SPC 407
Sheet 8
Compressible Flow – Review

1. The thrust developed by the engine of a Boeing 777 is about 380 kN. Assuming choked flow in the nozzles, determine the mass flow rate of air through the nozzle. Take the ambient conditions to be 220 K and 40 kPa.

   Solution:

   Assumptions 1. Air is an ideal gas with constant specific heats.
   2. Flow of combustion gases through the nozzle is isentropic.
   3. Choked flow conditions exist at the nozzle exit.
   4. The velocity of gases at the nozzle inlet is negligible.

   Properties The gas constant of air is R = 0.287 kPa.m^3/kg-K, and it can also be used for combustion gases. The specific heat ratio of combustion gases is k = 1.33.

   The velocity at the nozzle exit is the sonic speed, which is determined to be

$$V = c = \sqrt{kRT} = \sqrt{(1.33)(0.287\ \mathrm{kJ/kg\cdot K})\left(\frac{1000\ \mathrm{m^2/s^2}}{1\ \mathrm{kJ/kg}}\right)(220\ \mathrm{K})} = 289.8\ \mathrm{m/s}$$

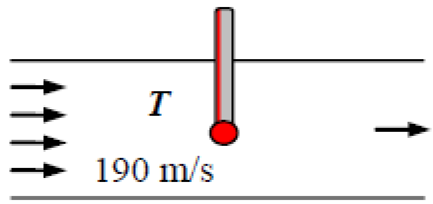
   Noting that thrust F is related to velocity by F = $\dot{m}$ V , the mass flow rate of combustion gases is determined to be

$$\dot{m} = \frac{F}{V} = \frac{380{,}000\ \mathrm{N}}{289.8\ \mathrm{m/s}}\left(\frac{1\ \mathrm{kg\cdot m/s^2}}{1\ \mathrm{N}}\right) = 1311\ \mathrm{kg/s} \cong \mathbf{1310\ kg/s}$$

   The combustion gases are mostly nitrogen (due to the 78% of N2 in air), and thus they can be treated as air
   with a good degree of approximation.

2. A stationary temperature probe inserted into a duct where air is flowing at 190 m/s reads 85°C. What is the actual temperature of the air?

Solution:



Assumptions

1. Air is an ideal gas with constant specific heats at room temperature.
2. The stagnation process is isentropic.

Properties The specific heat of air at room temperature is c_p = 1.005 kJ/kg-K.

The air that strikes the probe will be brought to a complete stop, and thus it will undergo a stagnation process. The thermometer will sense the temperature of this stagnated air, which is the stagnation temperature. The actual air temperature is determined from

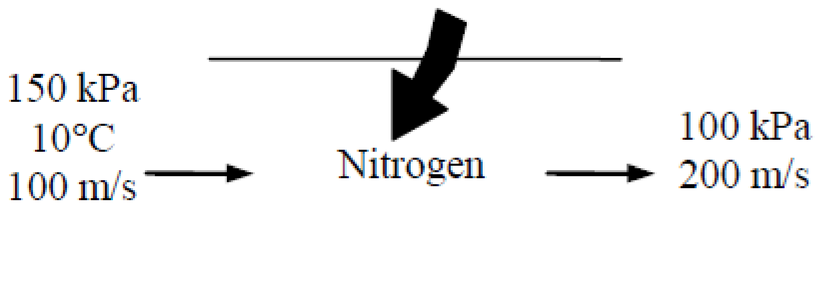
$$T = T_0 - \frac{V^2}{2c_p} = 85^\circ\text{C} - \frac{(190\ \text{m/s})^2}{2 \times 1.005\ \text{kJ/kg}\cdot\text{K}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{67.0^\circ C}$$

Temperature rise due to stagnation is very significant in high-speed flows, and should always be considered when compressibility effects are not negligible.

3. Nitrogen enters a steady-flow heat exchanger at 150 kPa, 108 C, and 100 m/s, and it receives heat in the amount of 150 kJ/kg as it flows through it. The nitrogen leaves the heat exchanger at 100 kPa with a velocity of 200 m/s. Determine the stagnation pressure and temperature of the nitrogen at the inlet and exit states.

Solution:

Nitrogen flows through a heat exchanger. The stagnation pressure and temperature of the nitrogen at the inlet and the exit states are to be determined.



Assumptions 1. Nitrogen is an ideal gas with constant specific heats.
2. Flow of nitrogen through the heat exchanger is isentropic.

