Sheet 4
Forced Vibration of Single Degree of Freedom System

Forced Excitation Vibration

1. A mass of **4 Kg** is supported by an elastic support causes a static deflection of **16 mm**. The system id immersed in a medium whose resistance at a velocity of one meter per second **60 N** and is acted upon by a simple harmonic disturbing force of maximum value of **60 N**. at what speed of disturbing force will the amplitude of vibration be maximum? What is then the maximum amplitude and what is the phase difference with disturbing force?

2. A vibrating system has a spring scale of **588 KN/m**, a damping coefficient of **11 KN.s/m**, and a mass of **400 Kg**. It is excited by a periodically varying force **F= 980 sin 15πt N**. Calculate the amplitude of forced vibration. What will be the value of the amplitude at resonance?

3. A mass of **1000 Kg** having a radius of gyration of **61 cm**. is mounted at one end of a shaft **76 mm** diameter, and **3 m** long. The inertia of the mass at the other end is so large that the attachment of that end may be considered rigid. The smaller mass is subjected to a fluctuating torque which varies in a simple harmonic manner; the maximum value is **4500 N.m,** and the frequency of the application is **3.3 Hz**.
   Neglecting the inertia of the shaft itself, determine the amplitude of vibration at the smaller mass when the transient vibration has died away. Take the modulus of rigidity for the shaft material as **79 GPa**.

4. A machine weighing **2000 N** mounted on an elastic support was at resonance when running at **1000 rpm**. Its amplitude reached **8 mm** in **three** seconds. When viscous damping was added to the system the steady state amplitude was **0.15 mm**. Find the coefficient of damping.

5. A mass of **10 Kg** is hung from a spiral spring of stiffness **9.8 KN/m**. The vibration is controlled by a dashpot and it is found that the amplitude of vibration diminishes to **one-tenth** of its initial value in **two** complete oscillations. Assuming linear damping, find the damping coefficient at one meter per second and the ratio of the frequency of the damped vibration to that of the natural vibration. If a periodic force of **150 cos 25 t N** acts on the mass, find the amplitude of the forced oscillation. What would be the amplitude at resonance? What damping would be necessary in order to limit the amplitude at resonance to **25 mm** ?

Rotating unbalance

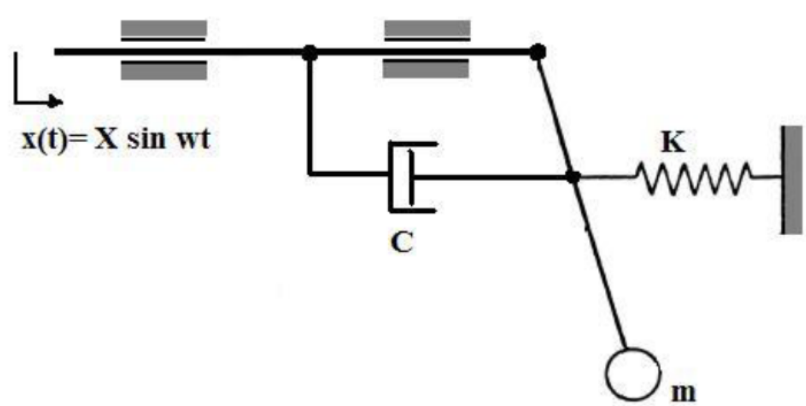
1. A rotating machine weighting **3000 N** and having a spring scale of **100 KN/m**, when the machine runs at **1300 rpm** the unbalance rotating force causes an amplitude of vibration of **2.5 mm** After the motor was shut off, the amplitude reduced to **0.8 mm** in five complete cycles Calculate the amplitude of vibration at **6000 rpm**.

2. A machine fixed to the floor at a workshop produced static deflection of **2 mm** under the machine. When the machine is working, there is an unbalance mass which produces a vertical alternating force whose frequency is equal to the speed of the machine. When the speed of the machine is **4 rps**, the amplitude of forced vibration of the floor is **1.25 mm**. If the floor is assumed to be elastic, and the damping is neglected, find the amplitude of forced vibration, when the speed is **8 rps**? At what speed does resonance occur?

3. A rotating machine having a mass of **2000 Kg** rests on spring with static deflection of **0.007m**. When the machine runs at **1000 rpm**, the unbalance rotating force is **3300 N**. The ratio of two consecutive amplitudes in free vibrations is **1** to **0.83**. Determine:-
   a. The dynamic amplitude at this speed.
   b. What will be happen to the amplitude if the speed of the machine is reduced?

