SPC 407

Supersonic & Hypersonic Fluid Dynamics

Lecture 7

November 13, 2016

DUCT FLOW WITH HEAT TRANSFER AND NEGLIGIBLE FRICTION (RAYLEIGH FLOW)

Consider steady one-dimensional flow of an ideal gas with constant specific heats through a constant-area duct with heat transfer, but with negligible friction. Such flows are referred to as Rayleigh flows

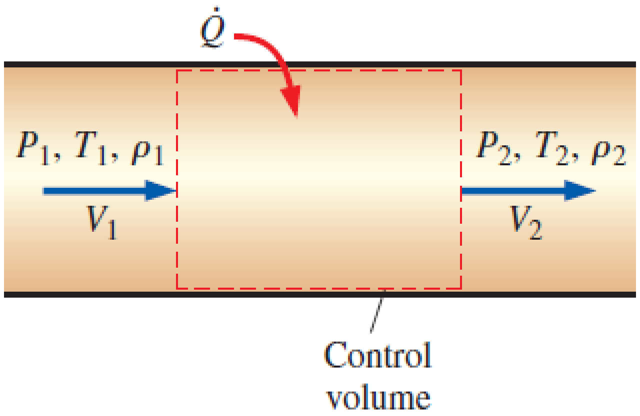


FIGURE 12–47
Control volume for flow in a constant-area duct with heat transfer and negligible friction.

Governing Equations Summary

$$\rho_1 V_1 = \rho_2 V_2$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$$

$$q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$

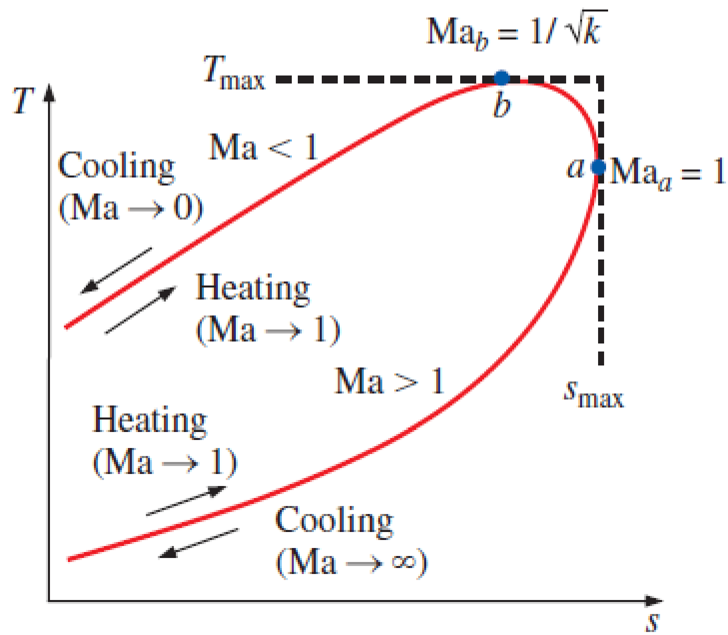


FIGURE 12–48

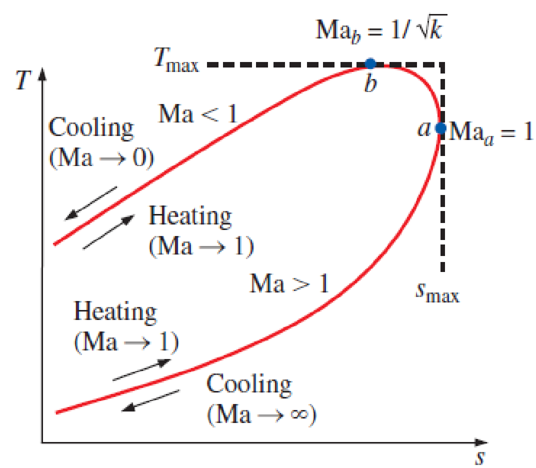
T-s diagram for flow in a constant-area duct with heat transfer and negligible friction (Rayleigh flow).

Governing Equations Summary

TABLE 12–3

The effects of heating and cooling on the properties of Rayleigh flow

	Heating		Cooling	
Property	Subsonic	Supersonic	Subsonic	Supersonic
Velocity, V	Increase	Decrease	Decrease	Increase
Mach number, Ma	Increase	Decrease	Decrease	Increase
Stagnation temperature, T_0	Increase	Increase	Decrease	Decrease
Temperature, T	Increase for Ma $< 1/k^{1/2}$ Decrease for Ma $> 1/k^{1/2}$	Increase	Decrease for Ma $< 1/k^{1/2}$ Increase for Ma $> 1/k^{1/2}$	Decrease
Density, ρ	Decrease	Increase	Increase	Decrease
Stagnation pressure, P_0	Decrease	Decrease	Increase	Increase
Pressure, P	Decrease	Increase	Increase	Decrease
Entropy, s	Increase	Increase	Decrease	Decrease



Governing Equations Summary

$$\frac{T_0}{T_0^*} = \frac{(k + 1)\mathrm{Ma}^2[2 + (k - 1)\mathrm{Ma}^2]}{(1 + k\mathrm{Ma}^2)^2}$$

$$\frac{P_0}{P_0^*} = \frac{k + 1}{1 + k\mathrm{Ma}^2}\left(\frac{2 + (k - 1)\mathrm{Ma}^2}{k + 1}\right)^{k/(k-1)}$$

$$\frac{T}{T^*} = \left(\frac{\mathrm{Ma}(1 + k)}{1 + k\mathrm{Ma}^2}\right)^2$$

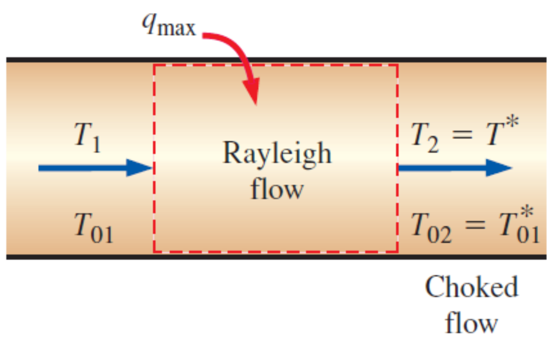
$$\frac{P}{P^*} = \frac{1 + k}{1 + k\mathrm{Ma}^2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \frac{(1 + k)\mathrm{Ma}^2}{1 + k\mathrm{Ma}^2}$$