Properties The properties of nitrogen are cp = 1.039 kJ/kg-K and k = 1.4.
The stagnation temperature and pressure of nitrogen at the inlet and the exit states are determined from

$$T_{01} = T_1 + \frac{V_1^2}{2c_p} = 10°\text{C} + \frac{(100\text{ m/s})^2}{2 \times 1.039\text{ kJ/kg}\cdot°\text{C}}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right) = \mathbf{14.8°C}$$

$$P_{01} = P_1\left(\frac{T_{01}}{T_1}\right)^{k/(k-1)} = (150\text{ kPa})\left(\frac{288.0\text{ K}}{283.2\text{ K}}\right)^{1.4/(1.4-1)} = \mathbf{159\ kPa}$$

From the energy balance relation in out system $E_{in} - E_{out} = \Delta E_{system}$ with work = 0

$$q_{\text{in}} = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + \Delta pe^{\nearrow 0}$$

$$150\text{ kJ/kg} = (1.039\text{ kJ/kg}\cdot°\text{C})(T_2 - 10°\text{C}) + \frac{(200\text{ m/s})^2 - (100\text{ m/s})^2}{2}\left(\frac{1\text{ kJ/kg}}{1000\text{ m}^2/\text{s}^2}\right)$$

$$T_2 = 139.9°\text{C}$$

and

$$T_{02} = T_2 + \frac{V_2^{\ 2}}{2c_p} = 139.9°\text{C} + \frac{(200\ \text{m/s})^2}{2 \times 1.039\ \text{kJ/kg}\cdot°\text{C}}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{159°C}$$

$$P_{02} = P_2\left(\frac{T_{02}}{T_2}\right)^{k/(k-1)} = (100\ \text{kPa})\left(\frac{432.3\ \text{K}}{413.1\ \text{K}}\right)^{1.4/(1.4-1)} = \mathbf{117\ kPa}$$

Note that the stagnation temperature and pressure can be very different than their thermodynamic counterparts when dealing with compressible flow.

4. A subsonic airplane is flying at a 5000-m altitude where the atmospheric conditions are 54 kPa and 256 K. A Pitot static probe measures the difference between the static and stagnation pressures to be 16 kPa. Calculate the speed of the airplane and the flight Mach number.

Solution:

A Pitot-static probe measures the difference between the static and stagnation pressures for a subsonic airplane. The speed of the airplane and the flight Mach number are to be determined.

Assumptions 1. Air is an ideal gas with a constant specific heat ratio.
2. The stagnation process is isentropic.

Properties The properties of air are R = 0.287 kJ/kg.K and k = 1.4.

The stagnation pressure of air at the specified conditions is

$$P_0 = P + \Delta P = 54 + 16 = 70\ \text{kPa}$$

Then,

$$\frac{P_0}{P} = \left(1 + \frac{(k-1)\text{Ma}^2}{2}\right)^{k/k-1} \longrightarrow \frac{70}{54} = \left(1 + \frac{(1.4-1)\text{Ma}^2}{2}\right)^{1.4/0.4}$$

It yields Ma = 0.620

The speed of sound in air at the specified conditions is

$$c = \sqrt{kRT} = \sqrt{(1.4)(0.287\ \text{kJ/kg}\cdot\text{K})(256\ \text{K})\left(\frac{1000\ \text{m}^2/\text{s}^2}{1\ \text{kJ/kg}}\right)} = 320.7\ \text{m/s}$$

Thus,

$$V = \text{Ma} \times c = (0.620)(320.7\ \text{m/s}) = \mathbf{199\ m/s}$$

Note that the flow velocity can be measured in a simple and accurate way by simply measuring pressure.

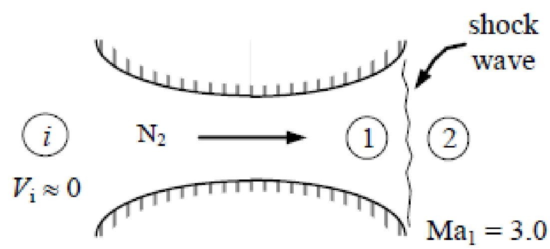
5. Nitrogen enters a converging–diverging nozzle at 620 kPa and 310 K with a negligible velocity, and it experiences a normal shock at a location where the Mach number is Ma = 3.0. Calculate the pressure, temperature, velocity, Mach number, and stagnation pressure downstream of the shock. Compare these results to those of air undergoing a normal shock at the same conditions.

Solution:

Assumptions

1. Nitrogen is an ideal gas with constant specific heats.
2. Flow through the nozzle is steady, one dimensional, and isentropic.
3. The nozzle is adiabatic.

Properties The properties of nitrogen are R = 0.297 kJ/kg-K and k = 1.4.



