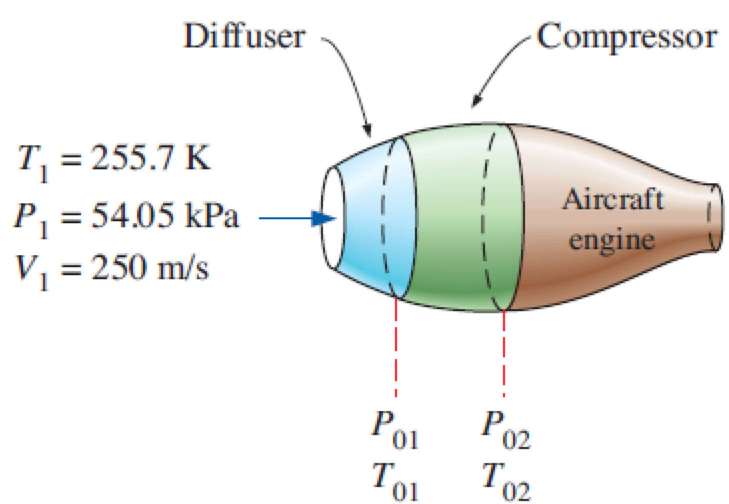
where

$$h_o = C_p T_o$$

$$(q_{\text{in}} - q_{\text{out}}) + (w_{\text{in}} - w_{\text{out}}) = c_p(T_{02} - T_{01}) + g(z_2 - z_1)$$

Example: Compression of High-Speed Air in an Aircraft

An aircraft is flying at a cruising speed of 250 m/s at an altitude of 5000 m where the atmospheric pressure is 54.05 kPa and the ambient air temperature is 255.7 K. The ambient air is first decelerated in a diffuser before it enters the compressor (Fig. 12–5). Approximating both the diffuser and the compressor to be isentropic, determine (*a*) the stagnation pressure at the compressor inlet and (*b*) the required compressor work per unit mass if the stagnation pressure ratio of the compressor is 8.



Example: Compression of High-Speed Air in an Aircraft

SOLUTION High-speed air enters the diffuser and the compressor of an aircraft. The stagnation pressure of the air and the compressor work input are to be determined.

Assumptions **1** Both the diffuser and the compressor are isentropic. **2** Air is an ideal gas with constant specific heats at room temperature.

Properties The constant-pressure specific heat c_p and the specific heat ratio k of air at room temperature are

$$c_p = 1.005 \text{ kJ/kg·K} \quad \text{and} \quad k = 1.4$$

Analysis (*a*) Under isentropic conditions, the stagnation pressure at the compressor inlet (diffuser exit) can be determined from Eq. 12–5. However, first we need to find the stagnation temperature T_{01} at the compressor inlet. Under the stated assumptions, T_{01} is determined from Eq. 12–4 to be

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 255.7 \text{ K} + \frac{(250 \text{ m/s})^2}{(2)(1.005 \text{ kJ/kg·K})}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right)$$
$$= 286.8 \text{ K}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (54.05 \text{ kPa})\left(\frac{286.8 \text{ K}}{255.7 \text{ K}}\right)^{1.4/(1.4-1)}$$
$$= \mathbf{80.77 \text{ kPa}}$$

Example: Compression of High-Speed Air in an Aircraft

(*b*) To determine the compressor work, we need to know the stagnation temperature of air at the compressor exit T_{02}. The stagnation pressure ratio across the compressor P_{02}/P_{01} is specified to be 8. Since the compression process is approximated as isentropic, T_{02} can be determined from the ideal-gas isentropic relation (Eq. 12–5):

$$T_{02} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{(k-1)/k} = (286.8\text{ K})(8)^{(1.4-1)/1.4} = 519.5\text{ K}$$

Disregarding potential energy changes and heat transfer, the compressor work per unit mass of air is determined from Eq. 12–8:

$$\begin{aligned} w_{\text{in}} &= c_p(T_{02} - T_{01}) \\ &= (1.005\text{ kJ/kg}\cdot\text{K})(519.5\text{ K} - 286.8\text{ K}) \\ &= \mathbf{233.9\text{ kJ/kg}} \end{aligned}$$