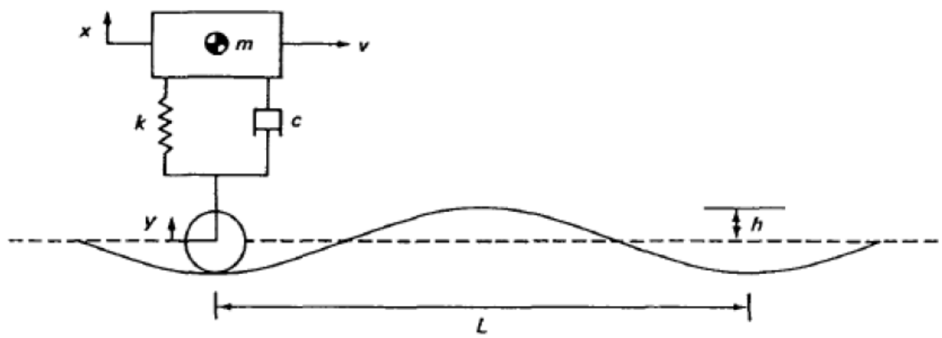
Supported Motion

1. For the system shown in figure, set up the equation of motion and find the steady state amplitude and phase angle.



2. A vehicle is traveling over a rough road represented in the figure. The springs are compressed **10.16 cm** under its weight. Find the critical speed when the

vehicle is traveling over a road with a profile approximated by a sin wave of amplitude **7.62 cm** and wave length of **14.63 m**. What will be the amplitude of vibration at **64.4 Km/hr** (neglecting damping)? What will happen to the amplitude if the dash pot having a damping factor **0.1** is introduced.



Measuring Instruments

1. a- Can a vibrometer be used as an accelerometer? Why?
   b- A seismometer having a spring stiffness of **2 KN/m** and viscous damping **50%** of the critical is used to measure a vibration of **2000 cpm**. If the error is not exceed **2 %,** which one of the following masses **(0.5, 1.0, 1.5, 2.0) Kg** is used in the instrument. It is desired to use this instrument as accelerometer by changing the spring, Find the new spring stiffness.

2. A vibrometer having a mass of **0.227 Kg** and a spring of **6.13 N/cm** stiffness, is used to measure vibration of **1200 cpm**. The recorded amplitude is **1.143 mm**. What is the actual amplitude? If damping is added, how will it affect the recorded amplitude?

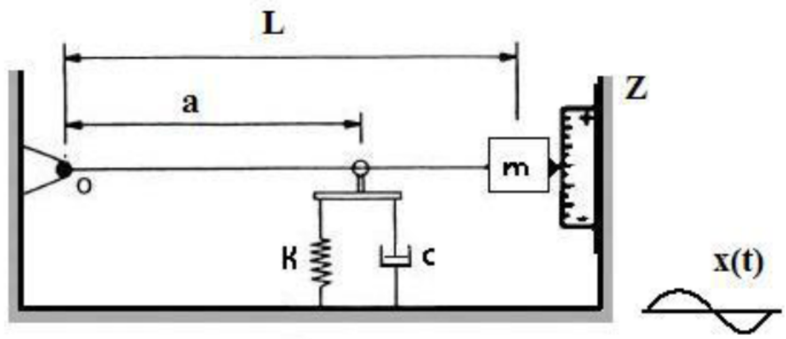
3. A machine element is vibrating with a motion given by,

   $$x_1 = 0.1 \sin 60\pi t + 0.05 \sin 120\pi t$$

   Determine the vibration recorded by using a seismic instrument, if the damping factor and frequency of the instrument is $\zeta = 0.65$, $f = 1500$ **cps**. Compare the vibration recorded with the actual acceleration of the machine element.

4. The instrument shown in figure is used to measure vertical vibrations. It consists of mass **"M"** placed at the end of a light bar (length **L**) which is pivoted at the other end. The spring **"K"** and a damper "C" are fixed at a distance "A" form the pivot. The base is placed on the vibrating object whose motion given by **x(t)=X sin ωt** and the amplitude of the mass **"Z"** is recorded:-
   c. Write down the equation of motion then find the relation between **"Z"** and **"X".**

d. If **M=46 gm**, **K=3500 N/m**, **ζ=0.5** and the instrument is used to measure the vibrations of a structure whose frequency is **1200 cpm**. Find the proper ratio for **"L/a"** so that the recorded amplitude will represent the displacement of the structure to within **2%** error.
e. What happens to the accuracy of the instrument if the damping is removed.



Vibration Isolation

1. A reciprocating engine is subjected to a fluctuating exciting force whose frequency is the same as the running speed of the engine. The engine and base weigh **2220 N** and are supported on a vibrating isolating mounting which has an equivalent spring constant of **520 N/cm** and a dash pot which is adjusted so that the damping is **20%** of the critical:
   a. Over what speed range in rpm is the amplitude of the fluctuating force transmitted to the foundation actually large than the excitation?
   b. Over what speed range is the transmitted force amplitude less than **20%** of the exciting force amplitude.