The inlet stagnation properties in this case are identical to the inlet properties since the inlet velocity is negligible. Assuming the flow before the shock to be isentropic,

$$P_{01} = P_i = 620\,\text{kPa}$$
$$T_{01} = T_i = 310\,\text{K}$$

Then,

$$T_1 = T_{01}\left(\frac{2}{2+(k-1)\text{Ma}_1^2}\right) = (310\,\text{K})\left(\frac{2}{2+(1.4-1)3^2}\right) = 110.7\,\text{K}$$

and

$$P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{k/(k-1)} = (620\,\text{kPa})\left(\frac{110.7}{310}\right)^{1.4/0.4} = 16.88\,\text{kPa}$$

The fluid properties after the shock (denoted by subscript 2) are related to those before the shock through the functions listed in Table A-14. For Ma_1 = 3.0 we read

$$\text{Ma}_2 = 0.4752 \cong \mathbf{0.475}, \quad \frac{P_{02}}{P_{01}} = 0.32834, \quad \frac{P_2}{P_1} = 10.333, \quad \text{and} \quad \frac{T_2}{T_1} = 2.679$$

Then the stagnation pressure P_{02}, static pressure P_2, and static temperature T_2, are determined to be

$$P_{02} = 0.32834 P_{01} = (0.32834)(620 \text{ kPa}) = 203.6 \text{ kPa} \cong \mathbf{204 \text{ kPa}}$$

$$P_2 = 10.333 P_1 = (10.333)(16.88 \text{ kPa}) = 174.4 \text{ kPa} \cong \mathbf{174 \text{ kPa}}$$

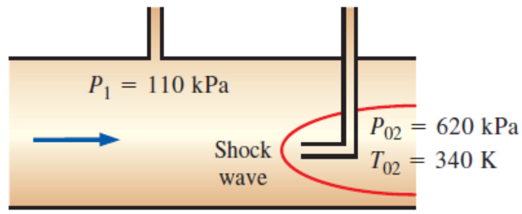
$$T_2 = 2.679 T_1 = (2.679)(110.7 \text{ K}) = 296.6 \text{ K} \cong \mathbf{297 \text{ K}}$$

The velocity after the shock can be determined from $V_2 = Ma_2 c_2$, where c_2 is the speed of sound at the exit conditions after the shock,

$$V_2 = \text{Ma}_2 c_2 = \text{Ma}_2 \sqrt{kRT_2} = (0.4752)\sqrt{(1.4)(0.297 \text{ kJ/kg} \cdot \text{K})(296.6 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 166.9 \text{ m/s} \cong \mathbf{167 \text{ m/s}}$$

For air at specified conditions k = 1.4 (same as nitrogen) and R = 0.287 kJ/kg·K. Thus the only quantity which will be different in the case of air is the velocity after the normal shock, which happens to be 164.0 m/s.

6. In compressible flow, velocity measurements with a Pitot probe can be grossly in error if relations developed for incompressible flow are used. Therefore, it is essential that compressible flow relations be used when evaluating flow velocity from Pitot probe measurements. Consider supersonic flow of air through a channel. A probe inserted into the flow causes a shock wave to occur upstream of the probe, and it measures the stagnation pressure and temperature to be 620 kPa and 340 K, respectively. If the static pressure upstream is 110 kPa, determine the flow velocity.



Solution:

The flow velocity of air in a channel is to be measured using a Pitot-static probe, which causes a shock wave to occur. For measured values of static pressure before the shock and stagnation pressure and temperature after the shock, the flow velocity before the shock is to be determined.

Assumptions 1. Air is an ideal gas with constant specific heats.
2. Flow through the nozzle is steady and one-dimensional.

Properties The specific heat ratio of air at room temperature is k= 1.4.

The nose of the probe is rounded (instead of being pointed), and thus it will cause a bow shock wave to form. Bow shocks are difficult to analyze. But they are normal to the body at the nose, and thus we can approximate them as normal shocks in the vicinity of the probe. It is given that the static pressure before the shock is P_1 = 110 kPa, and the stagnation pressure and temperature after the shock are P_{02} = 620 kPa, and T_{02} = 340 K. Noting that the stagnation temperature remains constant, we have

$$T_{01} = T_{02} = 340\ \text{K}$$

Also,

$$\frac{P_{02}}{P_1} = \frac{620\ \text{kPa}}{110\ \text{kPa}} = 5.6364 \approx 5.64$$

The fluid properties after the shock are related to those before the shock through the functions listed in Table A-14.

For P_{02}/P_{01}= 5.64 we read

$$\text{Ma}_1 = 2.0, \quad \text{Ma}_2 = 0.5774, \quad \frac{P_{02}}{P_{01}} = 0.7209, \quad \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = 2.6667,$$

Then the stagnation pressure and temperature before the shock become
$P_{01} = P_{02}/0.7209 = (620 \text{ kPa})/0.7209 = 860 \text{ kPa}$

$$P_{01} = P_{02}/0.7209 = (620 \text{ kPa})/0.7209 = 860 \text{ kPa}$$

$$T_1 = T_{01}\left(\frac{P_1}{P_{01}}\right)^{(k-1)/k} = (340 \text{ K})\left(\frac{110 \text{ kPa}}{860 \text{ kPa}}\right)^{(1.4-1)/1.4} = 188.9 \text{ K}$$

