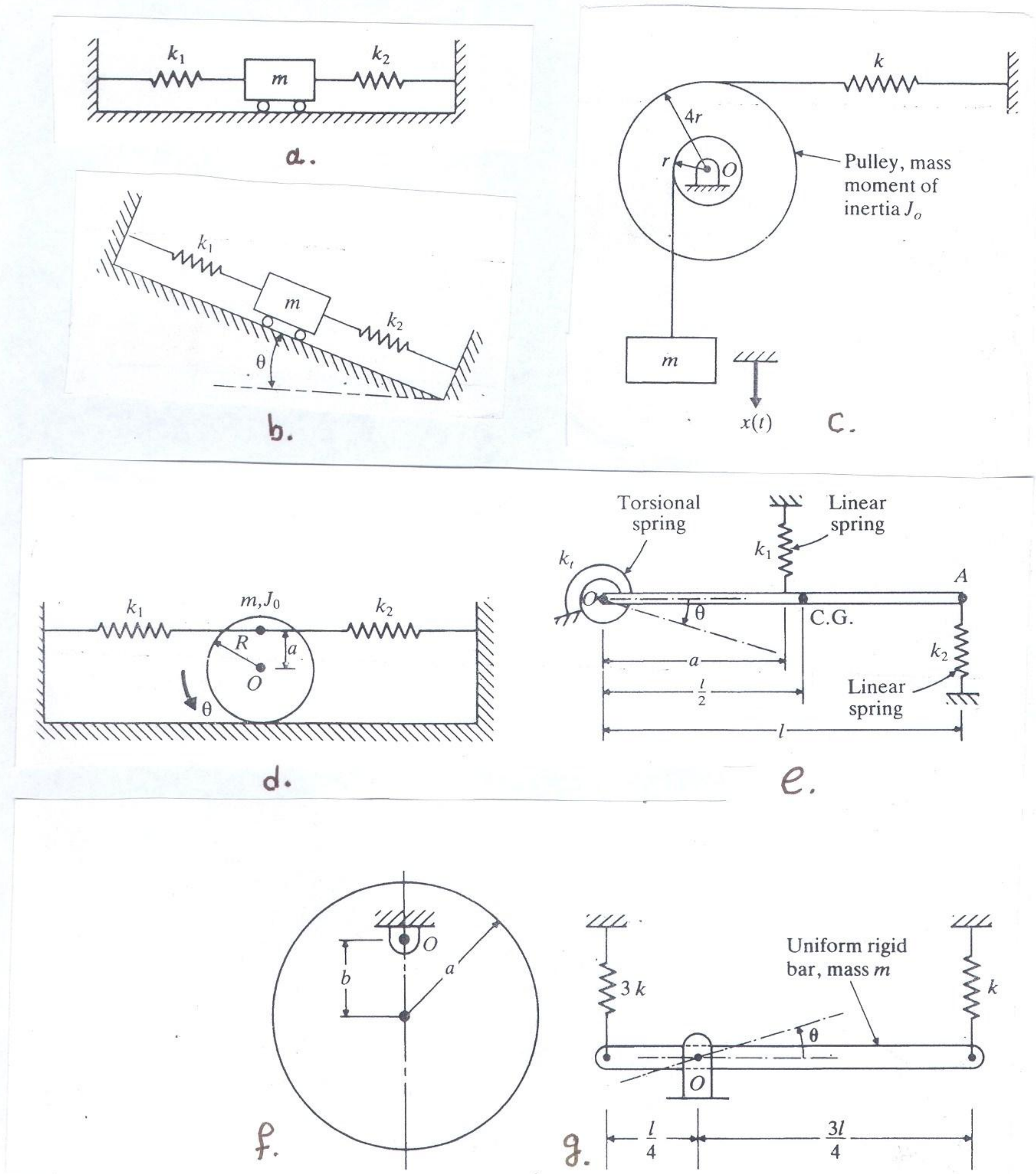
Sheet 3
Free Vibration of Single Degree of Freedom System

