SPC 407

Supersonic & Hypersonic Fluid Dynamics

Lecture 6

November 6, 2016

DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

The essential features of such complex flows can still be captured by a simple analysis by modeling the generation or absorption of thermal energy as heat transfer through the duct wall at the same rate and disregarding any changes in chemical composition. This simplified problem is still too complicated for an elementary treatment of the topic since the flow may involve friction, variations in duct area, and multidimensional effects. In this section, we limit our consideration to one-dimensional flow in a duct of constant cross-sectional area with negligible frictional effects.

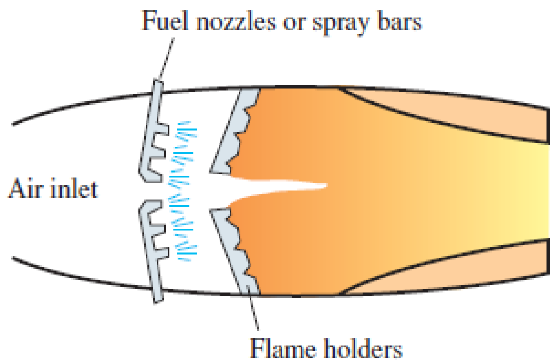


FIGURE 12–46
Many practical compressible flow problems involve combustion, which may be modeled as heat gain through the duct wall.

DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

Consider steady one-dimensional flow of an ideal gas with constant specific heats through a constant-area duct with heat transfer, but with negligible friction. Such flows are referred to as Rayleigh flows

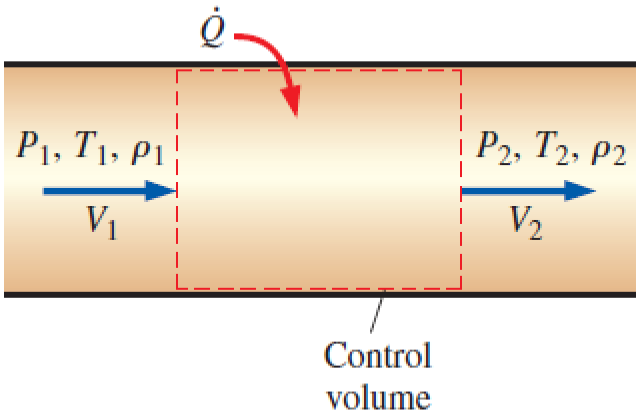
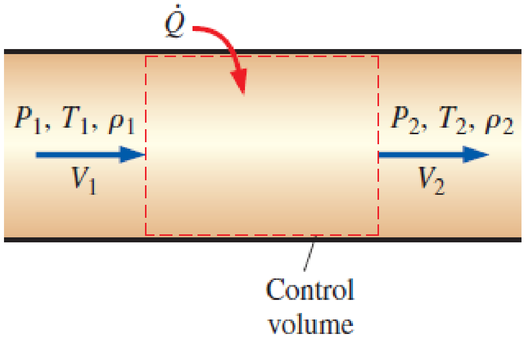


FIGURE 12–47
Control volume for flow in a constant-area duct with heat transfer and negligible friction.

Governing Equations



Conservation of mass:

$$\dot{m}_1 = \dot{m}_2 \text{ or } \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Since we have a constant Cross Sectional Area, A

$$\rho_1 V_1 = \rho_2 V_2$$

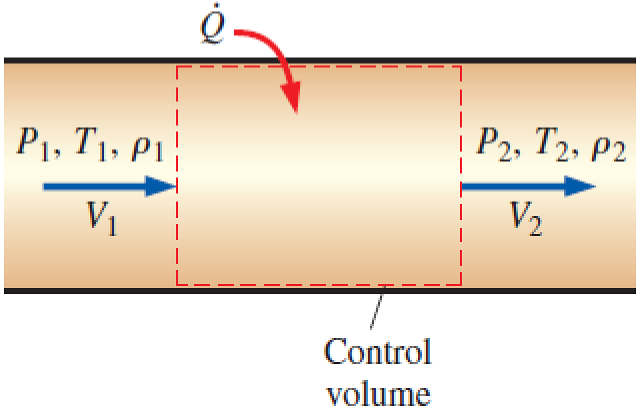
Linear momentum equation:

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1)$$

$$P_1 A_1 - P_2 A_2 = \dot{m} V_2 - \dot{m} V_1 \rightarrow P_1 - P_2 = (\rho_2 V_2) V_2 - (\rho_1 V_1) V_1$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

Governing Equations



Conservation of Energy:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{Q} + \dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \rightarrow q + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

Since $\Delta h = c_p \, \Delta T$ this equation reduces to

$$q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$q = h_{02} - h_{01} = c_p(T_{02} - T_{01})$$