2. An electric motor of mass **68 Kg** is mounted on an isolator block of mass **1200 Kg** and the natural frequency of the total assembly is **160 cpm** with a damping factor **0.1**, If there is an unbalance in the motor that results in a harmonic force **F=100 sin 31.4t**, Determine:
   a. The amplitude of block vibration.
   b. The force transmitted to the floor.
   c. The transmissibility.
   d. How to decrease the force transmitted by **50%**

3. A machine having a rotating unbalance is resting on an elastic support. The machine weights **2224 N**. When running at **1200 rpm** the system was at resonance and had an amplitude of **0.254 mm**. The amplitude dropped to **0.127**

mm when the machine was running at **2400 rpm**. Determine the transmitted force at the latter speed.

4. A rotating machine weighting **4000 N** is supported by a rubber pad having stiffness of **10000 N/m** when running at **2000 rpm** it vibrated with an amplitude of **0.17 mm** and force transmitted to foundation was **1000 N**. Find the transmitted force at **300 rpm**.

5. A spring mass system having a mass **M= 50Kg** and spring **K= 10 N/m**. the mass is affected by a harmonic force **F=60 sin ωt** and the amplitude of vibration was **4 mm**. Determine:-
   a. The transmitted force.
   b. The no. of cycles for **X= 2mm** if the force is removed.

6. A machine weighing **1000 N** has an operating speed of **3000 rpm**;
   a. Design the simplest support of the machine so that the transmitted force to the foundation must be less than **50%** of the force due to vibration of the machine.
   b. What is the speed range of the machine so that the transmitted force should not exceeds the vibrating force?
   c. What is the modification in the support of the machine if its speed reduced to **50%** of the operating speed so that the transmitted force should be less than twice the force of vibrations? And what is the transmitted force at the operating speed?

7. A vertical single-stage air compressor weighing **5000 N** is mounted on springs of **200 KN/m** stiffness; and dampers having damping factor **0.2**. The rotation parts are well balanced, and the equivalent reciprocating parts weigh **200 N**. The stroke is **200 mm**. Determine the dynamic amplitude of the vertical motion, the phase angle with respect to the excitation force, the transmissibility and the force transmitted to the foundation, if the compressor is operated at **200 rpm** and **600 rpm**.
   Have you any suggestions for better vibrational conditions (amplitude and transmissibility) of the compressor at both the two above mentioned speeds.

<u>Critical Speed of Shafts:</u>

1- A disc mounted on the middle of a light shaft weighs **140 N** and has an unbalance of **0.002 Kg.m**. The two bearings that support the shaft are not rigid and are assumed to have a stiffness of **$8x10^6$ N/m** each. When the shaft runs at **10000 rpm** the bearing deflection is **0.25 mm**.
   Find the shaft deflection at that speed.

2- A grinding wheel weighs **100 N** has an unbalance of **0.0015 Kg.m**. The wheel is mounted on the center of a weightless shaft has a stiffness of **5 MN/m**. The shaft is supported on two bearings each has a stiffness of **10 MN/m**.
   a. Find the force transmitted to the bearings and the bending stress on **200 mm** shaft length and **20 mm** diameter at **9000 rpm**.
   b. If the shaft deflection should not exceed **0.25 mm**. Find the speed range to be avoided.

3- A circular disc weighing **178 N** is mounted midway on a **15 mm** diameter steel shaft **916 mm** long and its Young's modulus is **200 GPa**. The center of gravity of the disc is **3 mm** from its geometric center. The unit rotates at **600 rpm** and the damping factor is estimated to be **0.05**. Determine the critical speed of the shaft and the bending stress at the operating speed. Where should the disc be located in order to increase the critical speed by **20%**. (the deflection of simply supported shaft subjected to concentrated load P, $\delta = a^2.b^2/ 3EIL$).

4- A disc-shaft-bearings assembly in a test to determine the stiffness of the shaft and bearing supports, vertical force of **712 N** applied at the rotor causes a total shaft deflection of **0.254 mm**. The disc weighs **76 N** and has an eccentricity of **0.1 mm**. damping is assumed to be **10%** of the critical. Find the speed range to be avoided, if the shaft deflection should not exceed **0.127** mm.

5- A circular disc of **2 Kg** is mounted midway on a **0.02 m** diameter shaft **0.75 m** in length. The center of gravity of the disc is **7 μ** from its geometric center. The unit is rotated at **1000 rpm**. and the damping factor is estimated to be **0.06**. Compare the dead load stress in the shaft with the stress at operating speed.

6- The impeller of a centrifugal pump has a mass **8 Kg** and is attached to a point at **0.24 m** from the shaft end. The shaft has Young's modulus **200 GPa**. The distance from center to center of the rigid bearings is **0.72 m** and the outside and inside diameters are respectively **0.035** and **0.02 m**. Find:
   i- The critical speed.
   ii- The maximum vibratory force transmitted to the bearings and the maximum bending stress in the shaft during **one** second at the critical speed. The eccentricity between center of impeller and the shaft is **0.035 mm**.
   iii- What will be the change in the critical speed and why if:
      a- elastic bearings are added instead of rigid bearings.
      b- adding a dashpot to the bearings.