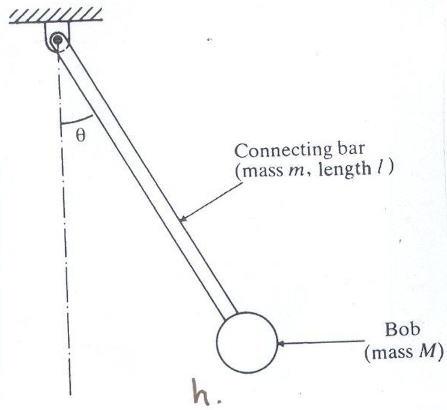
1. a. Draw the free-body diagram and derive the equation of motion using Newton's second law of motion for each of the systems shown in Figs. (1)

   b. Derive the equation of motion using the principle of conservation of energy for each of the systems shown in Figs. (1)

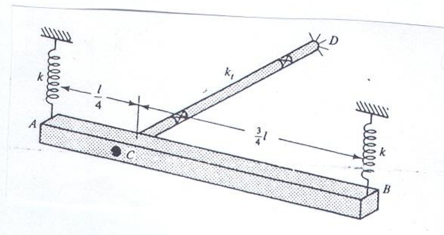
   C. find the natural frequency of vibration.



Figs. (1)



h.



i.

Figs. (1) Cont.

2. Three springs and a mass are attached to a rigid, weightless, bar PQ as shown in Fig. 2. Find the natural frequency of vibration of the system.

3. An automobile having a mass of 2000 kg deflects its suspension springs 0.02 m under static conditions. Determine the natural frequency of the automobile in the vertical direction by assuming damping to be negligible.

4. A heavy ring of mass moment of inertia 1.0 kg-m^2 is attached at the end of a two-layered hollow shaft of length 2 m (Fig. 3. If the two layers of the shaft are made of steel and brass, determine the natural time period of torsional vibration of the heavy ring.

5. A uniform slender rod of mass m and length l is hinged at point A and is attached to four linear springs and one torsional spring, as shown in Fig. 4. Find the natural frequency of the system if k = 2000 N/m, k_t = 1000 N-m/rad, m = 10 kg, and l = 5 m.

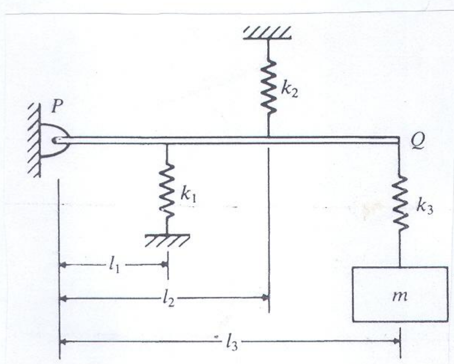


FIGURE 2.

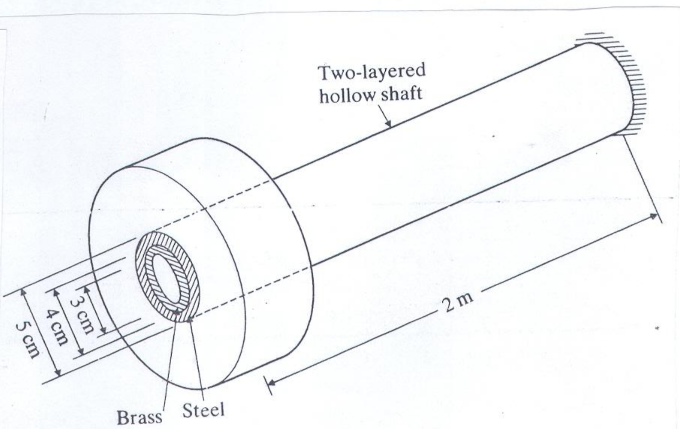


FIGURE 3.