Governing Equations

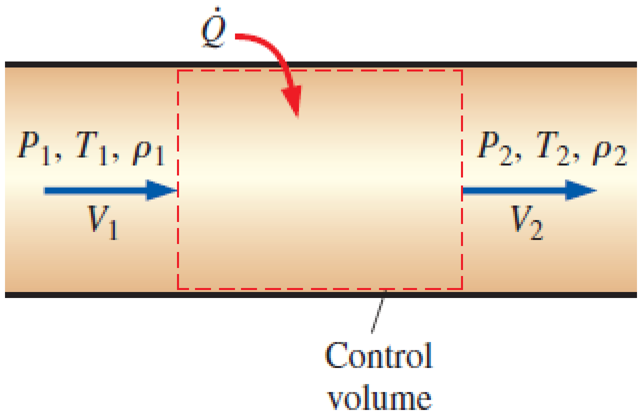
Second Law of Thermodynamic:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Equation of State

$$P = \rho R T$$

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$



Governing Equations Summary

$$\rho_1 V_1 = \rho_2 V_2$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$$q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$

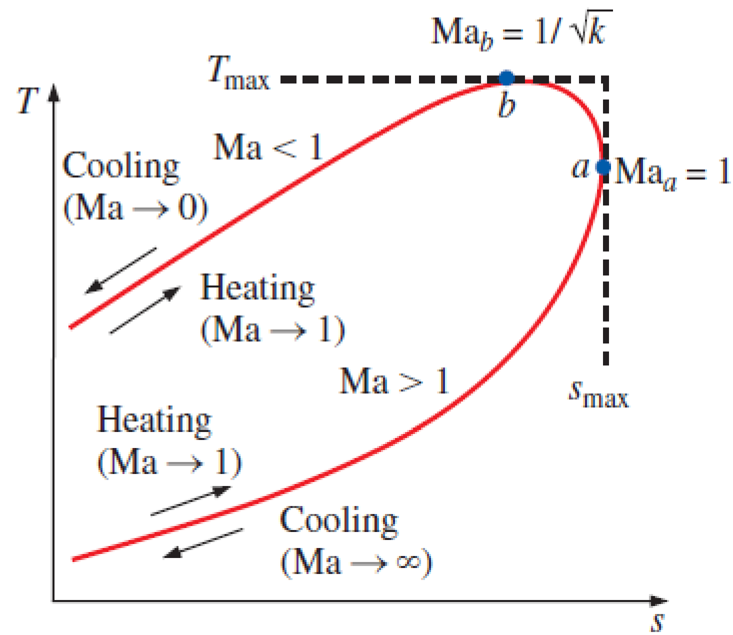


FIGURE 12–48

T-s diagram for flow in a constant-area duct with heat transfer and negligible friction (Rayleigh flow).

Governing Equations Summary

$$T_0 = T + V^2/2c_p$$

From the Continuity Equation

$$\rho V = \text{constant} \; = K$$

$$P + KV = \text{constant}$$

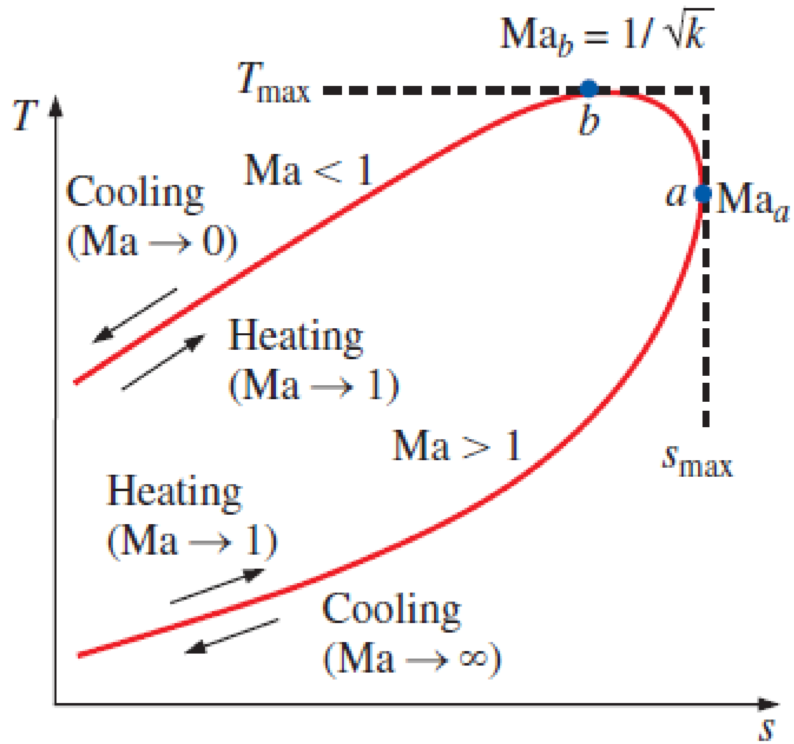


FIGURE 12–48

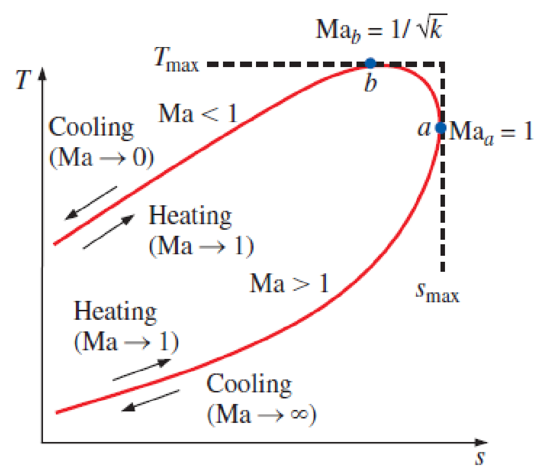
T-s diagram for flow in a constant-area duct with heat transfer and negligible friction (Rayleigh flow).