Speed of sound

An important parameter in the study of compressible flow is the **speed of sound** c

For an ideal gas it simplifies to

$$c = \sqrt{kRT}$$

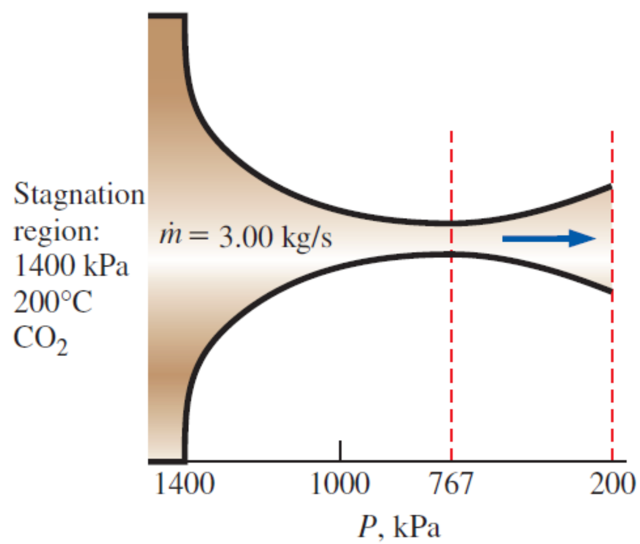
where k is the specific heat ratio of the gas and R is the specific gas constant. The ratio of the speed of the flow to the speed of sound is the dimensionless Mach number M

$$M = \frac{V}{c}$$

During fluid flow through many devices such as nozzles, diffusers, and turbine blade passages, flow quantities vary primarily in the flow direction only, and the flow can be approximated as one-dimensional isentropic flow with good accuracy.

Example Gas Flow through a Converging–Diverging Duct

Carbon dioxide flows steadily through a varying cross-sectional area duct such as a nozzle shown in Fig. 12–6 at a mass flow rate of 3.00 kg/s. The carbon dioxide enters the duct at a pressure of 1400 kPa and 200°C with a low velocity, and it expands in the nozzle to an exit pressure of 200 kPa. The duct is designed so that the flow can be approximated as isentropic. Determine the density, velocity, flow area, and Mach number at each location along the duct that corresponds to an overall pressure drop of 200 kPa.



Example Gas Flow through a Converging–Diverging Duct

SOLUTION Carbon dioxide enters a varying cross-sectional area duct at specified conditions. The flow properties are to be determined along the duct.

Assumptions **1** Carbon dioxide is an ideal gas with constant specific heats at room temperature. **2** Flow through the duct is steady, one-dimensional, and isentropic.

Properties For simplicity we use $c_p = 0.846$ kJ/kg·K and $k = 1.289$ throughout the calculations, which are the constant-pressure specific heat and specific heat ratio values of carbon dioxide at room temperature. The gas constant of carbon dioxide is $R = 0.1889$ kJ/kg·K.

Analysis We note that the inlet temperature is nearly equal to the stagnation temperature since the inlet velocity is small. The flow is isentropic, and thus the stagnation temperature and pressure throughout the duct remain constant. Therefore,

$$T_0 \cong T_1 = 200°\text{C} = 473\text{ K}$$

and

$$P_0 \cong P_1 = 1400\text{ kPa}$$

Example Gas Flow through a Converging–Diverging Duct

To illustrate the solution procedure, we calculate the desired properties at the location where the pressure is 1200 kPa, the first location that corresponds to a pressure drop of 200 kPa.