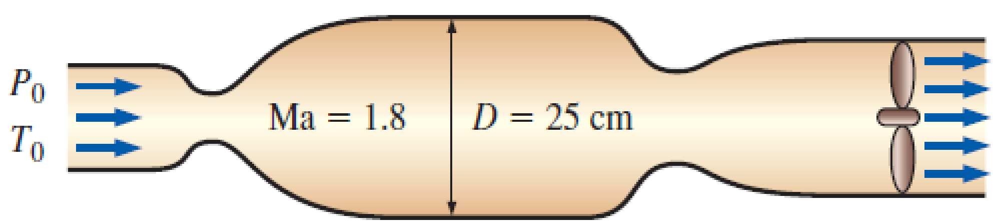
The flow velocity before the shock can be determined from $V_1 = Ma_1c_1$, where c_1 is the speed of sound before the shock,

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.4)(0.287 \text{ kJ/kg}\cdot\text{K})(188.9 \text{ K})\left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)} = 275.5 \text{ m/s}$$

$$V_1 = \text{Ma}_1 c_1 = 2(275.5 \text{ m/s}) = \mathbf{551 \text{ m/s}}$$

Discussion The flow velocity after the shock is V2 = V1/2.6667 = 551/2.6667 = 207 m/s. Therefore, the velocity measured by a Pitot-static probe would be very different that the flow velocity.

7. Design a 1-m-long cylindrical wind tunnel whose diameter is 25 cm operating at a Mach number of 1.8. Atmospheric air enters the wind tunnel through a converging– diverging nozzle where it is accelerated to supersonic velocities. Air leaves the tunnel through a converging–diverging diffuser where it is decelerated to a very low velocity before entering the fan section. Disregard any irreversibilities. Specify the temperatures and pressures at several locations as well as the mass flow rate of air at steady-flow conditions. Why is it often necessary to dehumidify the air before it enters the wind tunnel?



Solution:
The solution is left to the students.

8. A 10° half-angle wedge is placed in a "mystery flow" of unknown Mach number. Using a Schlieren system, the shock wave angle is measured as 44". What is the free-stream Mach number?

From the θ- β - M chart, for θ = 10° and β = 44°, we have

$$M_1 = 1.8$$

This technique has actually been used in some experiments for the measurement of Mach number. However, it is usually more accurate and efficient to use a Pitot tube to measure Mach number.

9. Consider a 15° half-angle wedge at zero angle of attack. Calculate the pressure coefficient on the wedge surface in a Mach 3 flow of air.

Solution:

The pressure coefficient is defined as

$$C_p = \frac{p - p_\infty}{q_\infty}$$

where p_∞ is the free-stream pressure and q_∞ is the free-stream dynamic pressure, defined by $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$. For a calorically perfect gas, q_∞ can also be expressed in terms of p_∞ and M_∞ as

$$q_\infty \equiv \tfrac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\frac{\gamma p_\infty}{\gamma p_\infty}\rho_\infty V_\infty^2 = \frac{\gamma p_\infty}{2}\frac{V_\infty^2}{a_\infty^2} = \frac{\gamma}{2}p_\infty M_\infty^2$$

Thus, the pressure coefficient can be written as

$$C_p = \frac{p - p_\infty}{\frac{\gamma}{2}p_\infty M_\infty^2} = \frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

In terms of the nomenclature being used in this chapter, where the free-stream properties in front of the shock are denoted by a subscript 1, then C_p is written as

$$C_p = \frac{2}{\gamma M_1^2}\left(\frac{p_2}{p_1} - 1\right)$$

For $M_1 = 3$ and $\theta = 15°$, we have from the θ-β-M diagram $\beta = 32.2°$. Hence

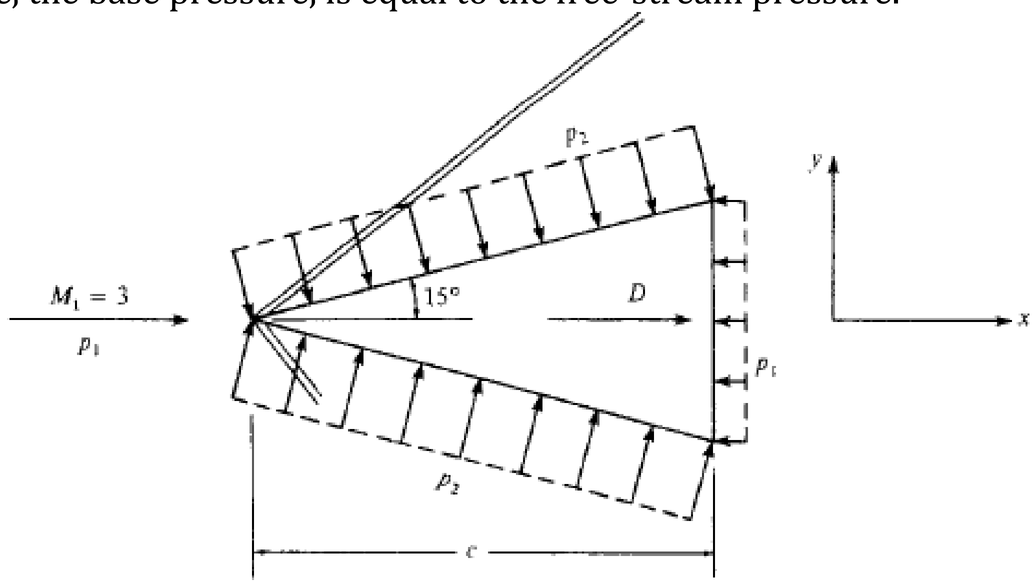
$$M_{n_1} = M_1 \sin\beta = 3 \sin 32.2 = 1.6$$

