

SPC 407
Sheet 1 - Solution
Compressible Flow - Governing Equations

1. A high-speed aircraft is cruising in still air. How does the temperature of air at the nose of the aircraft differ from the temperature of air at some distance from the aircraft?

Solution:

The temperature of the air rises as it approaches the nose because of the stagnation process.

In the frame of reference moving with the aircraft, the air decelerates from high speed to zero at the nose (stagnation point), and this causes the air temperature to rise.

2. In air-conditioning applications, the temperature of air is measured by inserting a probe into the flow stream. Thus, the probe actually measures the stagnation temperature. Does this cause any significant error?

Solution:

No, there is not significant error, because the velocities encountered in air-conditioning applications are very low, and thus the static and the stagnation temperatures are practically identical.

If the air stream were supersonic, however, the error would indeed be significant.

3. Air flows through a device such that the stagnation pressure is 0.6 MPa, the stagnation temperature is 400 °C, and the velocity is 570 m/s. Determine the static pressure and temperature of the air at this state.

Solution:

Assumptions

1. The stagnation process is isentropic.
2. Air is an ideal gas.

The properties of air at an anticipated average temperature of 600 K are $C_p = 1.051 \text{ kJ/kg}\cdot\text{K}$ and $k = 1.376$.

The static temperature and pressure of air are determined from:

$$T = T_0 - \frac{V^2}{2c_p} = 673.2 - \frac{(570 \text{ m/s})^2}{2 \times 1.051 \text{ kJ/kg} \cdot \text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 518.6 \text{ K} \approx \mathbf{519 \text{ K}}$$

$$P_2 = P_{02} \left(\frac{T_2}{T_{02}} \right)^{k/(k-1)} = (0.6 \text{ MPa}) \left(\frac{518.6 \text{ K}}{673.2 \text{ K}} \right)^{1.376/(1.376-1)} = \mathbf{0.231 \text{ MPa}}$$

4. Air at 320 K is flowing in a duct at a velocity of (a) 1, (b) 10, (c) 100, and (d) 1000 m/s. Determine the temperature that a stationary probe inserted into the duct will read for each case.

Solution:

Assumptions: The stagnation process is isentropic.

The air which strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature, T_0 . It is determined from

$$T_0 = T + \frac{V^2}{2c_p}$$

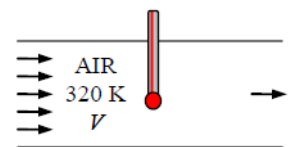
The results for each case are calculated below:

$$(a) \quad T_0 = 320 \text{ K} + \frac{(1 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg} \cdot \text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = \mathbf{320.0 \text{ K}}$$

$$(b) \quad T_0 = 320 \text{ K} + \frac{(10 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg} \cdot \text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = \mathbf{320.1 \text{ K}}$$

$$(c) \quad T_0 = 320 \text{ K} + \frac{(100 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg} \cdot \text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = \mathbf{325.0 \text{ K}}$$

$$(d) \quad T_0 = 320 \text{ K} + \frac{(1000 \text{ m/s})^2}{2 \times 1.005 \text{ kJ/kg} \cdot \text{K}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = \mathbf{817.5 \text{ K}}$$



Note that the stagnation temperature is nearly identical to the thermodynamic temperature at low velocities, but the difference between the two is significant at high velocities.

5. Calculate the stagnation temperature and pressure for the following substances flowing through a duct: (a) helium with $c_p = 5.1926 \text{ kJ/kg}\cdot\text{K}$ and $k = 1.667$ at 0.25 MPa , 50°C , and 240 m/s ; (b) nitrogen with $c_p = 1.039 \text{ kJ/kg}\cdot\text{K}$ and $k = 1.4$ at 0.15 MPa , 50°C , and 300 m/s ; and (c) steam gas with $c_p = 1.865 \text{ kJ/kg}\cdot\text{K}$ and $k = 1.329$ at 0.1 MPa , 350°C , and 480 m/s .

