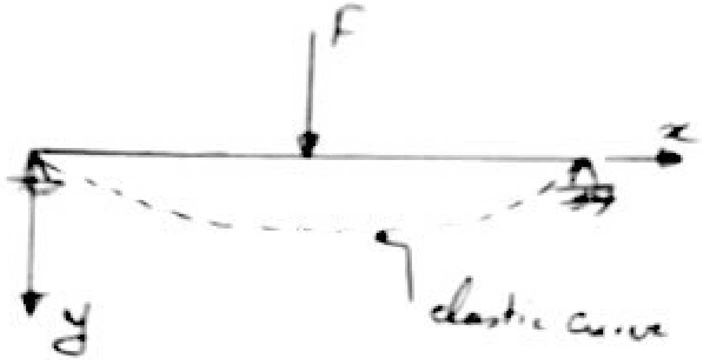
Deflection

Double integration method

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$



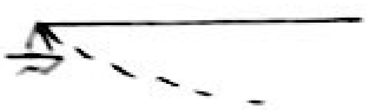
this is the equation of the elastic curve

$$\frac{dy}{dx} = f_1(x) + C_1$$

$$y = f_2(x) + C_1 x + C_2$$

where C_1 & C_2 are constants and can be obtained from boundary conditions

Boundary Conditions



simple support

$y = 0.0$

$\frac{dy}{dx} \neq 0.0$

fixed support

$\frac{dy}{dx} = 0.0$

& $y = 0.0$

Then

$$y = \frac{1}{EI}\left[\frac{wa^2}{4}x^2 + \frac{w}{24}x^4 - \frac{wa}{6}x^3\right]$$

(2) $a \leq x \leq a+b$

$M_x = 0.0$

$$EI\frac{d^2y}{dx^2} = 0.0$$

$$EI\frac{dy}{dx} = C_3$$

$$EI\,y = C_3\,x + C_4$$

Common boundary Conditions

at $x = a \longrightarrow \left.\frac{dy}{dx}\right)_1 = \left.\frac{dy}{dx}\right)_2$

& $\longrightarrow y)_1 = y)_2$

Then $C_3 = \_\_$ & $C_4 = \_\_$

$$EI\frac{dy}{dx} = -\frac{wL}{4}x^2 + \frac{w}{6}x^3 + \frac{wL^3}{24}$$

$$EIy = -\frac{wL}{12}x^3 + \frac{w}{24}x^4 + \frac{wL^3}{24}x + C_2$$

2nd boundary Condition

at $x = 0.0 \longrightarrow y = 0.0$

$$\boxed{0.0 = C_2}$$

Then

$$y = \frac{1}{EI}\left[\frac{-wLx^3}{12} + \frac{wx^4}{24} + \frac{wL^3}{24}x\right]$$

but y_{max} at $x = \frac{L}{2}$

$$\therefore y_{max} = \frac{1}{EI}\left[\frac{-wL\left(\frac{L}{2}\right)^3}{12} + \frac{w\left(\frac{L}{2}\right)^4}{24} + \frac{wL^3}{24}\frac{L}{2}\right]$$