Governing Equations Summary

TABLE 12–3

The effects of heating and cooling on the properties of Rayleigh flow

	Heating		*Cooling*	
Property	Subsonic	Supersonic	Subsonic	Supersonic
Velocity, V	Increase	Decrease	Decrease	Increase
Mach number, Ma	Increase	Decrease	Decrease	Increase
Stagnation temperature, T_0	Increase	Increase	Decrease	Decrease
Temperature, T	Increase for $\mathrm{Ma} < 1/k^{1/2}$ Decrease for $\mathrm{Ma} > 1/k^{1/2}$	Increase	Decrease for $\mathrm{Ma} < 1/k^{1/2}$ Increase for $\mathrm{Ma} > 1/k^{1/2}$	Decrease
Density, ρ	Decrease	Increase	Increase	Decrease
Stagnation pressure, P_0	Decrease	Decrease	Increase	Increase
Pressure, P	Decrease	Increase	Increase	Decrease
Entropy, s	Increase	Increase	Decrease	Decrease



Extremes of Rayleigh Line

Consider the *T-s* diagram of Rayleigh flow, as shown in Fig. 12–50. Using the differential forms of the conservation equations and property relations, show that the Mach number is $\mathrm{Ma}_a = 1$ at the point of maximum entropy (point *a*), and $\mathrm{Ma}_b = 1\sqrt{k}$ at the point of maximum temperature (point *b*).

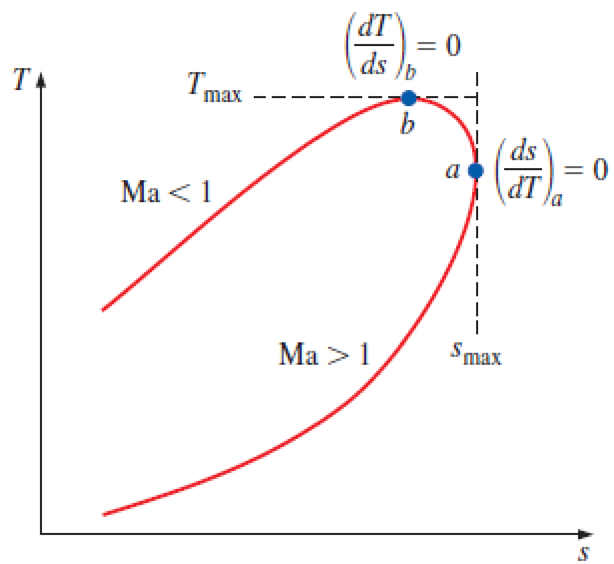


FIGURE 12–50
The *T-s* diagram of Rayleigh flow

Extremes of Rayleigh Line

SOLUTION It is to be shown that $\mathrm{Ma}_a = 1$ at the point of maximum entropy and $\mathrm{Ma}_b = 1\sqrt{k}$ at the point of maximum temperature on the Rayleigh line.

Assumptions The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid.

Analysis The differential forms of the continuity (ρV = constant), momentum [rearranged as $P + (\rho V)V$ = constant], ideal gas ($P = \rho RT$), and enthalpy change ($\Delta h = c_p\, \Delta T$) equations are expressed as

$$\rho V = \text{constant} \quad \rightarrow \quad \rho\, dV + V\, d\rho = 0 \quad \rightarrow \quad \frac{d\rho}{\rho} = -\frac{dV}{V} \tag{1}$$

$$P + (\rho V)V = \text{constant} \quad \rightarrow \quad dP + (\rho V)\, dV = 0 \quad \rightarrow \quad \frac{dP}{dV} = -\rho V \tag{2}$$

$$P = \rho RT \quad \rightarrow \quad dP = \rho R\, dT + RT\, d\rho \quad \rightarrow \quad \frac{dP}{P} = \frac{dT}{T} + \frac{d\rho}{\rho} \tag{3}$$

The differential form of the entropy change relation (Eq. 12–40) of an ideal gas with constant specific heats is

$$ds = c_p \frac{dT}{T} - R\frac{dP}{P} \tag{4}$$

Extremes of Rayleigh Line

Substituting Eq. 3 into Eq. 4 gives

$$ds = c_p \frac{dT}{T} - R\left(\frac{dT}{T} + \frac{d\rho}{\rho}\right) = (c_p - R)\frac{dT}{T} - R\frac{d\rho}{\rho} = \frac{R}{k-1}\frac{dT}{T} - R\frac{d\rho}{\rho} \qquad (5)$$

since

$$c_p - R = c_v \quad \rightarrow \quad kc_v - R = c_v \quad \rightarrow \quad c_v = R/(k-1)$$