Chocked Rayleigh Flow

It is clear from the earlier discussions that subsonic Rayleigh flow in a duct may accelerate to sonic velocity (Ma = 1) with heating.
What happens if we continue to heat the fluid? Does the fluid continue to accelerate to supersonic velocities? An examination of the Rayleigh line indicates that the fluid at the critical state of Ma = 1 cannot be accelerated to supersonic velocities by heating. Therefore, the flow is *choked*.
This is analogous to not being able to accelerate a fluid to supersonic velocities in a converging nozzle by simply extending the converging flow section.
If we keep heating the fluid, we will simply move the critical state further downstream and reduce the flow rate since fluid density at the critical state will now be lower.
Therefore, for a given inlet state, the corresponding critical state fixes the maximum possible heat transfer for steady flow.

$$q_{\max} = h_0^* - h_{01} = c_p(T_0^* - T_{01})$$



Chocked Rayleigh Flow

Further heat transfer causes choking and thus the inlet state to change (e.g., inlet velocity will decrease), and the flow no longer follows the same Rayleigh line.

Cooling the subsonic Rayleigh flow reduces the velocity, and the Mach number approaches zero as the temperature approaches absolute zero. Note that the stagnation temperature T_0 is maximum at the critical state of Ma = 1.

In supersonic Rayleigh flow, heating decreases the flow velocity. Further heating simply increases the temperature and moves the critical state farther downstream, resulting in a reduction in the mass flow rate of the fluid.

ADIABATIC DUCT FLOW WITH FRICTION (FANNO FLOW)

Consider steady, one-dimensional, adiabatic flow of an ideal gas with constant specific heats through a constant-area duct with significant frictional effects. Such flows are referred to as **Fanno flows.**

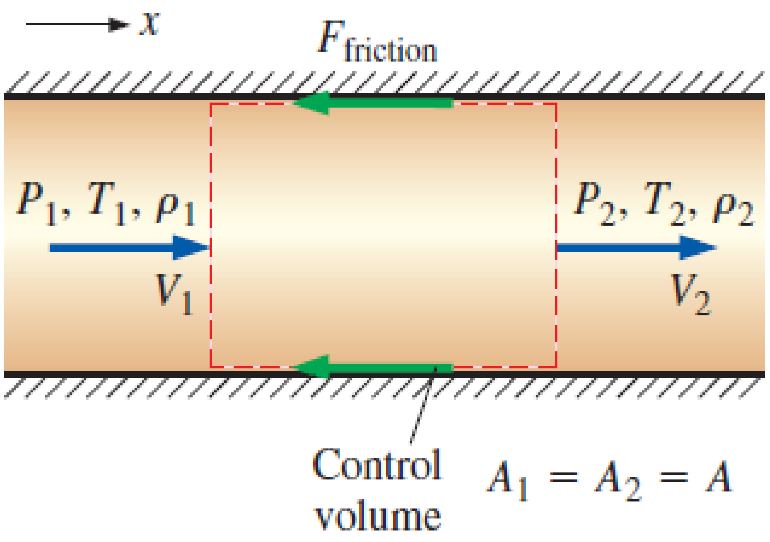


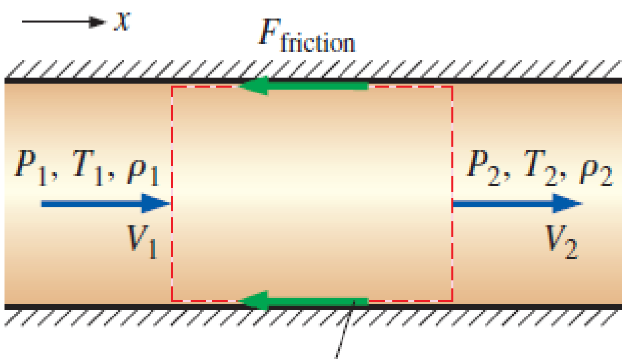
FIGURE 12–55
Control volume for adiabatic flow in a constant-area duct with friction.

Governing Equations

Conservation of mass:

$$\dot{m}_1 = \dot{m}_2 \text{ or } \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Since we have a constant Cross Sectional Area, A



$$\rho_1 V_1 = \rho_2 V_2$$

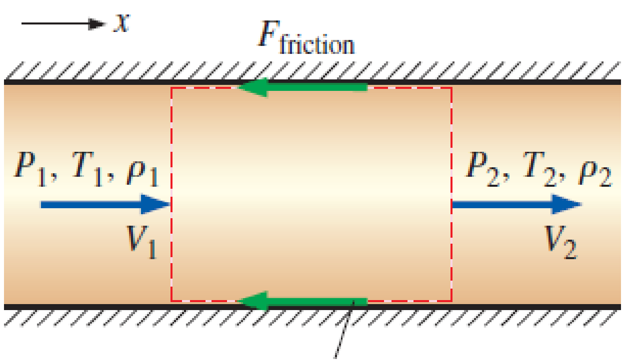
Linear momentum equation:

$$P_1 A - P_2 A - F_{\text{friction}} = \dot{m} V_2 - \dot{m} V_1$$

$$P_1 - P_2 - \frac{F_{\text{friction}}}{A} = (\rho_2 V_2) V_2 - (\rho_1 V_1) V_1$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{friction}}}{A}$$

Governing Equations



Conservation of Energy:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \rightarrow \quad h_{01} = h_{02} \quad \rightarrow \quad h_0 = h + \frac{V^2}{2} = \text{constant}$$

Since $\Delta h = c_p \, \Delta T$ this equation reduces to

$$T_1 + \frac{V_1^2}{2c_p} = T_2 + \frac{V_2^2}{2c_p} \quad \rightarrow \quad T_{01} = T_{02} \quad \rightarrow \quad T_0 = T + \frac{V^2}{2c_p} = \text{constant}$$

Therefore, the stagnation enthalpy h_0 and stagnation temperature T_0 remain constant during Fanno flow.