From Table A.2, for $M_{n_1} = 1.6$: $p_2/p_1 = 2.82$. Thus,

$$C_p = \frac{2}{(1.4)(3)^2}(2.82 - 1) = \boxed{0.289}$$

Note: For this example, we can deduce that C_p is strictly a function of γ and M_1.

10. Consider a 15° half-angle wedge at zero angle of attack in a Mach 3 flow of air. Calculate the drag coefficient. Assume that the pressure exerted over the base of the wedge, the base pressure, is equal to the free-stream pressure.



Solution:
The physical picture is sketched in Fig. The drag is the net force in the x direction; is exerted perpendicular to the top and bottom faces, and p_1 is exerted over the base. The chord length of the wedge is c. Consider a unit span of the wedge, i.e.. a length of unity perpendicular to the xy plane. The drag per unit span, denoted by D', is

$$D' = 2\left[\frac{(c)(1)}{\cos 15^\circ} p_2\right] \sin 15^\circ - (2c \tan 15^\circ) p_1$$

By definition, the drag coefficient is

$$c_d \equiv \frac{D'}{q_\infty S}$$

where S is the planform area (the projected area seen by viewing the wedge from the top). Thus, S = (c) (1). Hence

$$c_d = \frac{D'}{q_\infty c}$$

From Problem 9, we saw that

$$q_\infty = \frac{\gamma}{2} p_\infty M_\infty^2 = \frac{\gamma}{2} p_1 M_1^2$$

Thus.

$$c_d = \frac{2D'}{\gamma p_1 M_1^2 (c)(1)}$$

Or

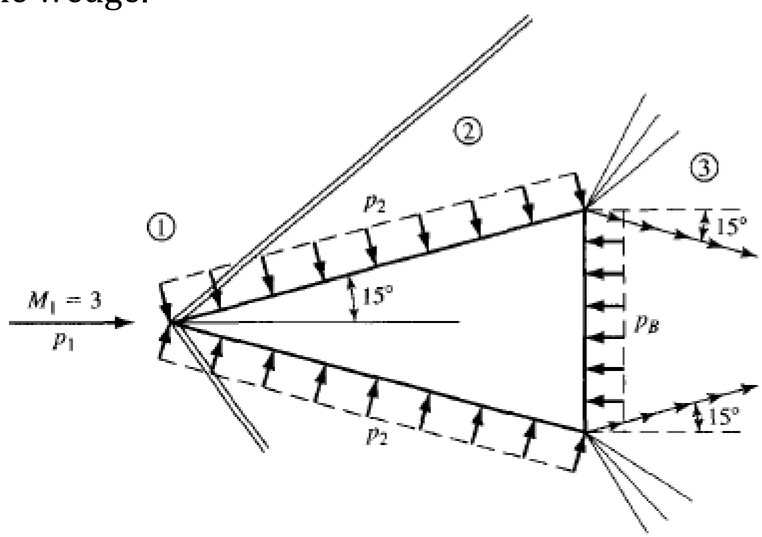
$$c_d = \frac{2}{\gamma p_1 M_1^2 c}\left[\frac{(2)(c)(1)}{\cos 15^\circ} p_2 \sin 15^\circ - (2c \tan 15^\circ) p_1\right]$$

$$= \frac{4}{\gamma p_1 M_1^2}(p_2 - p_1)\tan 15^\circ = \frac{4}{\gamma M_1^2}\left(\frac{p_2}{p_1} - 1\right)\tan 15^\circ$$

From Problem 9, which deals with the same wedge at the same flow conditions, we have $p_2/p_1 = 2.82$. Thus

$$c_d = \frac{4}{(1.4)(3)^2}(2.82 - 1)\tan 15^\circ = \boxed{0.155}$$

11. Consider the 15° half-angle wedge shown in Fig. We make the assumptions that (1) the flow separates at the comers, with the streamlines trailing downstream of the corners deflected toward the base at an angle of 15° from the horizontal, as shown in Fig, and (2) the base pressure p_B is the arithmetic average between the pressure downstream of the expansion waves, p_3, and the freestream pressure, p_1. i.e., $p_B = 1/2(p_3 + p_l)$. We emphasize that both assumptions are purely arbitrary; they represent a qualitative model of the flow with arbitrary numbers, and do not necessarily reflect the actual quantitative flowfield values that actually exist in the base flow region. Based on the model flow sketched in Fig. calculate the drag coefficient of the wedge.