$$\boxed{y_{max} = \frac{5}{384}\frac{wL^4}{EI}}$$

$$M_x = Fx - F(x - 125) = 125F$$

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = -\frac{125F}{EI}$$

$$\frac{dy}{dx} = -\frac{125F}{EI}x + C_3$$

$$y = -\frac{125F}{2EI}x^2 + C_3 x + C_4$$

<u>boundary Conditions</u>

at $x = 250 \longrightarrow \frac{dy}{dx} = 0.0$

$$0.0 = -125\frac{F}{EI} * 250 + C_3$$

$$\boxed{C_3 = 2*(125)^2\frac{F}{EI}}$$

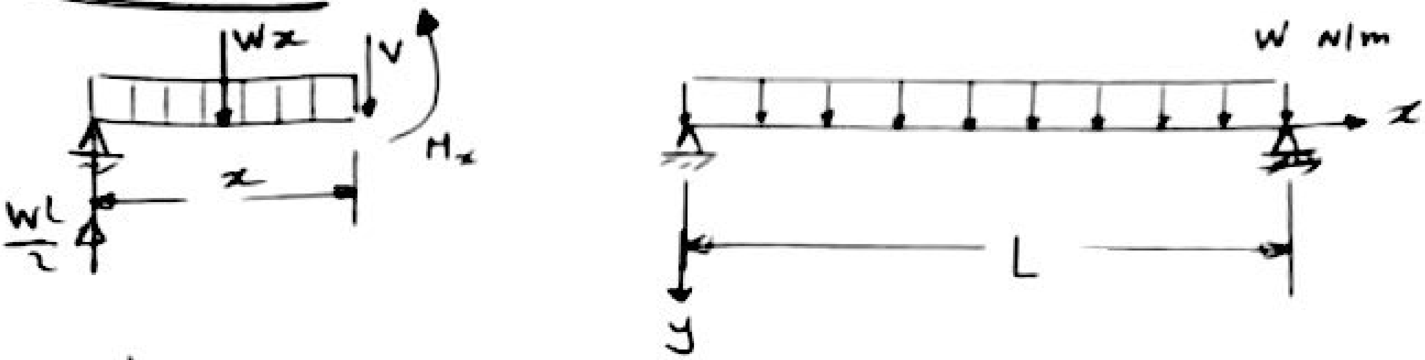
$$\therefore y = -\frac{125F}{2EI}x^2 + \frac{2(125)^2}{EI}Fx + C_4 \longrightarrow 2$$

<u>Common boundary Condition</u>

at $x = 125$ $\left.\frac{dy}{dx}\right)_1 = \left.\frac{dy}{dx}\right)_2$

$$-\frac{F(125)^2}{2EI} + C_1 = -\frac{125F}{EI}*125 + 2*(125)^2\frac{F}{EI}$$

Example 2



$$\frac{WL}{2}x - Wx\frac{x}{2} - M_x = 0.0$$

$$\boxed{M_x = \frac{WL}{2}x - W\frac{x^2}{2}}$$

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI}$$

$$EI\frac{d^2y}{dx^2} = -\frac{WL}{2}x + W\frac{x^2}{2}$$

$$EI\frac{dy}{dx} = -\frac{WL}{4}x^2 + \frac{W}{6}x^3 + C_1$$

1st boundary Condition

at $x = \frac{L}{2} \longrightarrow \frac{dy}{dx} = 0.0$

$$\therefore \quad 0.0 = -\frac{WL}{4}\frac{L^2}{4} + \frac{W}{6}\frac{L^3}{8} + C_1$$

$$\boxed{C_1 = \frac{WL^3}{24}}$$

$$\therefore \quad C_1 = \frac{3}{2}\frac{(125)^2 F}{EI}$$

at $\quad x = 125 \qquad y)_1 = y)_2$

$$-\frac{F(125)^3}{6EI} + \frac{3}{2}\frac{(125)^2 F}{EI} = -\frac{(125)^3 F}{2EI} + \frac{2(125)^3}{EI} F + C_4$$