Dividing both sides of Eq. 5 by dT and combining with Eq. 1,

$$\frac{ds}{dT} = \frac{R}{T(k-1)} + \frac{R}{V}\frac{dV}{dT} \qquad (6)$$

Dividing Eq. 3 by dV and combining it with Eqs. 1 and 2 give, after rearranging,

$$\frac{dT}{dV} = \frac{T}{V} - \frac{V}{R} \qquad (7)$$

Substituting Eq. 7 into Eq. 6 and rearranging,

$$\frac{ds}{dT} = \frac{R}{T(k-1)} + \frac{R}{T - V^2/R} = \frac{R(kRT - V^2)}{T(k-1)(RT - V^2)} \qquad (8)$$

Setting $ds/dT = 0$ and solving the resulting equation $R(kRT - V^2) = 0$ for V give the velocity at point a to be

$$V_a = \sqrt{kRT_a} \quad \text{and} \quad \mathrm{Ma}_a = \frac{V_a}{c_a} = \frac{\sqrt{kRT_a}}{\sqrt{kRT_a}} = 1 \qquad (9)$$

Extremes of Rayleigh Line

Therefore, sonic conditions exist at point a, and thus the Mach number is 1.

Setting $dT/ds = (ds/dT)^{-1} = 0$ and solving the resulting equation $T(k-1) \times (RT - V^2) = 0$ for velocity at point b give

$$V_b = \sqrt{RT_b} \quad \text{and} \quad \mathrm{Ma}_b = \frac{V_b}{c_b} = \frac{\sqrt{RT_b}}{\sqrt{kRT_b}} = \frac{1}{\sqrt{k}} \tag{10}$$

Therefore, the Mach number at point b is $\mathrm{Ma}_b = 1\sqrt{k}$. For air, $k = 1.4$ and thus $\mathrm{Ma}_b = 0.845$.

Discussion Note that in Rayleigh flow, sonic conditions are reached as the entropy reaches its maximum value, and maximum temperature occurs during subsonic flow.

Effect of Heat Transfer on Flow Velocity

Starting with the differential form of the energy equation, show that the flow velocity increases with heat addition in subsonic Rayleigh flow, but decreases in supersonic Rayleigh flow.

SOLUTION It is to be shown that flow velocity increases with heat addition in subsonic Rayleigh flow and that the opposite occurs in supersonic flow.

Assumptions **1** The assumptions associated with Rayleigh flow are valid. **2** There are no work interactions and potential energy changes are negligible.

Analysis Consider heat transfer to the fluid in the differential amount of δq. The differential forms of the energy equations are expressed as

$$\delta q = dh_0 = d\left(h + \frac{V^2}{2}\right) = c_p\, dT + V\, dV \qquad \text{(1)}$$

Dividing by $c_p T$ and factoring out dV/V give

$$\frac{\delta q}{c_p T} = \frac{dT}{T} + \frac{V\, dV}{c_p T} = \frac{dV}{V}\left(\frac{V}{dV}\,\frac{dT}{T} + \frac{(k-1)V^2}{kRT}\right) \qquad \text{(2)}$$

Effect of Heat Transfer on Flow Velocity

where we also used $c_p = kR/(k-1)$. Noting that $\mathrm{Ma}^2 = V^2/c^2 = V^2/kRT$ and using Eq. 7 for dT/dV from Example 12–12 give

$$\frac{\delta q}{c_pT} = \frac{dV}{V}\left(\frac{V}{T}\left(\frac{T}{V} - \frac{V}{R}\right) + (k-1)\mathrm{Ma}^2\right) = \frac{dV}{V}\left(1 - \frac{V^2}{TR} + k\,\mathrm{Ma}^2 - \mathrm{Ma}^2\right) \tag{3}$$

Canceling the two middle terms in Eq. 3 since $V^2/TR = k\,\mathrm{Ma}^2$ and rearranging give the desired relation,

$$\frac{dV}{V} = \frac{\delta q}{c_pT}\frac{1}{(1-\mathrm{Ma}^2)} \tag{4}$$

In subsonic flow, $1 - \mathrm{Ma}^2 > 0$ and thus heat transfer and velocity change have the same sign. As a result, heating the fluid ($\delta q > 0$) increases the flow velocity while cooling decreases it. In supersonic flow, however, $1 - \mathrm{Ma}^2 < 0$ and heat transfer and velocity change have opposite signs. **As a result, heating the fluid ($\delta q > 0$) decreases the flow velocity while cooling increases it** (Fig. 12–51).

Discussion Note that heating the fluid has the opposite effect on flow velocity in subsonic and supersonic Rayleigh flows.

Property Relations for Rayleigh Flow

Since
$$\mathrm{Ma} = V/c = V/\sqrt{kRT}$$

Thus
$$V = \mathrm{Ma}\sqrt{kRT}$$

Since
$$P = \rho RT$$

Substitute in the momentum equation $P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$

We get
$$P_1 + kP_1\mathrm{Ma}_1^2 = P_2 + kP_2\mathrm{Ma}_2^2$$

Which can be rearranged as
$$\frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2}$$

Property Relations for Rayleigh Flow

Again utilizing
$$V = \mathrm{Ma}\sqrt{kRT}$$

The continuity equation
$$\rho_1 V_1 = \rho_2 V_2$$

is expressed as
$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{\mathrm{Ma}_2\sqrt{kRT_2}}{\mathrm{Ma}_1\sqrt{kRT_1}} = \frac{\mathrm{Ma}_2\sqrt{T_2}}{\mathrm{Ma}_1\sqrt{T_1}}$$