Solution:

From Problem 10, we have these results for the leading edge shock wave and properties in region 2 behind the shock: θ= 15°, β = 32.2°, M_{n1} = 1.6, p_2/p_1 = 2.82. From Table A.2, we obtain M_{n2}= 0.6684. Hence,

$$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = \frac{0.6684}{\sin(32.2 - 15)} = 2.26$$

From Table A.1, for M_2 = 2.26, p_{o2}/p_2= 11.75. From Table A.5, for M_2 = 2.26, ν_2 = 33.27. Examining Fig., the flow expands from region 2 to region 3 through a total deflection angle of 15 + 15 = 30°. Hence,

$$\nu_3 = 33.27 + 30 = 63.27^\circ$$

From Table A.5, for ν_2 = 63.27" we obtain M_3 = 3.82. From Table A.l, for M_3 = 3.82, p_{o3}/p_3 = 119.1. Hence,

$$\frac{p_3}{p_1} = \frac{p_3}{p_{o_3}} \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_2} \frac{p_2}{p_1} = \left(\frac{1}{119.1}\right)(1)(11.75)(2.82) = 0.278$$

Assume $p_B = 0.5*(p_1 + p_3)$. Hence

$$\frac{p_B}{p_1} = \frac{1}{2}\left(1 + \frac{p_3}{p_1}\right) = \frac{1}{2}(1 + 0.278) = 0.639$$

From Problem 10, the drag coefficient for the wedge, with the base pressure now denoted by p_B, is given by

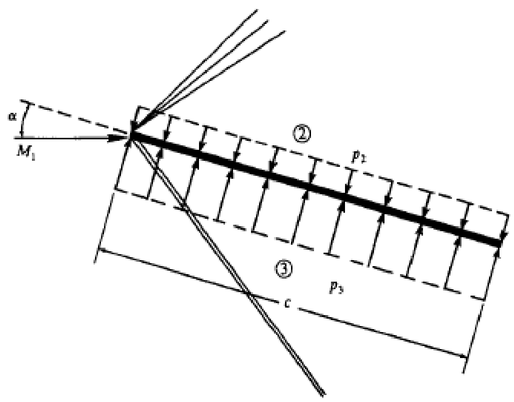
$$c_d = \frac{4}{\gamma p_1 M_1^2}(p_2 - p_B)\tan 15^\circ$$

$$= \frac{4}{\gamma M_1^2}\left(\frac{p_2}{p_1} - \frac{p_B}{p_1}\right)\tan 15^\circ$$

$$= \frac{4}{(1.4)(3)^2}(2.82 - 0.639)\tan 15^\circ = \boxed{0.186}$$

The value of c_d obtained from Problem 10 was the lower value of 0.155. The present example indicates that a 36 percent reduction in base pressure results in a 20 percent increase in drag coefficient.

12. Consider an infinitely thin flat plate at a 5° angle of attack in a Mach 2.6 free stream. Calculate the lift and drag coefficients.



Solution:

From Table A.5, for $M_1 = 2.6$: $\nu_1 = 41.41°$. Thus, from Eq. (4.45)

$$\nu_2 = \nu_1 + \alpha = 41.41 + 5 = 46.41°$$

From Table A.5, for $\nu_2 = 46.41°$: $M_2 = 2.85$. From Table A.1, for $M_1 = 2.6$: $p_{o_1}/p_1 = 19.95$. From Table A.1, for $M_2 = 2.85$: $p_{o_2}/p_2 = 29.29$. Hence

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \frac{1}{29.29}(1)(19.95) = 0.681$$