Governing Equations

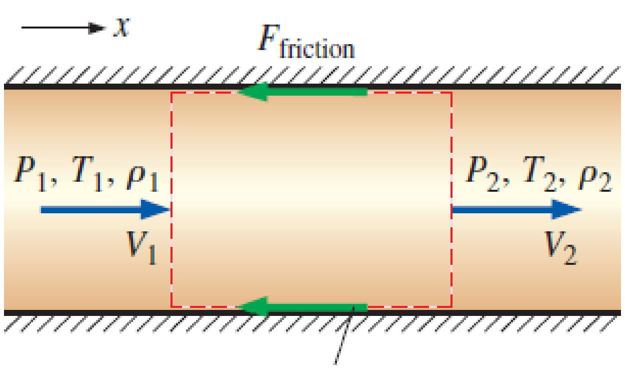
Second Law of Thermodynamic:

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Equation of State

$$P = \rho R T$$

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$



Governing Equations Summary

$$\rho_1 V_1 = \rho_2 V_2$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{frictior}}}{A}$$

$$T_1 + \frac{V_1^2}{2c_p} = T_2 + \frac{V_2^2}{2c_p}$$

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}$$

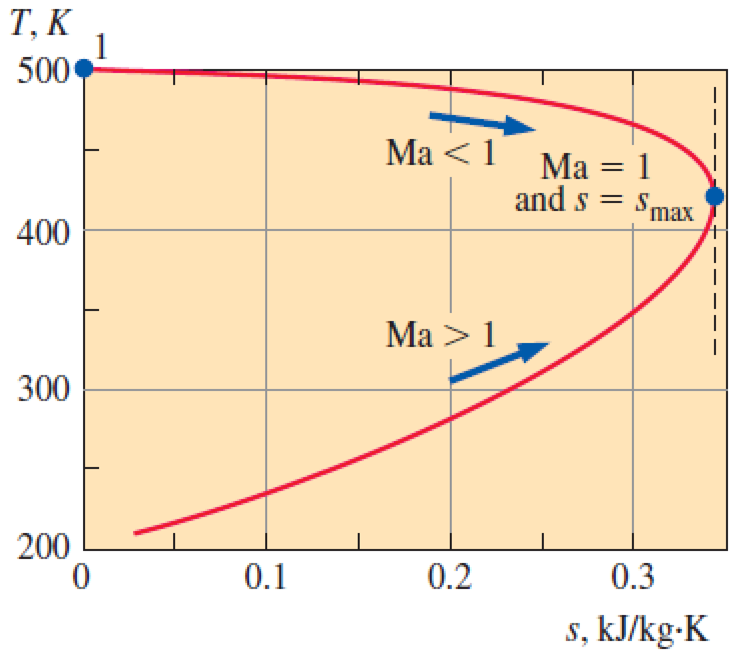


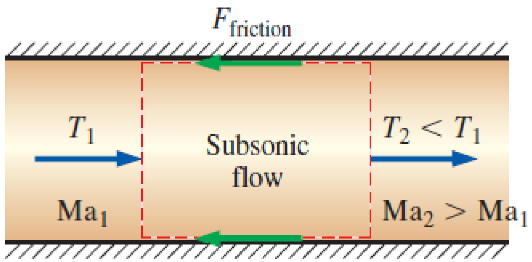
FIGURE 12–56
T-s diagram for adiabatic frictional flow in a constant-area duct (Fanno flow). Numerical values are for air with $k = 1.4$ and inlet conditions of $T_1 = 500$ K, $P_1 = 600$ kPa, $V_1 = 80$ m/s, and an assigned value of $s_1 = 0$.

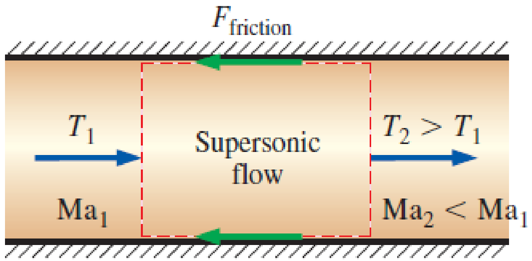
Governing Equations Summary

TABLE 12–4

The effects of friction on the properties of Fanno flow

Property	Subsonic	Supersonic
Velocity, V	Increase	Decrease
Mach number, Ma	Increase	Decrease
Stagnation temperature, T_0	Constant	Constant
Temperature, T	Decrease	Increase
Density, ρ	Decrease	Increase
Stagnation pressure, P_0	Decrease	Decrease
Pressure, P	Decrease	Increase
Entropy, s	Increase	Increase





Property Relations for Fanno Flow

Continuity Equation

The differential form of the continuity equation is obtained by differentiating the continuity relation

$$\rho V = \text{constant}$$

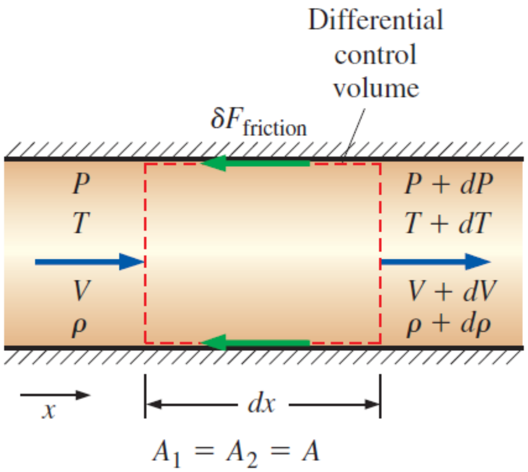
Thus

$$\rho\, dV + V\, d\rho = 0 \quad \rightarrow \quad \frac{d\rho}{\rho} = -\frac{dV}{V}$$