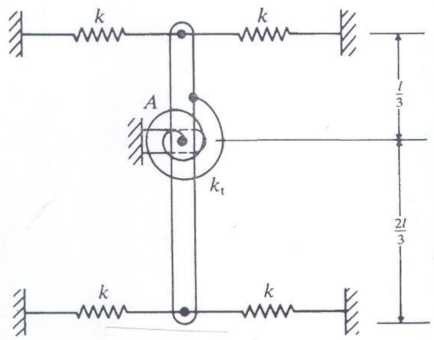


FIGURE 4.

6. A sphere of weight W and radius r rolls without slipping on a cylindrical surface as shown. For small angles of θ, determine (a) the differential equation of motion using the concept of dynamic equilibrium and (b) the undamped natural frequency f_n in terms of the parameters. Note that point A on the sphere moves to A' and that the angle ϕ is the absolute angular displacement of the sphere. The centroidal mass moment of inertia $\bar{I}$ of a sphere is $2mr^2/5$.

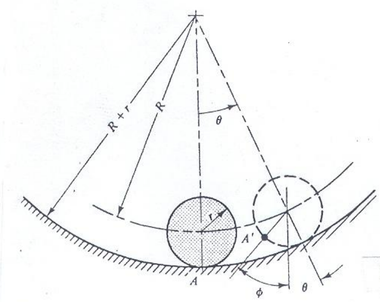


FIGURE 5.

7. A rigid block of mass M is mounted on four elastic supports, as shown in Fig. 6. A mass m drops from a height l and adheres to the rigid block without rebounding. If the spring constant of each elastic support is k, find the natural frequency of vibration of the system (a) without the mass m, and (b) with the mass m. Also find the resulting motion of the system in case (b).

8. A bungee jumper weighing 160 lb ties one end of an elastic rope of length 200 ft and stiffness 10 lb/in to a bridge and the other end to himself and jumps from the bridge (Fig. 7). Assuming the bridge to be rigid, determine the vibratory motion of the jumper about his static equilibrium position.

9. An acrobat weighing 120 lb walks on a tightrope, as shown in Fig. 8. If the natural frequency of vibration in the given position, in vertical direction, is 10 rad/s, find the tension in the rope.

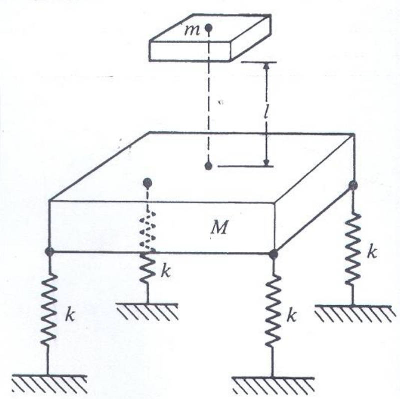


FIGURE 6.

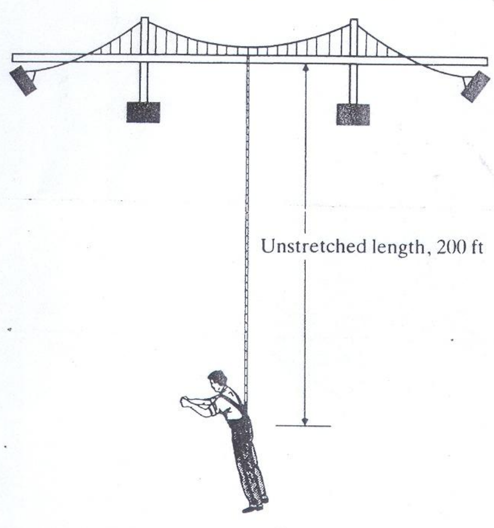


FIGURE 7.