From the θ-β-M diagram, for $M_1 = 2.6$ and $\theta = \alpha = 5°$: $\beta = 26.5°$. Thus

$$M_{n_1} = M_1 \sin\beta = 2.6 \sin 26.5° = 1.16$$

From Table A.2, for $M_{n_1} = 1.16$: $p_3/p_1 = 1.403$. From Fig. 4.36, the lift per unit span L' is

$$L' = (p_3 - p_2)c\cos\alpha$$

The drag per unit span D' is

$$D' = (p_3 - p_2)c\sin\alpha$$

Recalling that $q_1 = (\gamma/2)p_1 M_1^2$, we have

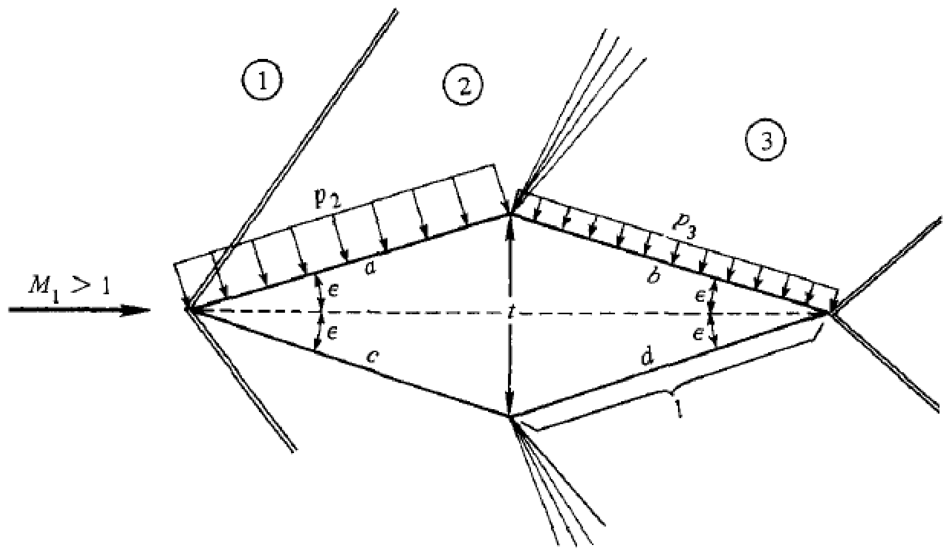
$$c_l = \frac{L'}{q_1 c} = \frac{2}{\gamma M_1^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\cos\alpha$$

$$= \frac{2}{(1.4)(2.6)^2}(1.403 - 0.681)\cos 5° = \boxed{0.152}$$

$$c_d = \frac{D'}{q_1 c} = \frac{2}{\gamma M_1^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\sin\alpha$$

$$= \frac{2}{(1.4)(2.6)^2}(1.403 - 0.681)\sin 5° = \boxed{0.0133}$$

13. Calculate the lift and drag (in pounds) on a symmetrical diamond airfoil of semiangle ε = 15 (see Fig.) at an angle of attack to the free stream of 5° when the upstream Mach number and pressure are 2.0 and 2116 Ib/ft^2, respectively. The maximum thickness of the airfoil is t = 0.5 ft. Assume a unit length of 1 ft in the span direction (perpendicular to the page in Fig.).



Solution:

$$\ell = \frac{t/2}{\sin\varepsilon} = \frac{0.25}{\sin(15^\circ)} = 0.966 \text{ ft}$$

For face (a): When M_1 = 2.0 and θ = 10°, β = 39.2°

$$M_{n_1} = M_1 \sin\beta = (2.0)\sin 39.2^\circ = 1.264$$

$$M_{n_2} = 0.8049$$

$$p_2 = \frac{p_2}{p_1} p_1 = (1.698)(2116) = 3593 \text{ lb/ft}^2$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.8049}{\sin(39.12 - 10)} = 1.65$$

$$\nu_2 = 16.34^\circ$$

For face (b): $\nu_3 = \nu_2 + \theta = 16.34 + 30 = 46.34^\circ$

$\therefore M_3 = 2.83.$ Also $\frac{p_{o_3}}{p_3} = 28.41$

$$p_3 = \frac{p_3}{p_{o_3}} \frac{p_{o_2}}{p_2} \; p_2 = \left(\frac{1}{28.41}\right)(4.579)(3595) = 579 \text{ lb/ft}^2$$

For face (c): When $M_1 = 2.0$ and $\theta = 20°$, **=** 53.5

$$M_{n_1} = M_1 \sin\beta = (2) \sin 53.5° = 1.61$$

$$M_{n_4} = 0.6655$$

$$p_4 = \frac{p_4}{p_1} \; p_1 = (2.857)(2116) = 6045 \text{ lb/ft}^2$$

$$M_4 = \frac{M_{n_4}}{\sin(\beta - \theta)} = \frac{0.66559}{\sin(53.5 - 20)} = 1.21$$

$$\nu_4 = 3.806°$$

For face (d): $\nu_5 = \nu_4 + \theta = 3.806 + 30 = 33.81°$

$M_5 = 2.28.$ Also $\frac{p_{o_5}}{p_5} = 12.12$

$$p_5 = \frac{p_5}{p_{o_5}} \frac{p_{o_4}}{p_4} \; p_2 = \left(\frac{1}{12.12}\right)(2.457)(6045) = 1225 \text{ lb/ft}^2$$