Momentum Equation

The differential form of the Momentum equation is given by

$$PA_c - (P + dP)A - \delta F_{\text{friction}} = \dot{m}(V + dV) - \dot{m}V$$



which yields

$$-dPA - \delta F_{\text{friction}} = \rho A V\, dV \qquad \text{or} \qquad dP + \frac{\delta F_{\text{friction}}}{A} + \rho V\, dV = 0$$

Property Relations for Fanno Flow

The friction force is related to the wall shear stress and the local friction factor *f* by

$$\delta F_{\text{friction}} = \tau_w \, dA_s = \tau_w p \, dx = \left(\frac{f_x}{8}\rho V^2\right)\frac{4A}{D_h}dx = \frac{f_x}{2}\frac{A\,dx}{D_h}\rho V^2$$

where *dx* is the length of the flow section, *p* is the perimeter, and *Dh* = 4*A*/*p* is the hydraulic diameter of the duct

Substituting,

$$dP + \frac{\rho V^2 f_x}{2D_h}dx + \rho V\,dV = 0$$

Since

$$\mathrm{Ma} = V/c = V/\sqrt{kRT}$$

Thus

$$V = \mathrm{Ma}\sqrt{kRT}$$

Since

$$P = \rho RT$$

Property Relations for Fanno Flow

We have

$$\rho V^2 = \rho k R T \mathrm{Ma}^2 = kP\mathrm{Ma}^2 \text{ and } \rho V = kP\mathrm{Ma}^2/V$$

Substituting,

$$\frac{1}{k\mathrm{Ma}^2}\frac{dP}{P} + \frac{f_x}{2D_h}dx + \frac{dV}{V} = 0$$

Energy Equation

$$T_0 = \text{constant or } T + V^2/2c_p = \text{constant}$$

$$T_0 = T\left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right) = \text{constant}$$

Differentiating and arranging

$$\frac{dT}{T} = -\frac{2(k-1)\mathrm{Ma}^2}{2 + (k-1)\mathrm{Ma}^2}\frac{d\mathrm{Ma}}{\mathrm{Ma}}$$

Property Relations for Fanno Flow

Mach Number

$$\mathrm{Ma} = V/c = V/\sqrt{kRT} \qquad V^2 = \mathrm{Ma}^2 kRT.$$

Differentiating and rearranging give

$$2V\,dV = 2\mathrm{Ma}kRT\,d\mathrm{Ma} + kR\mathrm{Ma}^2\,dT$$

$$2V\,dV = 2\frac{V^2}{\mathrm{Ma}}\,d\mathrm{Ma} + \frac{V^2}{T}\,dT$$

Dividing each term by $2V^2$ and rearranging,

$$\frac{dV}{V} = \frac{d\mathrm{Ma}}{\mathrm{Ma}} + \frac{1}{2}\frac{dT}{T}$$

Substituting by dT/T

$$\frac{dV}{V} = \frac{d\mathrm{Ma}}{\mathrm{Ma}} - \frac{(k-1)\mathrm{Ma}^2}{2 + (k-1)\mathrm{Ma}^2}\frac{d\mathrm{Ma}}{\mathrm{Ma}} \quad \text{or} \quad \frac{dV}{V} = \frac{2}{2 + (k-1)\mathrm{Ma}^2}\frac{d\mathrm{Ma}}{\mathrm{Ma}}$$

Property Relations for Fanno Flow

Ideal Gas

$$P = \rho RT,$$

Differentiating

$$dP = \rho R\, dT + RT\, d\rho \quad \rightarrow \quad \frac{dP}{P} = \frac{dT}{T} + \frac{d\rho}{\rho}$$

$$\frac{dP}{P} = \frac{dT}{T} - \frac{dV}{V}$$

$$\frac{dP}{P} = -\frac{2 + 2(k-1)\mathrm{Ma}^2}{2 + (k-1)\mathrm{Ma}^2}\frac{d\mathrm{Ma}}{\mathrm{Ma}}$$

differential equation for the variation of the Mach number with *x* as

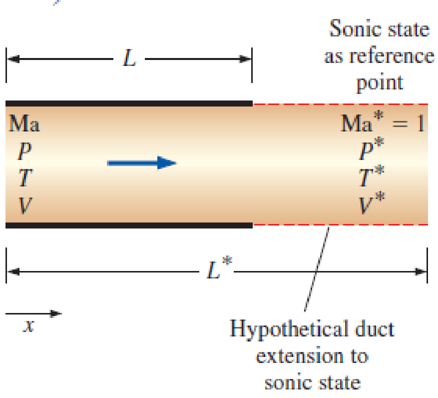
$$\frac{f_x}{D_h}dx = \frac{4(1 - \mathrm{Ma}^2)}{k\mathrm{Ma}^3\,[2 + (k-1)\mathrm{Ma}^2]}\, d\mathrm{Ma}$$

Property Relations for Fanno Flow

Considering that all Fanno flows tend to Ma = 1, it is again convenient to use the critical point (i.e., the sonic state) as the reference point and to express flow properties relative to the critical point properties, even if the actual flow never reaches the critical point. Integrating Eq. from any state (Ma = Ma and $x = x$) to the critical state (Ma=1 and $x = x_{cr}$) gives

$$\frac{fL^*}{D_h} = \frac{1 - \mathrm{Ma}^2}{k\mathrm{Ma}^2} + \frac{k+1}{2k}\ln\frac{(k+1)\mathrm{Ma}^2}{2+(k-1)\mathrm{Ma}^2}$$

where f is the average friction factor between x and x_{cr}, which is assumed to be constant, and $L^* = x_{cr} - x$ is the channel length required for the Mach number to reach unity under the influence of wall friction. Therefore, L^* represents the distance between a given section where the Mach number is Ma and a section (an imaginary section if the duct is not long enough to reach Ma = 1) where sonic conditions occur.