Then the ideal-gas relation
$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$

Becomes
$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} = \left(\frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2}\right)\left(\frac{\mathrm{Ma}_2\sqrt{T_2}}{\mathrm{Ma}_1\sqrt{T_1}}\right)$$

Solving Eq. for the temperature ratio T_2/T_1 gives
$$\frac{T_2}{T_1} = \left(\frac{\mathrm{Ma}_2(1 + k\mathrm{Ma}_1^2)}{\mathrm{Ma}_1(1 + k\mathrm{Ma}_2^2)}\right)^2$$

Property Relations for Rayleigh Flow

Substituting this relation into Eq. 12–59

$$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{\mathrm{Ma}_2\sqrt{kRT_2}}{\mathrm{Ma}_1\sqrt{kRT_1}} = \frac{\mathrm{Ma}_2\sqrt{T_2}}{\mathrm{Ma}_1\sqrt{T_1}}$$

gives the density or velocity ratio as

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{\mathrm{Ma}_1^2(1 + k\mathrm{Ma}_2^2)}{\mathrm{Ma}_2^2(1 + k\mathrm{Ma}_1^2)}$$

Flow properties at sonic conditions are usually easy to determine, and thus the critical state corresponding to Ma = 1 serves as a convenient reference point in compressible flow. Taking state 2 to be the sonic state ($\mathrm{Ma}_2 = 1$, and superscript * is used) and state 1 to be any state (no subscript)

$$\frac{T_2}{T_1} = \left(\frac{\mathrm{Ma}_2(1 + k\mathrm{Ma}_1^2)}{\mathrm{Ma}_1(1 + k\mathrm{Ma}_2^2)}\right)^2 \qquad \frac{P_2}{P_1} = \frac{1 + k\mathrm{Ma}_1^2}{1 + k\mathrm{Ma}_2^2}$$

$$\frac{P}{P^*} = \frac{1 + k}{1 + k\mathrm{Ma}^2} \qquad \frac{T}{T^*} = \left(\frac{\mathrm{Ma}(1 + k)}{1 + k\mathrm{Ma}^2}\right)^2 \quad \text{and} \quad \frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1 + k)\mathrm{Ma}^2}{1 + k\mathrm{Ma}^2}$$

Property Relations for Rayleigh Flow

Similar relations can be obtained for dimensionless stagnation temperature and stagnation pressure as follows:

$$\frac{T_0}{T_0^*} = \frac{T_0}{T}\frac{T}{T^*}\frac{T^*}{T_0^*} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)\left(\frac{\mathrm{Ma}(1+k)}{1+k\mathrm{Ma}^2}\right)^2\left(1 + \frac{k-1}{2}\right)^{-1}$$

which simplifies to

$$\frac{T_0}{T_0^*} = \frac{(k+1)\mathrm{Ma}^2[2 + (k-1)\mathrm{Ma}^2]}{(1+k\mathrm{Ma}^2)^2}$$

Also,

$$\frac{P_0}{P_0^*} = \frac{P_0}{P}\frac{P}{P^*}\frac{P^*}{P_0^*} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{k/(k-1)}\left(\frac{1+k}{1+k\mathrm{Ma}^2}\right)\left(1 + \frac{k-1}{2}\right)^{-k/(k-1)}$$

Which simplifies to

$$\frac{P_0}{P_0^*} = \frac{k+1}{1+k\mathrm{Ma}^2}\left(\frac{2+(k-1)\mathrm{Ma}^2}{k+1}\right)^{k/(k-1)}$$

Rayleigh Flow Equation Summary and tables

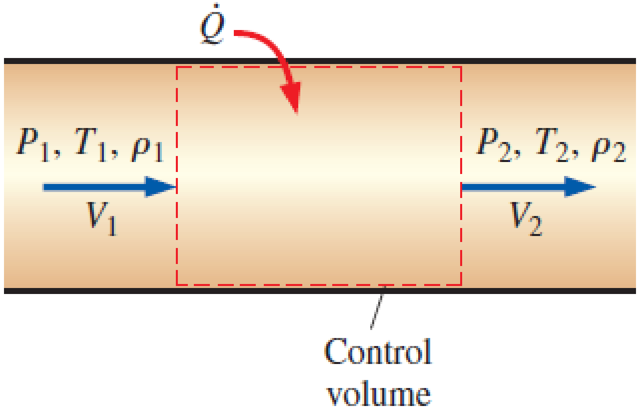
$$\frac{T_0}{T_0^*} = \frac{(k+1)\mathrm{Ma}^2[2+(k-1)\mathrm{Ma}^2]}{(1+k\mathrm{Ma}^2)^2}$$

$$\frac{P_0}{P_0^*} = \frac{k+1}{1+k\mathrm{Ma}^2}\left(\frac{2+(k-1)\mathrm{Ma}^2}{k+1}\right)^{k/(k-1)}$$

$$\frac{T}{T^*} = \left(\frac{\mathrm{Ma}(1+k)}{1+k\mathrm{Ma}^2}\right)^2$$

$$\frac{P}{P^*} = \frac{1+k}{1+k\mathrm{Ma}^2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1+k)\mathrm{Ma}^2}{1+k\mathrm{Ma}^2}$$