Lift is the component of the total aerodynamic force in the y-direction:

$$L(\text{per unit span}) = \ell \; [(p_4\text{-}p_3) \cos 20° + (p_5\text{-}p_2) \cos 10°]$$

$$= 0.966 \; [(6045\text{-}579) \cos 20° + (1225\text{-}3593 \cos 10°]$$

$$= 0.966 \; (5136 - 2332) = \boxed{2708 \text{ lb per foot of span}}$$

Drag is the component of the total aerodynamic force in the x-direction:

$$D(\text{per unit span}) = \ell \; [(p_4\text{-}p_3) \sin 20° + (p_2\text{-}p_5) \sin 10°]$$

$$= 0.99 \; (1869 + 411) = \boxed{2202 \text{ lb per foot of span}}$$

14. Consider a flat plate with a chord length (from leading to trailing edge) of 1 m. The free-stream flow properties are $M_1 = 3$, $p_1 = 1$ atm, and $T_1 = 270$ K. Tabulate and plot on graph paper these properties as functions of angle of attack from 0 to 30° (use increments of 5°):
    a. Pressure on the top surface
    b. Pressure on the bottom surface
    c. Temperature on the top surface
    d. Temperature on the bottom surface
    e. Lift per unit span
    f. Drag per unit span
    g. Lift/drag ratio

Solution:

α	β	M_{n_1}	$\frac{p_2}{p_1}$	p_2 (atm)	$\frac{T_2}{T_1}$	T_2 (°K)
0	19.6	1	1	1	1	270
5	23.1	1.18	1.458	1.458	1.115	301
10	27.4	1.38	2.055	2.055	1.242	335
15	32.2	1.60	2.820	2.820	1.388	375
20	37.8	1.84	3.783	3.783	1.562	422
25	44.0	2.08	4.881	4.881	1.754	474
30	52.0	2.36	6.331	6.331	2.002	541

For $M_1 = 3$, $\nu_1 = 49.76$, $\nu_3 = \nu_1 + \alpha$, $p_{o_1}/p_1 = 36.73$, $T_{o_1}/T_1 = 2.8$

α	ν_3	M_3	$\frac{p_{o_3}}{p_3}$	p_3 (atm)	$\frac{T_{o_3}}{T_3}$	T_3 (°K)
0	49.76	3	36.73	1	2.8	270
5	54.76	3.27	54.78	0.670	3.139	241
10	59.76	3.58	85.40	0.430	3.563	212
15	64.76	3.92	136.4.	0.269	4.073	186
20	69.76	4.32	230.6	0.159	4.732	160
25	74.76	4.78	407.8	0.090	5.57	136
30	79.76	5.32	762.8	0.048	6.66	114

Per unit span: $L = c\,(p_2 - p_3)\cos\alpha$, $D = c\,(p_2 - p_3)\cos\alpha$, 1 atm = 1.01×10^5 N/m^2

α	$(p_2 - p_3)$ N/m^2	L N/m	D N/m	L/D
0	0	0	0	-
5	$7.94 \mathrm{X} 10^4$	7.93×10^4	6.94×10^3	11.4
10	1.64×10^5	1.62×10^5	2.85×10^4	5.68
15	2.58×10^5	2.49×10^5	6.68×10^4	3.73
20	3.66×10^5	3.44×10^5	1.25×10^5	2.75
25	4.84×10^5	4.39×10^5	2.05×10^5	2.14
30	6.35×10^5	5.50×10^5	3.18×10^5	1.73

15. Calculate the drag coefficient for a wedge with a 20° half-angle at Mach 4. Assume the base pressure is free-stream pressure.

Solution:

$$D' = 2\left[\frac{c}{\cos\ 20°}\right] p_2 \sin 20° - p_\infty\, 2\, c\, \mathrm{Tan}\ 20° = 2\, c\, (p_2 - p_\infty)\, \mathrm{Tan}\ 20°$$

$$c_d = \frac{D'}{q_\infty S} = \frac{D'}{\frac{\gamma}{2} p_\infty M_\infty{}^2 c}$$

$$c_d = \frac{2c(p_2 - p_\infty)\mathrm{Tan}\ 20°}{\frac{\gamma}{2} p_\infty M_\infty{}^2 c} = \frac{4}{\gamma M_\infty{}^2}\left(\frac{p_2}{p_\infty}\right) \mathrm{Tan}\ 20°$$

From the θ-β-M diagram, for $\theta = 20°$ and $M_4 = 4$, we have $\beta = 32.5°$.

$$M_{n,1} = M_1 \sin\beta = 4 \sin 32.5° = 4\ (0.5373) = 2.149$$

From Table A.2, for $M_{n,1} = 2.149$, $\frac{p_2}{p_\infty} = 5.226$

$$c_d = \frac{4}{(1.4)(4)^2}\ (5.226 - 1)\ (0.364) = \boxed{0.275}$$