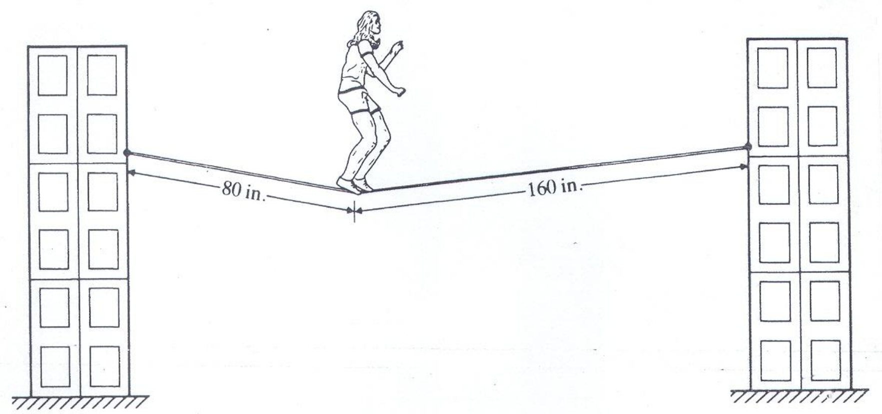


FIGURE 8.

10. A square platform *PQRS* and a car that it is supporting have a combined mass of *M*. The platform is suspended by four elastic wires from a fixed point *O*, as indicated in Fig. 9. The vertical distance between the point of suspension *O* and the horizontal equilibrium position of the platform is *h*. If the side of the platform is *a* and the stiffness of each wire is *k*, determine the period of vertical vibration of the platform.

11. The inclined manometer, shown in Fig. 10, is used to measure pressure. If the total length of mercury in the tube is *L*, find an expression for the natural frequency of oscillation of the mercury.

12. The crate, of mass 250 kg, hanging from a helicopter (shown in Fig. 11(a)) can be modeled as shown in Fig. 11(b). The rotor blades of the helicopter rotate at 300 rpm. Find the diameter of the steel cables so that the natural frequency of vibration of the crate is at least twice the frequency of the rotor blades.

13. A building frame is modeled by four identical steel columns, of weight *w* each, and a rigid floor of weight *W*, as shown in Fig.12. The columns are fixed at the ground and have a bending rigidity of *EI* each. Determine the natural frequency of horizontal vibration of the building frame by assuming the connection between the floor and the columns to be (a) pivoted as shown in Fig.12.(a), and (b) fixed against rotation as shown in Fig.12.(b). Include the effect of self weights of the columns.

14. A helical spring of stiffness *k* is cut into two halves and a mass *m* is connected to the two halves as shown in Fig. 13(a). The natural time period of this system is found to be 0.5 sec. If an identical spring is cut so that one part is 1/4 and the other part 3/4 of the original length, and the mass *m* is connected to the two parts as shown in Fig. 13(b), what would be the natural period of the system?

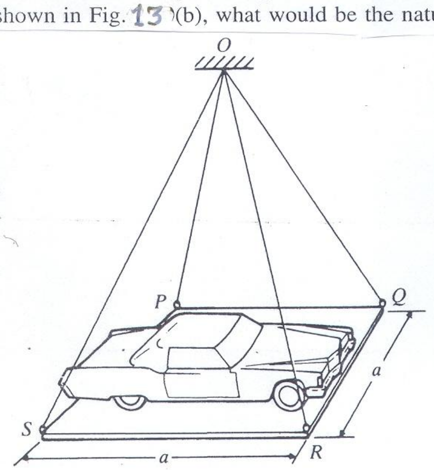


FIGURE 9.

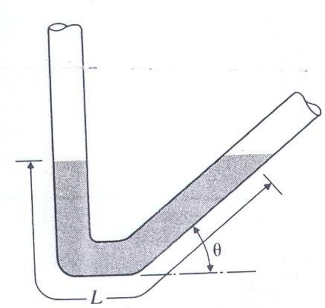


FIGURE 10.