Rayleigh Flow Equation Summary and tables

$$\frac{T_0}{T_0^*} = \frac{(k+1)\mathrm{Ma}^2[2+(k-1)\mathrm{Ma}^2]}{(1+k\mathrm{Ma}^2)^2}$$

$$\frac{P_0}{P_0^*} = \frac{k+1}{1+k\mathrm{Ma}^2}\left(\frac{2+(k-1)\mathrm{Ma}^2}{k+1}\right)^{k/(k-1)}$$

$$\frac{T}{T^*} = \left(\frac{\mathrm{Ma}(1+k)}{1+k\mathrm{Ma}^2}\right)^2$$

$$\frac{P}{P^*} = \frac{1+k}{1+k\mathrm{Ma}^2}$$

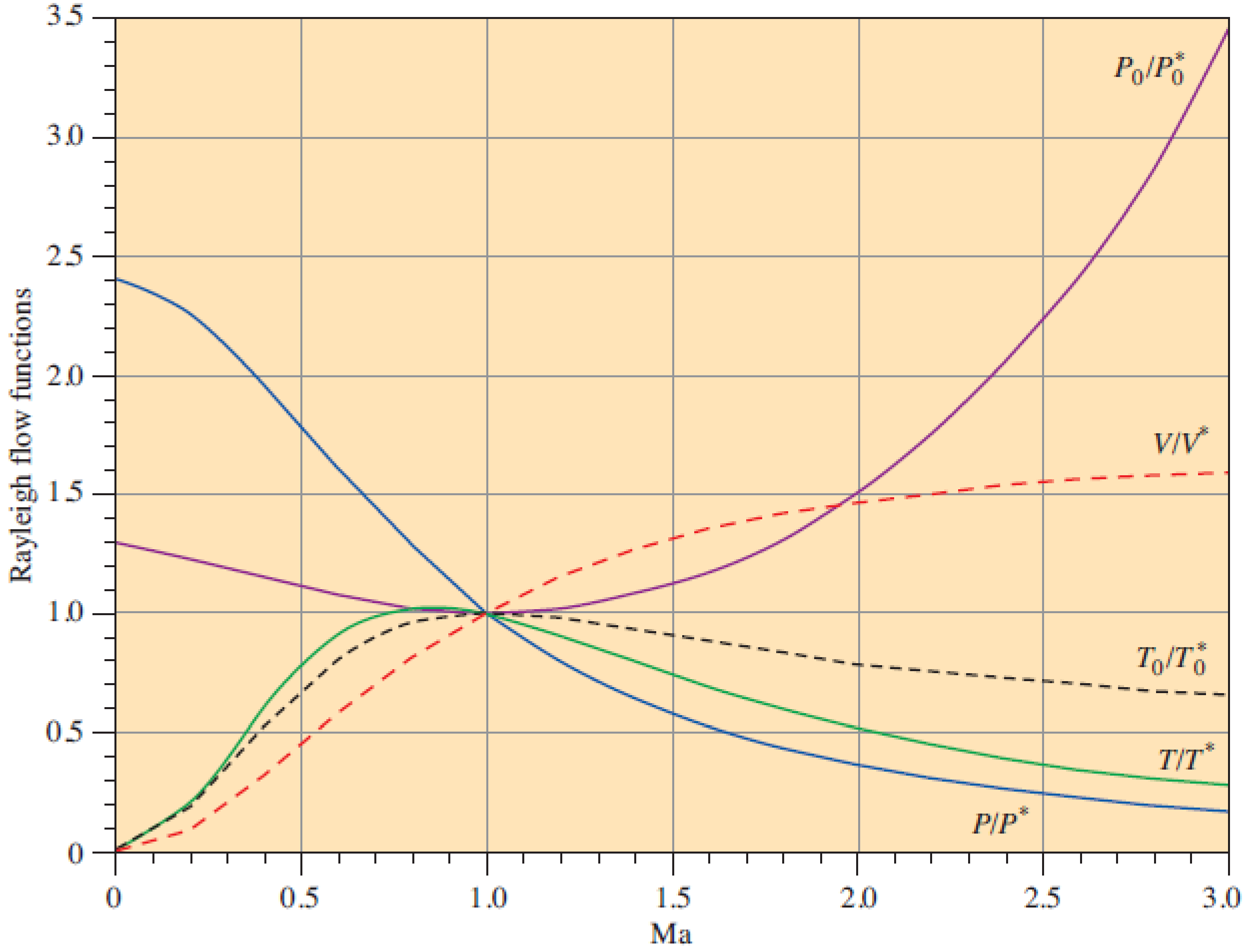
$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1+k)\mathrm{Ma}^2}{1+k\mathrm{Ma}^2}$$