Solution:

Assumptions

1. The stagnation process is isentropic.
2. Helium and nitrogen are ideal gases.

- a. Helium can be treated as an ideal gas. Then the stagnation temperature and pressure of helium are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50^\circ\text{C} + \frac{(240 \text{ m/s})^2}{2 \times 5.1926 \text{ kJ/kg}\cdot^\circ\text{C}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 55.5^\circ\text{C}$$

$$P_0 = P \left(\frac{T_0}{T} \right)^{k/(k-1)} = (0.25 \text{ MPa}) \left(\frac{328.7 \text{ K}}{323.2 \text{ K}} \right)^{1.667/(1.667-1)} = 0.261 \text{ MPa}$$

- b. Nitrogen can be treated as an ideal gas. Then the stagnation temperature and pressure of nitrogen are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 50^\circ\text{C} + \frac{(300 \text{ m/s})^2}{2 \times 1.039 \text{ kJ/kg}\cdot^\circ\text{C}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 93.3^\circ\text{C}$$

$$P_0 = P \left(\frac{T_0}{T} \right)^{k/(k-1)} = (0.15 \text{ MPa}) \left(\frac{366.5 \text{ K}}{323.2 \text{ K}} \right)^{1.4/(1.4-1)} = 0.233 \text{ MPa}$$

- c. Steam can be treated as an ideal gas. Then the stagnation temperature and pressure of steam are determined from

$$T_0 = T + \frac{V^2}{2c_p} = 350^\circ\text{C} + \frac{(480 \text{ m/s})^2}{2 \times 1.865 \text{ kJ/kg}\cdot^\circ\text{C}} \left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 411.8^\circ\text{C} = 685 \text{ K}$$

$$P_0 = P \left(\frac{T_0}{T} \right)^{k/(k-1)} = (0.1 \text{ MPa}) \left(\frac{685 \text{ K}}{623.2 \text{ K}} \right)^{1.329/(1.329-1)} = 0.147 \text{ MPa}$$

Note that the stagnation properties can be significantly different than thermodynamic properties.

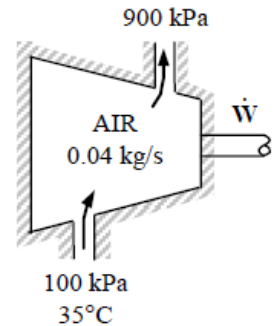
6. Air enters a compressor with a stagnation pressure of 100 kPa and a stagnation temperature of 358 °C, and it is compressed to a stagnation pressure of 900 kPa. Assuming the compression process to be isentropic, determine the power input to the compressor for a mass flow rate of 0.04 kg/s.

Solution:

Assumptions

1. The compressor is isentropic.
2. Air is an ideal gas.

The exit stagnation temperature of air T_{02} is determined from



$$T_{02} = T_{01} \left(\frac{P_{02}}{P_{01}} \right)^{(k-1)/k} = (308.2 \text{ K}) \left(\frac{900}{100} \right)^{(1.4-1)/1.4} = 577.4 \text{ K}$$

From the energy balance on the compressor,

$$\dot{W}_{\text{in}} = \dot{m}(h_{20} - h_{01})$$

$$\dot{W}_{\text{in}} = \dot{m}c_p(T_{02} - T_{01}) = (0.04 \text{ kg/s})(1.005 \text{ kJ/kg} \cdot \text{K})(577.4 - 308.2) \text{ K} = \mathbf{10.8 \text{ kW}}$$

7. Products of combustion enter a gas turbine with a stagnation pressure of 0.75 MPa and a stagnation temperature of 690 °C, and they expand to a stagnation pressure of 100 kPa. Taking $c_p = 1.157 \text{ kJ/kg} \cdot \text{K}$, $k = 1.33$ and $R = 0.287 \text{ kJ/kg} \cdot \text{K}$ for the products of combustion, and assuming the expansion process to be isentropic, determine the power output of the turbine per unit mass flow.

Solution:

Assumptions:

1. The expansion process is isentropic.
2. Products of combustion are ideal gases.

The exit stagnation temperature T_{02} is determined to be

$$T_{02} = T_{01} \left(\frac{P_{02}}{P_{01}} \right)^{(k-1)/k} = (963.2 \text{ K}) \left(\frac{0.1}{0.75} \right)^{(1.33-1)/1.33} = 584.2 \text{ K}$$

Also,

$$\begin{aligned}c_p = kc_v = k(c_p - R) &\longrightarrow c_p = \frac{kR}{k-1} \\ &= \frac{1.33(0.287 \text{ kJ/kg} \cdot \text{K})}{1.33-1} \\ &= 1.157 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

From the energy balance on the turbine,

$$-w_{\text{out}} = (h_{20} - h_{01})$$

$$w_{\text{out}} = c_p(T_{01} - T_{02}) = (1.157 \text{ kJ/kg} \cdot \text{K})(963.2 - 584.2)\text{K} = 438.5 \text{ kJ/kg} \cong \mathbf{439 \text{ kJ/kg}}$$