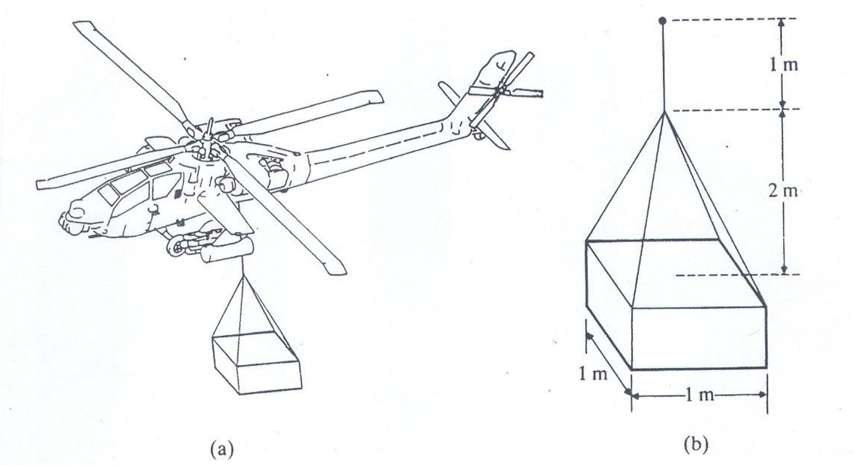


FIGURE 11.

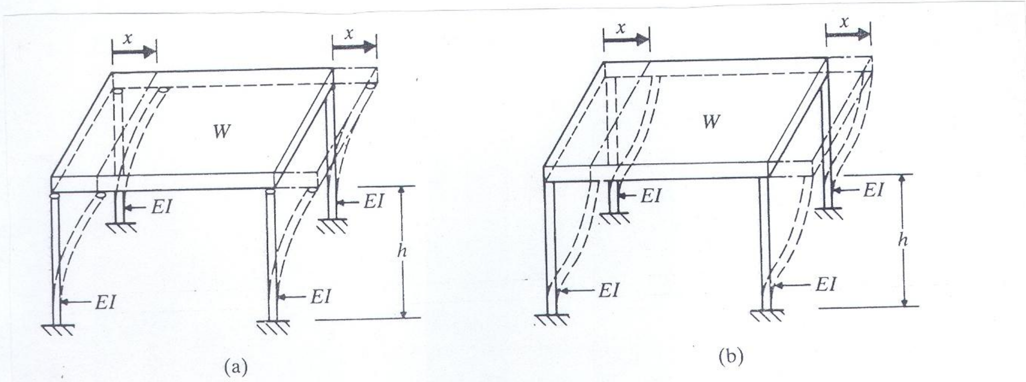


FIGURE 12.

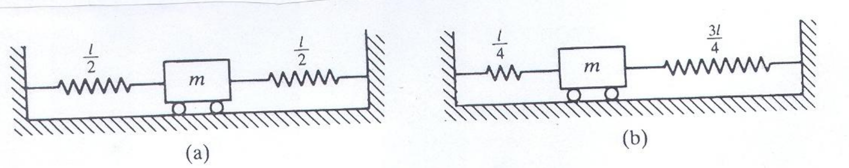


FIGURE 13.

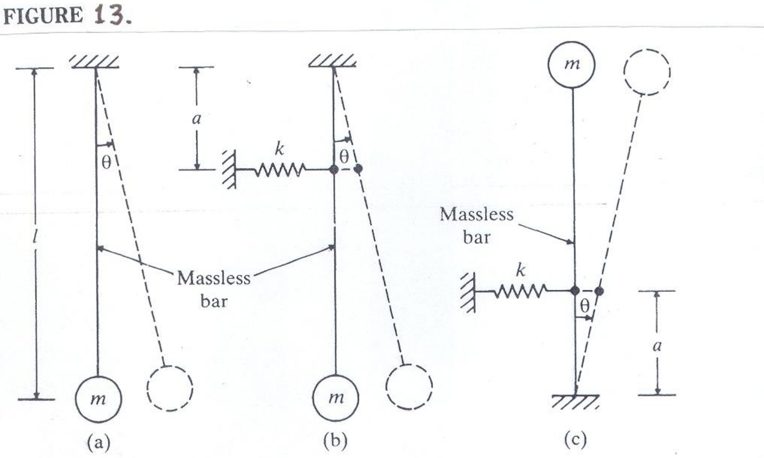


FIGURE 14.