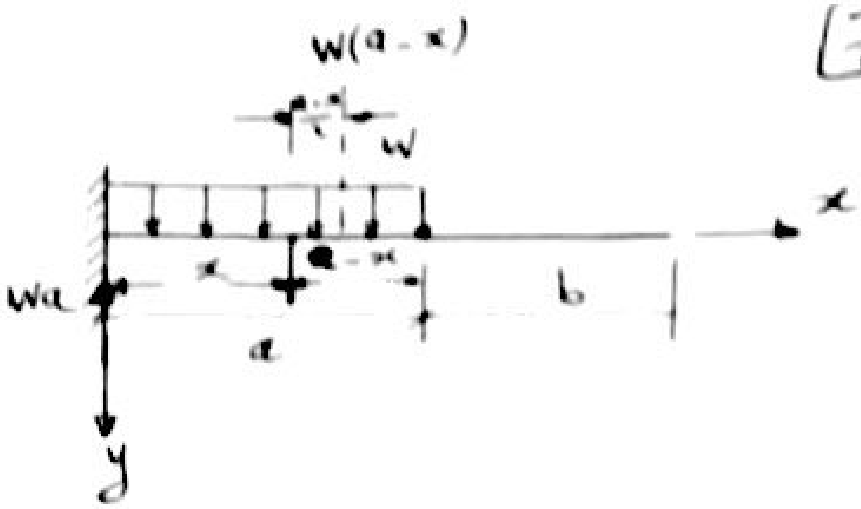
$$C_4 = -\frac{(125)^3 F}{6EI}$$

$y_{max} = y)_2$ at $x = 250$

$$\therefore y_{max} = -\frac{125 F}{2EI}(250)^2 + \frac{2(125)^2}{EI} F (250) - \frac{1}{6}\frac{(125)^3 F}{EI}$$

$$\boxed{y_{max} = \frac{11(125)^3 F}{6EI}}$$

Example ③



(1) $0.0 \leq x \leq a$

$$M_x = -w(a-x)\left(\frac{a-x}{2}\right)$$

$$EI\frac{d^2y}{dx^2} = -M_x$$

$$= w(a-x)\left(\frac{a-x}{2}\right) = \frac{wa^2}{2} - wax + \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{wa^2}{2}x + \frac{wx^3}{6} - \frac{wa}{2}x^2 + C_1$$

but at $x = 0.0 \longrightarrow \frac{dy}{dx} = 0.0$

$$\boxed{0.0 = C_1}$$

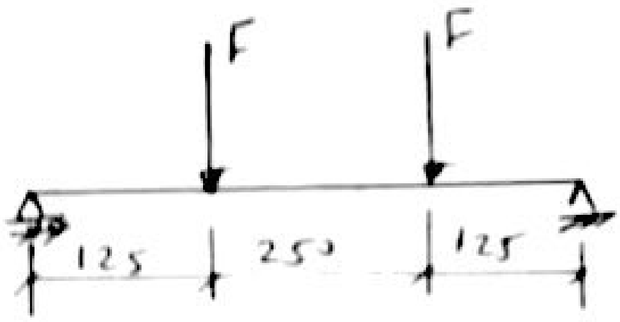
$$\therefore EI\frac{dy}{dx} = \frac{wa^2}{2}x + \frac{wx^3}{6} - \frac{wa}{2}x^2$$

$$EI\,y = \frac{wa^2}{4}x^2 + \frac{wx^4}{24} - \frac{wa}{6}x^3 + C_2$$

but at $x = 0.0 \longrightarrow y = 0.0$

$$\boxed{C_2 = 0.0}$$

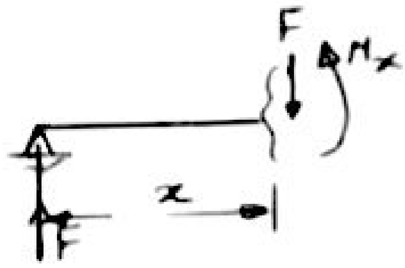
Example ①



Req y_{max}

sol^n

(1) for $0.0 \leq x \leq 125$



$$M_x = F x$$

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = -\frac{Fx}{EI}$$

$$\frac{dy}{dx} = \frac{-Fx^2}{2EI} + C_1$$

$$y = -\frac{Fx^3}{6EI} + C_1 x + C_2$$

Boundary Conditions

at $x = 0.0$ $\quad y = 0.0 \longrightarrow \boxed{C_2 = 0.0}$

Then

$$y = -\frac{Fx^3}{6EI} + C_1 x \longrightarrow (1)$$

(2) for $125 \leq x \leq 250$