Property Relations for Fanno Flow

The actual duct length *L* between two sections where the Mach numbers are Ma1 and Ma2 can be determined from

$$\frac{fL}{D_h} = \left(\frac{fL^*}{D_h}\right)_1 - \left(\frac{fL^*}{D_h}\right)_2$$

If *f* is approximated as constant for the entire duct (including the hypothetical extension part to the sonic state), then Eq. simplifies to

$$L = L_1^* - L_2^* \quad (f = \text{constant})$$

The friction factor depends on the Reynolds number

$$\text{Re} = \rho V D_h / \mu$$

any change in Re is due to the variation of viscosity with temperature.
f can be calculated from the Moody chart or Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Property Relations for Rayleigh Flow

Relations for other flow properties can be determined similarly by integrating the *dP/P*, *dT/T*, and *dV/V* relations

$$\frac{P}{P^*} = \frac{1}{\mathrm{Ma}}\left(\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{T}{T^*} = \frac{k+1}{2+(k-1)\mathrm{Ma}^2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \mathrm{Ma}\left(\frac{k+1}{2+(k-1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{\mathrm{Ma}}\left(\frac{2+(k-1)\mathrm{Ma}^2}{k+1}\right)^{(k+1)/[2(k-1)]}$$

Fanno Flow Equation Summary and tables

$$T_0 = T_0^*$$

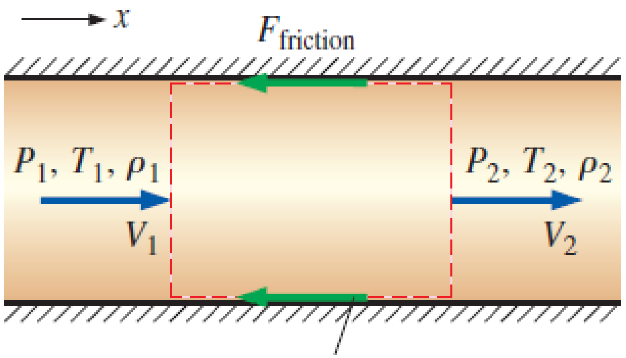
$$\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{\mathrm{Ma}}\left(\frac{2 + (k-1)\mathrm{Ma}^2}{k+1}\right)^{(k+1)/2(k-1)}$$

$$\frac{T}{T^*} = \frac{k+1}{2 + (k-1)\mathrm{Ma}^2}$$

$$\frac{P}{P^*} = \frac{1}{\mathrm{Ma}}\left(\frac{k+1}{2 + (k-1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \mathrm{Ma}\left(\frac{k+1}{2 + (k-1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{fL^*}{D} = \frac{1 - \mathrm{Ma}^2}{k\mathrm{Ma}^2} + \frac{k+1}{2k}\ln\frac{(k+1)\mathrm{Ma}^2}{2 + (k-1)\mathrm{Ma}^2}$$



Fanno Flow Equation Summary and tables

$$T_0 = T_0^*$$

$$\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{\mathrm{Ma}}\left(\frac{2 + (k-1)\mathrm{Ma}^2}{k+1}\right)^{(k+1)/2(k-1)}$$

$$\frac{T}{T^*} = \frac{k+1}{2 + (k-1)\mathrm{Ma}^2}$$

$$\frac{P}{P^*} = \frac{1}{\mathrm{Ma}}\left(\frac{k+1}{2 + (k-1)\mathrm{Ma}^2}\right)^{1/2}$$

$$\frac{V}{V^*} = \frac{\rho^*}{\rho} = \mathrm{Ma}\left(\frac{k+1}{2 + (k-1)\mathrm{Ma}^2}\right)^{1/2}$$

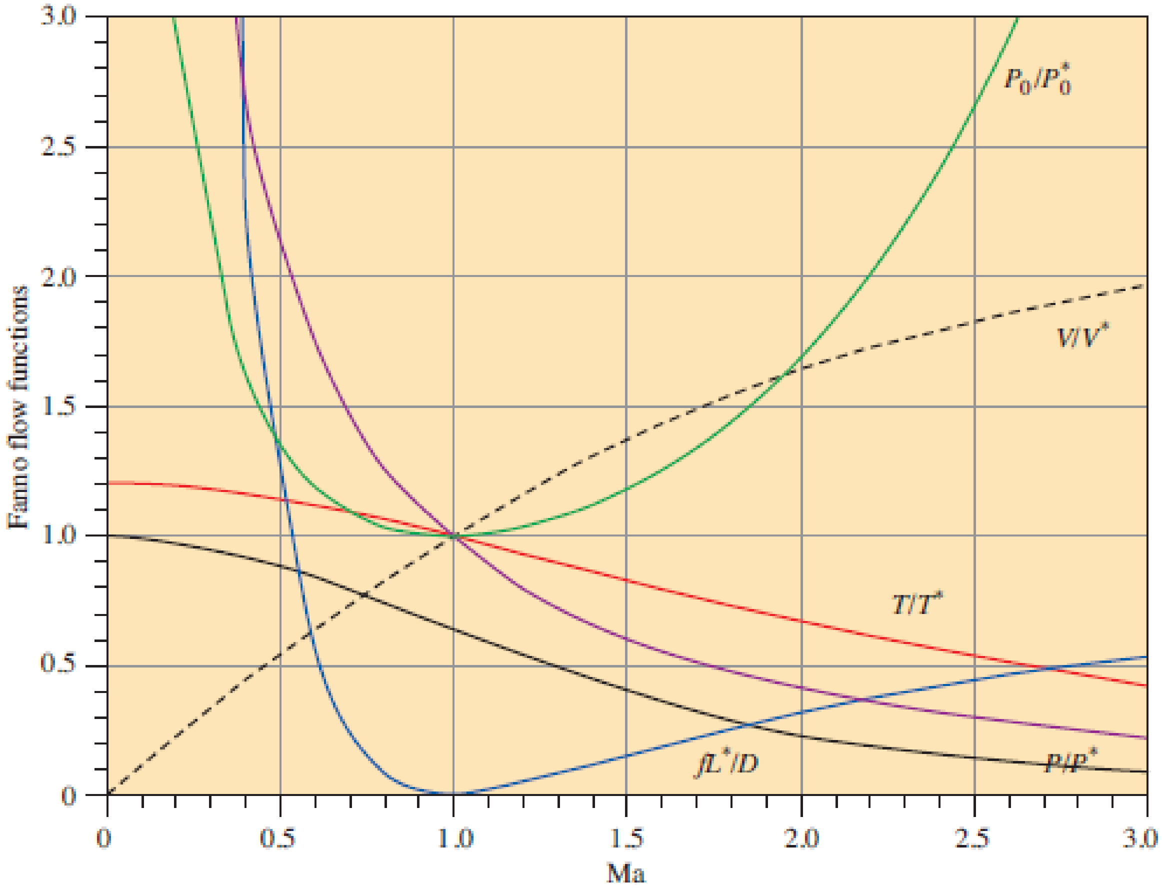
$$\frac{fL^*}{D} = \frac{1 - \mathrm{Ma}^2}{k\mathrm{Ma}^2} + \frac{k+1}{2k}\ln\frac{(k+1)\mathrm{Ma}^2}{2 + (k-1)\mathrm{Ma}^2}$$