From Eq. 12–5,

$$T = T_0\left(\frac{P}{P_0}\right)^{(k-1)/k} = (473\text{ K})\left(\frac{1200\text{ kPa}}{1400\text{ kPa}}\right)^{(1.289-1)/1.289} = 457\text{ K}$$

From Eq. 12–4,

$$V = \sqrt{2c_p(T_0 - T)}$$

$$= \sqrt{2(0.846\text{ kJ/kg·K})(473\text{ K} - 457\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^3}{1\text{ kJ/kg}}\right)}$$

$$= 164.5\text{ m/s} \cong \mathbf{164\text{ m/s}}$$

From the ideal-gas relation,

$$\rho = \frac{P}{RT} = \frac{1200\text{ kPa}}{(0.1889\text{ kPa·m}^3/\text{kg·K})(457\text{ K})} = \mathbf{13.9\text{ kg/m}^3}$$

Example Gas Flow through a Converging–Diverging Duct

From the mass flow rate relation,

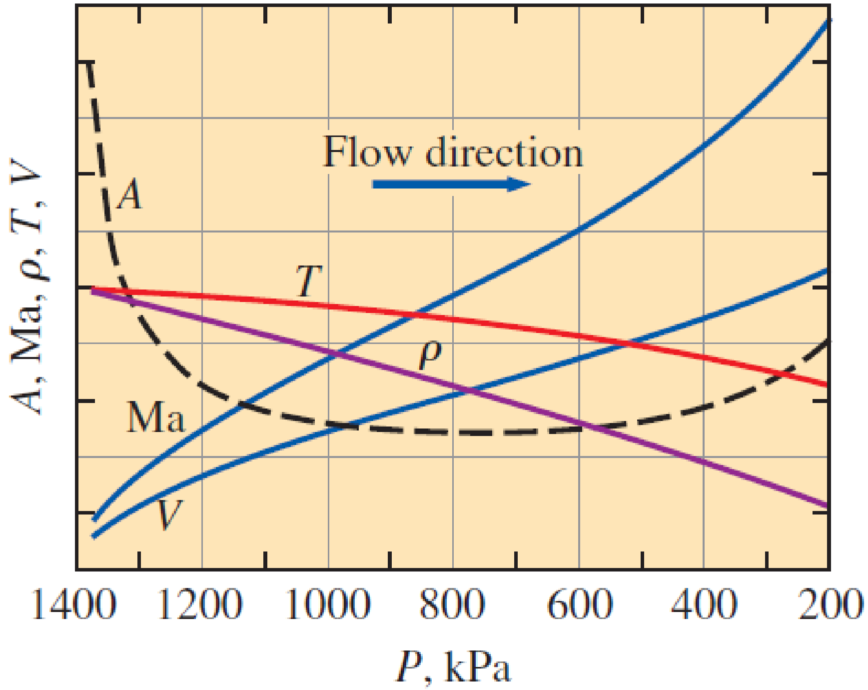
$$A = \frac{\dot{m}}{\rho V} = \frac{3.00 \text{ kg/s}}{(13.9 \text{ kg/m}^3)(164.5 \text{ m/s})} = 13.1 \times 10^{-4} \text{ m}^2 = \mathbf{13.1 \text{ cm}^2}$$

From Eqs. 12–11 and 12–12,

$$c = \sqrt{kRT} = \sqrt{(1.289)(0.1889 \text{ kJ/kg}\cdot\text{K})(457 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 333.6 \text{ m/s}$$

$$\text{Ma} = \frac{V}{c} = \frac{164.5 \text{ m/s}}{333.6 \text{ m/s}} = \mathbf{0.493}$$

Example Gas Flow through a Converging–Diverging Duct



Variation of fluid properties in flow direction in the duct described in Example 12–2 for $\dot{m}$ = 3 kg/s = constant

P, kPa	T, K	V, m/s	ρ, kg/m^3	c, m/s	A, cm^2	Ma
1400	473	0	15.7	339.4	∞	0
1200	457	164.5	13.9	333.6	13.1	0.493
1000	439	240.7	12.1	326.9	10.3	0.736
800	417	306.6	10.1	318.8	9.64	0.962
767*	413	317.2	9.82	317.2	9.63	1.000
600	391	371.4	8.12	308.7	10.0	1.203
400	357	441.9	5.93	295.0	11.5	1.498
200	306	530.9	3.46	272.9	16.3	1.946