TABLE A–15

Rayleigh flow functions for an ideal gas with $k = 1.4$

Ma	T_0/T_0^*	P_0/P_0^*	T/T^*	P/P^*	V/V^*
0.0	0.0000	1.2679	0.0000	2.4000	0.0000
0.1	0.0468	1.2591	0.0560	2.3669	0.0237
0.2	0.1736	1.2346	0.2066	2.2727	0.0909
0.3	0.3469	1.1985	0.4089	2.1314	0.1918
0.4	0.5290	1.1566	0.6151	1.9608	0.3137
0.5	0.6914	1.1141	0.7901	1.7778	0.4444
0.6	0.8189	1.0753	0.9167	1.5957	0.5745
0.7	0.9085	1.0431	0.9929	1.4235	0.6975
0.8	0.9639	1.0193	1.0255	1.2658	0.8101
0.9	0.9921	1.0049	1.0245	1.1246	0.9110
1.0	1.0000	1.0000	1.0000	1.0000	1.0000
1.2	0.9787	1.0194	0.9118	0.7958	1.1459
1.4	0.9343	1.0777	0.8054	0.6410	1.2564
1.6	0.8842	1.1756	0.7017	0.5236	1.3403
1.8	0.8363	1.3159	0.6089	0.4335	1.4046
2.0	0.7934	1.5031	0.5289	0.3636	1.4545
2.2	0.7561	1.7434	0.4611	0.3086	1.4938
2.4	0.7242	2.0451	0.4038	0.2648	1.5252
2.6	0.6970	2.4177	0.3556	0.2294	1.5505
2.8	0.6738	2.8731	0.3149	0.2004	1.5711
3.0	0.6540	3.4245	0.2803	0.1765	1.5882

Rayleigh Flow Equation Summary and tables



Example: Rayleigh Flow in a Tubular Combustor

A combustion chamber consists of tubular combustors of 15-cm diameter. Compressed air enters the tubes at 550 K, 480 kPa, and 80 m/s (Fig. 12–54). Fuel with a heating value of 42,000 kJ/kg is injected into the air and is burned with an air–fuel mass ratio of 40. Approximating combustion as a heat transfer process to air, determine the temperature, pressure, velocity, and Mach number at the exit of the combustion chamber.

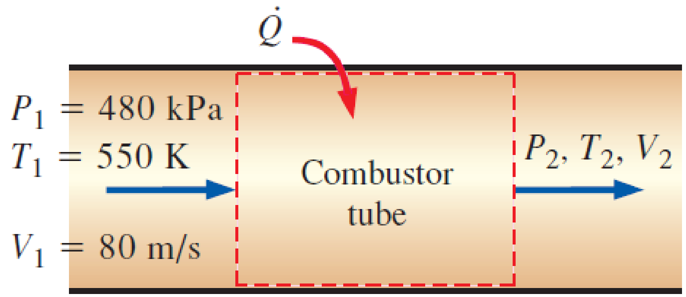


FIGURE 12–54
Schematic of the combustor tube analyzed in Example 12–14.

Example: Rayleigh Flow in a Tubular Combustor

SOLUTION Fuel is burned in a tubular combustion chamber with compressed air. The exit temperature, pressure, velocity, and Mach number are to be determined.

Assumptions **1** The assumptions associated with Rayleigh flow (i.e., steady one-dimensional flow of an ideal gas with constant properties through a constant cross-sectional area duct with negligible frictional effects) are valid. **2** Combustion is complete, and it is treated as a heat addition process, with no change in the chemical composition of the flow. **3** The increase in mass flow rate due to fuel injection is disregarded.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The inlet density and mass flow rate of air are

$$\rho_1 = \frac{P_1}{RT_1} = \frac{480\ \text{kPa}}{(0.287\ \text{kJ/kg}\cdot\text{K})(550\ \text{K})} = 3.041\ \text{kg/m}^3$$

$$\dot{m}_{\text{air}} = \rho_1 A_1 V_1 = (3.041\ \text{kg/m}^3)\,[\pi(0.15\ \text{m})^2/4](80\ \text{m/s}) = 4.299\ \text{kg/s}$$

Example: Rayleigh Flow in a Tubular Combustor

The mass flow rate of fuel and the rate of heat transfer are

$$\dot{m}_{\text{fuel}} = \frac{\dot{m}_{\text{air}}}{\text{AF}} = \frac{4.299 \text{ kg/s}}{40} = 0.1075 \text{ kg/s}$$

$$\dot{Q} = \dot{m}_{\text{fuel}} \text{HV} = (0.1075 \text{ kg/s})(42{,}000 \text{ kJ/kg}) = 4514 \text{ kW}$$

$$q = \frac{\dot{Q}}{\dot{m}_{\text{air}}} = \frac{4514 \text{ kJ/s}}{4.299 \text{ kg/s}} = 1050 \text{ kJ/kg}$$

The stagnation temperature and Mach number at the inlet are

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 550 \text{ K} + \frac{(80 \text{ m/s})^2}{2(1.005 \text{ kJ/kg·K})}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 553.2 \text{ K}$$

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg·K})(550 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 470.1 \text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{80 \text{ m/s}}{470.1 \text{ m/s}} = 0.1702$$

Example: Rayleigh Flow in a Tubular Combustor

The exit stagnation temperature is, from the energy equation $q = c_p(T_{02} - T_{01})$,

$$T_{02} = T_{01} + \frac{q}{c_p} = 553.2\text{ K} + \frac{1050\text{ kJ/kg}}{1.005\text{ kJ/kg}\cdot\text{K}} = 1598\text{ K}$$

