Alexandria University
Faculty of Engineering
Mechanical Engineering Department
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جامعة الاسكندرية
كلية الهندسة
قسم الهندسة الميكانيكية
يناير 2017

Optimum Design — **التصميم الأمثل**
Fourth Year — **السنة الرابعة**
Time Allowed: 3 Hour — **الزمن: 3 ساعات**

Answer the following questions

QUESTION ONE (20 points):

Graphically solve the optimal design problem given below.

$$\begin{aligned} &\text{Maximize} && f(x_1, x_2) = -x_1 - 4x_2 + 6 \\ &\text{Subject to} && 1 \le x_1 \le 4 \\ & && 1.3 \le x_2 \le 3.3 \\ & && x_1 + x_2 = 4 \end{aligned}$$

Show all constrains (with hatch marks to indicate the infeasible side. Name the feasible region such as (ABCD). Show at least two contours of $f(x)$. Find the Optimal Solution and Label it on the graph.

QUESTION TWO (30 points):

A beam-column of rectangular cross section is required to carry an axial load of 112 N and a transverse load of 45 N, as shown in Fig. 1. It is to be designed to avoid the possibility of yielding and buckling and for minimum weight. Formulate the optimization problem by assuming that the beam-column can bend only in the vertical (xy) plan. Assume the material to be steel with a specific weight of 8.3 kg/m^3, Young's modulus of 206 GPa, and a yield stress of 206 MPa. The width of the beam (b) is required to be at least 0.0127 m, and not greater than twice the depth (d). Also, find the solution of the problem graphically. Hint: The compressive stress in the beam-column due to P_y is P_y/bd and that due to P_x is

$$\frac{P_x l d}{2 I_{zz}} = \frac{6 P_x l}{b d^2}$$

The critical axial buckling load (force) is given by

$$(P_y)_{\text{cri}} = \frac{\pi^2 E I_{zz}}{4l^2} = \frac{\pi^2 E b d^3}{48 l^2}$$

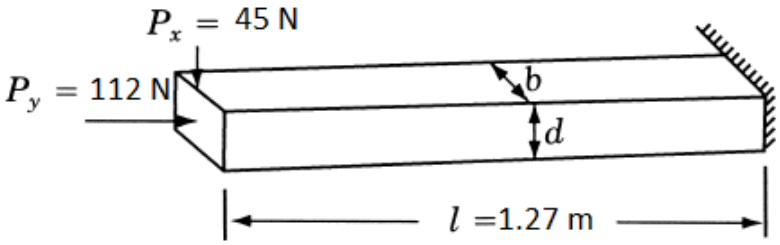

Fig. 1 Beam-column

QUESTION THREE (30 points):

Write optimality conditions and find stationary points for the following function. Also, determine the local minimum, local maximum, and inflection points for the Find also the corresponding objective function value at those points.

$$f(x) = x_1 + \frac{10}{x_1 x_2} + 5x_2$$

QUESTION FOUR (30 points):

Use the KKT necessary and sufficiency conditions to obtain the solution(s) to the following constrained optimization problem:

Minimize

$$f(x) = (x_1 - 1)^2 + x_2^2$$

subject to

$$-x_1 + x_2^2 \geq 0$$

End of Exam