15. A mass m is attached at the end of a bar of negligible mass and is made to vibrate in three different configurations, as indicated in Fig. 14. Find the configuration corresponding to the highest natural frequency.

16. Derive the equation of motion and find the natural frequency of vibration of each of the systems shown in Figs. 15.

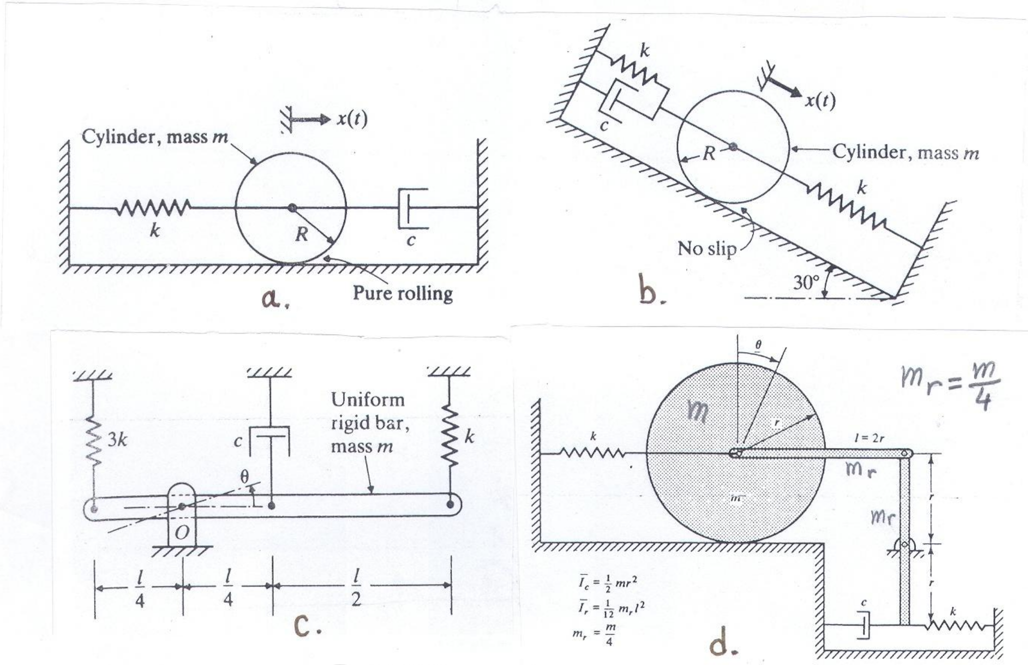


FIGURE 15.

17. A shock absorber is to be designed to limit its overshoot to 15 percent of its initial displacement when released. Find the damping ratio ζ_0 required. What will be the overshoot if ζ is made equal to (a) $\frac{3}{4}\zeta_0$, and (b) $\frac{5}{4}\zeta_0$?

18. The free vibration response of an electric motor of weight 500 N mounted on different types of foundations are shown in Figs. 16 (a) and (b). Identify the following in each case: (i) the nature of damping provided by the foundation; (ii) the spring constant and damping coefficient of the foundation; and (iii) the undamped and damped natural frequencies of the electric motor.