TABLE A–16

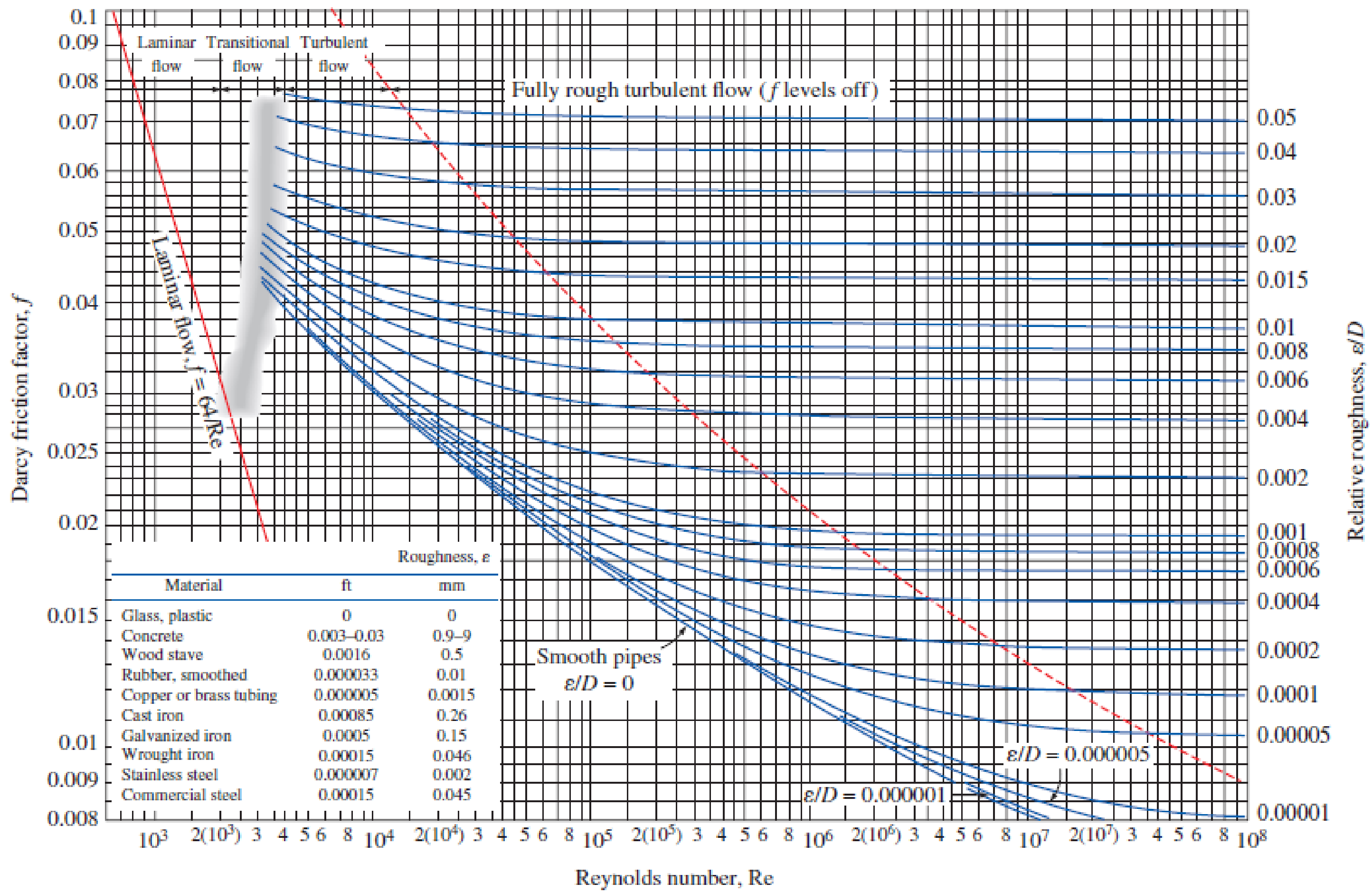
Fanno flow functions for an ideal gas with $k = 1.4$

Ma	P_0/P_0^*	T/T^*	P/P^*	V/V^*	fL^*/D
0.0	∞	1.2000	∞	0.0000	∞
0.1	5.8218	1.1976	10.9435	0.1094	66.9216
0.2	2.9635	1.1905	5.4554	0.2182	14.5333
0.3	2.0351	1.1788	3.6191	0.3257	5.2993
0.4	1.5901	1.1628	2.6958	0.4313	2.3085
0.5	1.3398	1.1429	2.1381	0.5345	1.0691
0.6	1.1882	1.1194	1.7634	0.6348	0.4908
0.7	1.0944	1.0929	1.4935	0.7318	0.2081
0.8	1.0382	1.0638	1.2893	0.8251	0.0723
0.9	1.0089	1.0327	1.1291	0.9146	0.0145
1.0	1.0000	1.0000	1.0000	1.0000	0.0000
1.2	1.0304	0.9317	0.8044	1.1583	0.0336
1.4	1.1149	0.8621	0.6632	1.2999	0.0997
1.6	1.2502	0.7937	0.5568	1.4254	0.1724
1.8	1.4390	0.7282	0.4741	1.5360	0.2419
2.0	1.6875	0.6667	0.4082	1.6330	0.3050
2.2	2.0050	0.6098	0.3549	1.7179	0.3609
2.4	2.4031	0.5576	0.3111	1.7922	0.4099
2.6	2.8960	0.5102	0.2747	1.8571	0.4526
2.8	3.5001	0.4673	0.2441	1.9140	0.4898
3.0	4.2346	0.4286	0.2182	1.9640	0.5222