The maximum value of stagnation temperature T_0^* occurs at Ma = 1, and its value can be determined from Table A–15 or from Eq. 12–65. At $\text{Ma}_1 = 0.1702$ we read $T_0/T_0^* = 0.1291$. Therefore,

$$T_0^* = \frac{T_{01}}{0.1291} = \frac{553.2\text{ K}}{0.1291} = 4284\text{ K}$$

The stagnation temperature ratio at the exit state and the Mach number corresponding to it are, from Table A–15,

$$\frac{T_{02}}{T_0^*} = \frac{1598\text{ K}}{4284\text{ K}} = 0.3730 \rightarrow \text{Ma}_2 = 0.3142 \cong \mathbf{0.314}$$

Example: Rayleigh Flow in a Tubular Combustor

The Rayleigh flow functions corresponding to the inlet and exit Mach numbers are (Table A–15):

$$\mathrm{Ma}_1 = 0.1702: \quad \frac{T_1}{T^*} = 0.1541 \quad \frac{P_1}{P^*} = 2.3065 \quad \frac{V_1}{V^*} = 0.0668$$

$$\mathrm{Ma}_2 = 0.3142: \quad \frac{T_2}{T^*} = 0.4389 \quad \frac{P_2}{P^*} = 2.1086 \quad \frac{V_2}{V^*} = 0.2082$$

Then the exit temperature, pressure, and velocity are determined to be

$$\frac{T_2}{T_1} = \frac{T_2/T^*}{T_1/T^*} = \frac{0.4389}{0.1541} = 2.848 \rightarrow T_2 = 2.848T_1 = 2.848(550\text{ K}) = \mathbf{1570\ K}$$

$$\frac{P_2}{P_1} = \frac{P_2/P^*}{P_1/P^*} = \frac{2.1086}{2.3065} = 0.9142 \rightarrow P_2 = 0.9142P_1 = 0.9142(480\text{ kPa}) = \mathbf{439\ kPa}$$

$$\frac{V_2}{V_1} = \frac{V_2/V^*}{V_1/V^*} = \frac{0.2082}{0.0668} = 3.117 \rightarrow V_2 = 3.117V_1 = 3.117(80\text{ m/s}) = \mathbf{249\ m/s}$$

Discussion Note that the temperature and velocity increase and pressure decreases during this subsonic Rayleigh flow with heating, as expected. This problem can also be solved using appropriate relations instead of tabulated values, which can likewise be coded for convenient computer solutions.

ADIABATIC DUCT FLOW WITH FRICTION (FANNO FLOW)

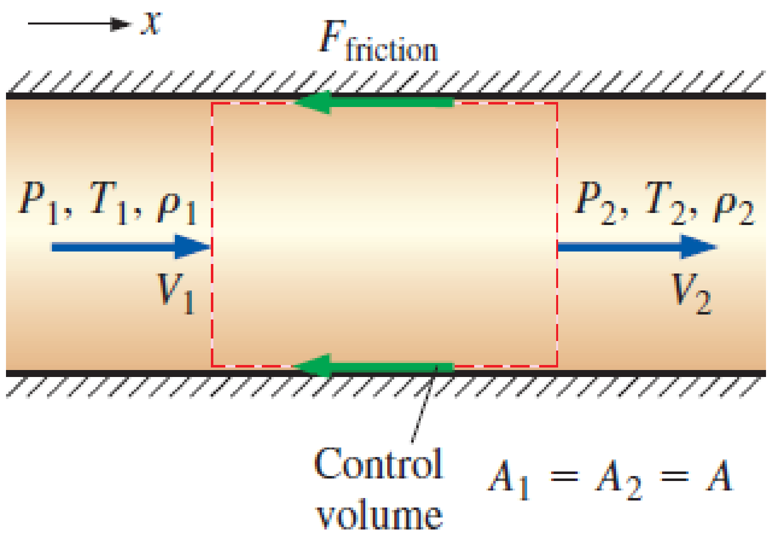


FIGURE 12–55
Control volume for adiabatic flow in a constant-area duct with friction.

DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

So far we have limited our consideration mostly to *isentropic flow*, also *called reversible adiabatic flow* since it involves no heat transfer and no irreversibilities such as friction. Many compressible flow problems encountered in practice involve chemical reactions such as combustion, nuclear reactions, evaporation, and condensation as well as heat gain or heat loss through the duct wall. Such problems are difficult to analyze exactly since they may involve significant changes in chemical composition during flow, and the conversion of latent, chemical, and nuclear energies to thermal energy

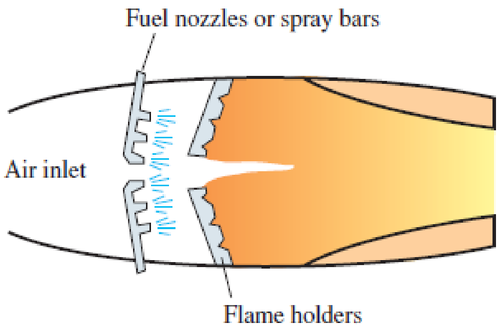


FIGURE 12–46
Many practical compressible flow problems involve combustion, which may be modeled as heat gain through the duct wall.