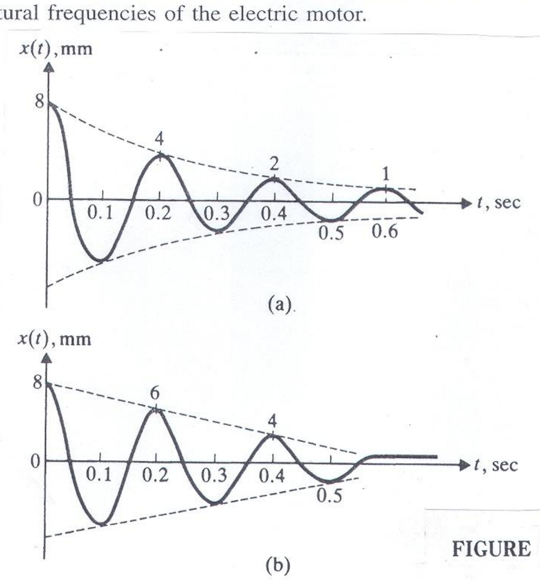


FIGURE 16

19. A locomotive car of mass 2000 kg traveling at a velocity $v = 10$ m/sec is stopped at the end of tracks by a spring-damper system, as shown in Fig. 17. If the stiffness of the spring is $k = 40$ N/mm and the damping constant is $c = 20$ N-s/mm, determine (a) the maximum displacement of the car after engaging the springs and damper and (b) the time taken to reach the maximum displacement.

20. A torsional pendulum has a natural frequency of 200 cycles/min when vibrating in vacuum. The mass moment of inertia of the disc is 0.2 kg-m^2. It is then immersed in oil and its natural frequency is found to be 180 cycles/min. Determine the damping constant. If the disc, when placed in oil, is given an initial displacement of 2°, find its displacement at the end of the first cycle.

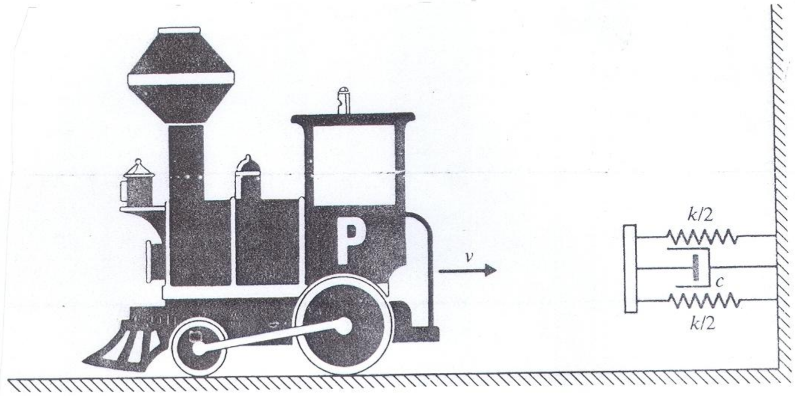


FIGURE 17.

21. The system shown in Fig. 18, has a natural frequency of 5 Hz for the following data: $m = 10$ kg, $J_0 = 5$ kg $-$ m^2, $r_1 = 10$ cm, $r_2 = 25$ cm. When the system is disturbed by giving it an initial displacement, the amplitude of free vibration is reduced by 80 percent in 10 cycles. Determine the values of k and c.

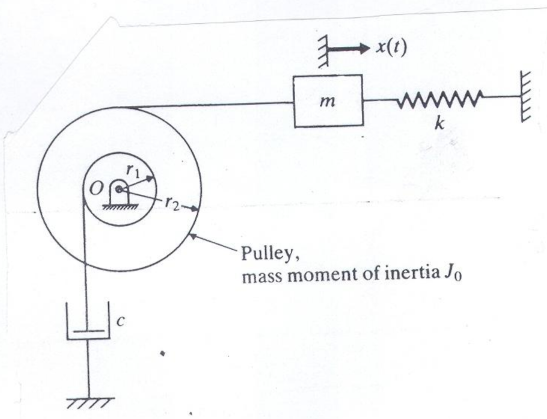


FIGURE 18.