Fanno Flow Equation Summary and tables



Moody Chart



Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

FIGURE A–12

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $h_L = f\frac{L}{D}\frac{V^2}{2g}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$.

Chocked Fanno Flow

It is clear from the previous discussions that friction causes subsonic Fanno flow in a constant-area duct to accelerate toward sonic velocity, and the Mach number becomes exactly unity at the exit for a certain duct length. This duct length is referred to as the **maximum length,** the **sonic length,** or the **critical length,** and is denoted by L^*. You may be curious to know what happens if we extend the duct length beyond L^*. In particular, does the flow accelerate to supersonic velocities? The answer to this question is a definite *no* since at Ma = 1 the flow is at the point of maximum entropy, and proceeding along the Fanno line to the supersonic region would require the entropy of the fluid to decrease—a violation of the second law of thermodynamics. (Note that the exit state must remain on the Fanno line to satisfy all conservation requirements.) Therefore, the flow is choked. This again is analogous to not being able to accelerate a gas to supersonic velocities in a converging nozzle by simply extending the converging flow section. If we extend the duct length beyond L^* anyway, we simply move the critical state further downstream and reduce the flow rate. This causes the inlet state to change (e.g., inlet velocity decreases), and the flow shifts to a different Fanno line. Further increase in duct length further decreases the inlet velocity and thus the mass flow rate.

Chocked Fanno Flow

Friction causes supersonic Fanno flow in a constant-area duct to decelerate and the Mach number to decrease toward unity. Therefore, the exit Mach number again becomes Ma = 1 if the duct length is L^*, as in subsonic flow. But unlike subsonic flow, increasing the duct length beyond L^* cannot
choke the flow since it is already choked. Instead, it causes a normal shock to occur at such a location that the continuing subsonic flow becomes sonic again exactly at the duct exit (Fig.). As the duct length increases, the
location of the normal shock moves further upstream. Eventually, the shock occurs at the duct inlet. Further increase in duct length moves the shock to the diverging section of the converging–diverging nozzle that originally generates the supersonic flow, but the mass flow rate still remains unaffected since the mass flow rate is fixed by the sonic conditions at the throat of the
nozzle, and it does not change unless the conditions at the throat change.

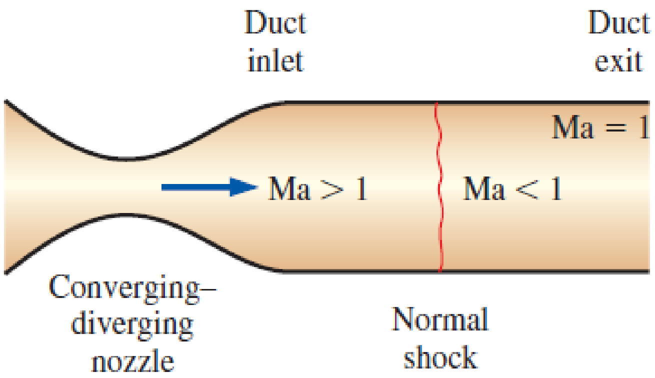
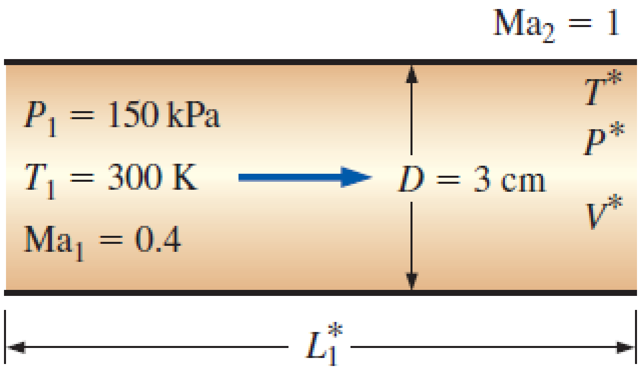


FIGURE 12–61
If duct length L is greater than L^*, supersonic Fanno flow is always sonic at the duct exit. Extending the duct will only move the location of the normal shock further upstream.

Example: Chocked Fanno Flow

Air enters a 3-cm-diameter smooth adiabatic duct at $\mathrm{Ma}_1 = 0.4$, $T_1 = 300$ K, and $P_1 = 150$ kPa (Fig. 12–62). If the Mach number at the duct exit is 1, determine the duct length and temperature, pressure, and velocity at the duct exit. Also determine the percentage of stagnation pressure lost in the duct.



Example: Chocked Fanno Flow

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, $R = 0.287$ kJ/kg·K, and $\nu = 1.58 \times 10^{-5}$ m²/s.

Analysis We first determine the inlet velocity and the inlet Reynolds number,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\ \text{kJ/kg·K})(300\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 347\ \text{m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 0.4(347\ \text{m/s}) = 139\ \text{m/s}$$

$$\text{Re}_1 = \frac{V_1 D}{\nu} = \frac{(139\ \text{m/s})(0.03\ \text{m})}{1.58 \times 10^{-5}\ \text{m}^2/\text{s}} = 2.637 \times 10^5$$

Example: Chocked Fanno Flow

The friction factor is determined from the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0}{3.7} + \frac{2.51}{2.637 \times 10^5\sqrt{f}}\right)$$

Its solution is

$$f = 0.0148$$

The Fanno flow functions corresponding to the inlet Mach number of 0.4 are (Table A–16):

$$\frac{P_{01}}{P_0^*} = 1.5901 \quad \frac{T_1}{T^*} = 1.1628 \quad \frac{P_1}{P^*} = 2.6958 \quad \frac{V_1}{V^*} = 0.4313 \quad \frac{fL_1^*}{D} = 2.3085$$

Noting that * denotes sonic conditions, which exist at the exit state, the duct length and the exit temperature, pressure, and velocity are determined to be

$$L_1^* = \frac{2.3085D}{f} = \frac{2.3085(0.03\text{ m})}{0.0148} = \mathbf{4.68\ m}$$

$$T^* = \frac{T_1}{1.1628} = \frac{300\text{ K}}{1.1628} = \mathbf{258\ K}$$

$$P^* = \frac{P_1}{2.6958} = \frac{150\text{ kPa}}{2.6958} = \mathbf{55.6\ kPa}$$

$$V^* = \frac{V_1}{0.4313} = \frac{139\text{ m/s}}{0.4313} = \mathbf{322\ m/s}$$

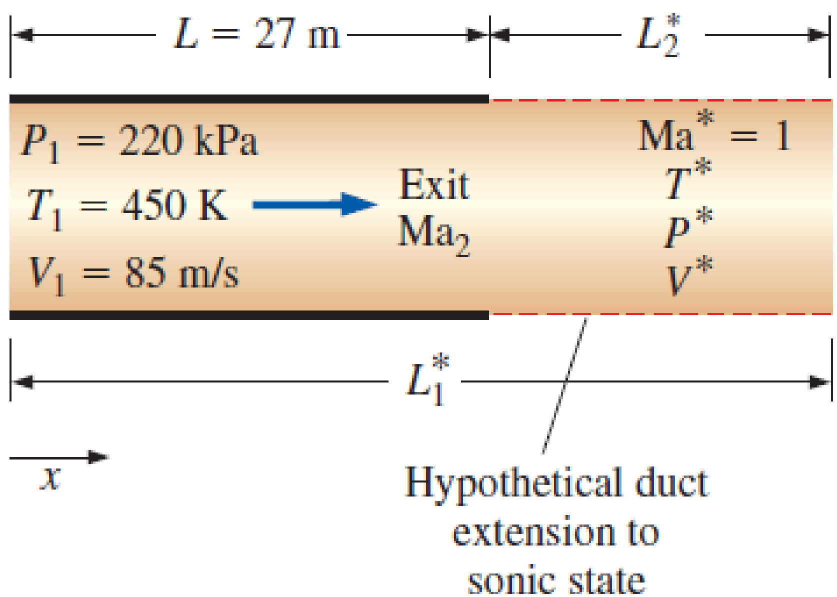
Example: Chocked Fanno Flow

Thus, for the given friction factor, the duct length must be 4.68 m for the Mach number to reach Ma = 1 at the duct exit. The fraction of inlet stagnation pressure P_{01} lost in the duct due to friction is

$$\frac{P_{01} - P_0^*}{P_{01}} = 1 - \frac{P_0^*}{P_{01}} = 1 - \frac{1}{1.5901} = 0.371 \quad \text{or} \quad \mathbf{37.1\%}$$

Example: Exit Conditions of Fanno Flow in a Duct

Air enters a 27-m-long 5-cm-diameter adiabatic duct at $V_1 = 85$ m/s, $T_1 = 450$ K, and $P_1 = 220$ kPa (Fig. 12–63). The average friction factor for the duct is estimated to be 0.023. Determine the Mach number at the duct exit and the mass flow rate of air.



Example: Exit Conditions of Fanno Flow in a Duct

Assumptions **1** The assumptions associated with Fanno flow (i.e., steady, frictional flow of an ideal gas with constant properties through a constant cross-sectional area adiabatic duct) are valid. **2** The friction factor is constant along the duct.

Properties We take the properties of air to be $k = 1.4$, $c_p = 1.005$ kJ/kg·K, and $R = 0.287$ kJ/kg·K.

Analysis The first thing we need to know is whether the flow is choked at the exit or not. Therefore, we first determine the inlet Mach number and the corresponding value of the function fL^*/D_h,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287\text{ kJ/kg·K})(450\text{ K})\left(\frac{1000\text{ m}^2/\text{s}^2}{1\text{ kJ/kg}}\right)} = 425\text{ m/s}$$

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{85\text{ m/s}}{425\text{ m/s}} = 0.200$$

Corresponding to this Mach number we read, from Table A–16, $(fL^*/D_h)_1 = 14.5333$. Also, using the actual duct length L, we have

$$\frac{fL}{D_h} = \frac{(0.023)(27\text{ m})}{0.05\text{ m}} = 12.42 < 14.5333$$

Example: Exit Conditions of Fanno Flow in a Duct

Therefore, flow is *not* choked and the exit Mach number is less than 1. The function fL^*/D_h at the exit state is calculated from Eq. 12–91,

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \frac{fL}{D_h} = 14.5333 - 12.42 = 2.1133$$

The Mach number corresponding to this value of fL^*/D is 0.42, obtained from Table A–16. Therefore, the Mach number at the duct exit is

$$\mathrm{Ma}_2 = \mathbf{0.420}$$

The mass flow rate of air is determined from the inlet conditions to be

$$\rho_1 = \frac{P_1}{RT_1} = \frac{220\ \mathrm{kPa}}{(0.287\ \mathrm{kJ/kg \cdot K})(450\ \mathrm{K})}\left(\frac{1\ \mathrm{kJ}}{1\ \mathrm{kPa \cdot m^3}}\right) = 1.703\ \mathrm{kg/m^3}$$

$$\dot{m}_{\mathrm{air}} = \rho_1 A_1 V_1 = (1.703\ \mathrm{kg/m^3})\,[\pi(0.05\ \mathrm{m})^2/4]\,(85\ \mathrm{m/s}) = \mathbf{0.284\ kg/s}$$