OPTIMUM DESIGN

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Lecture 1

OPTIMAL DESIGN

Definition:

The development and use of analytical and computer methods to provide an optimal design of a product or process with minimal computational effort.

That's right…

The thing we design will be optimal AND the methods we use will be optimal.

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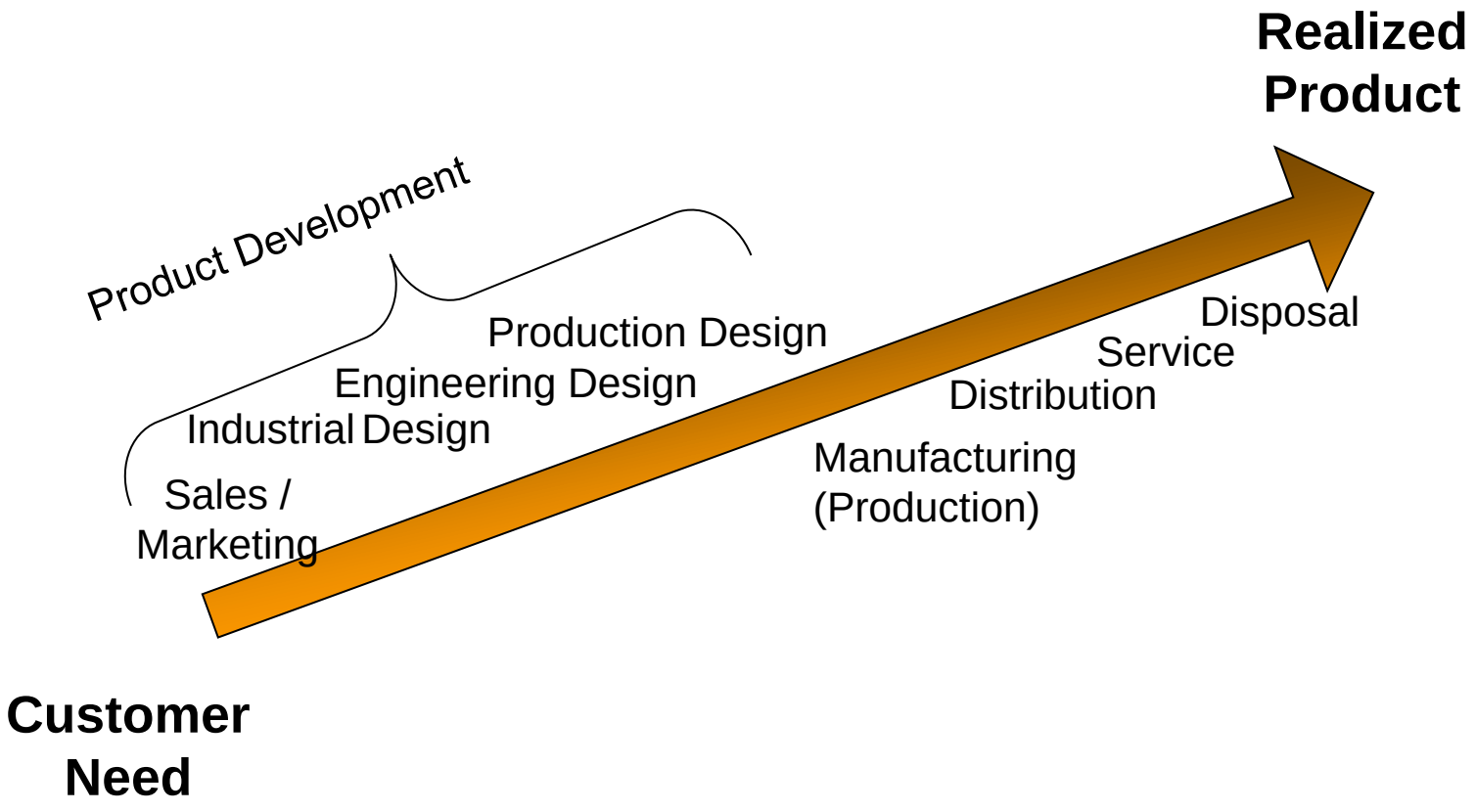
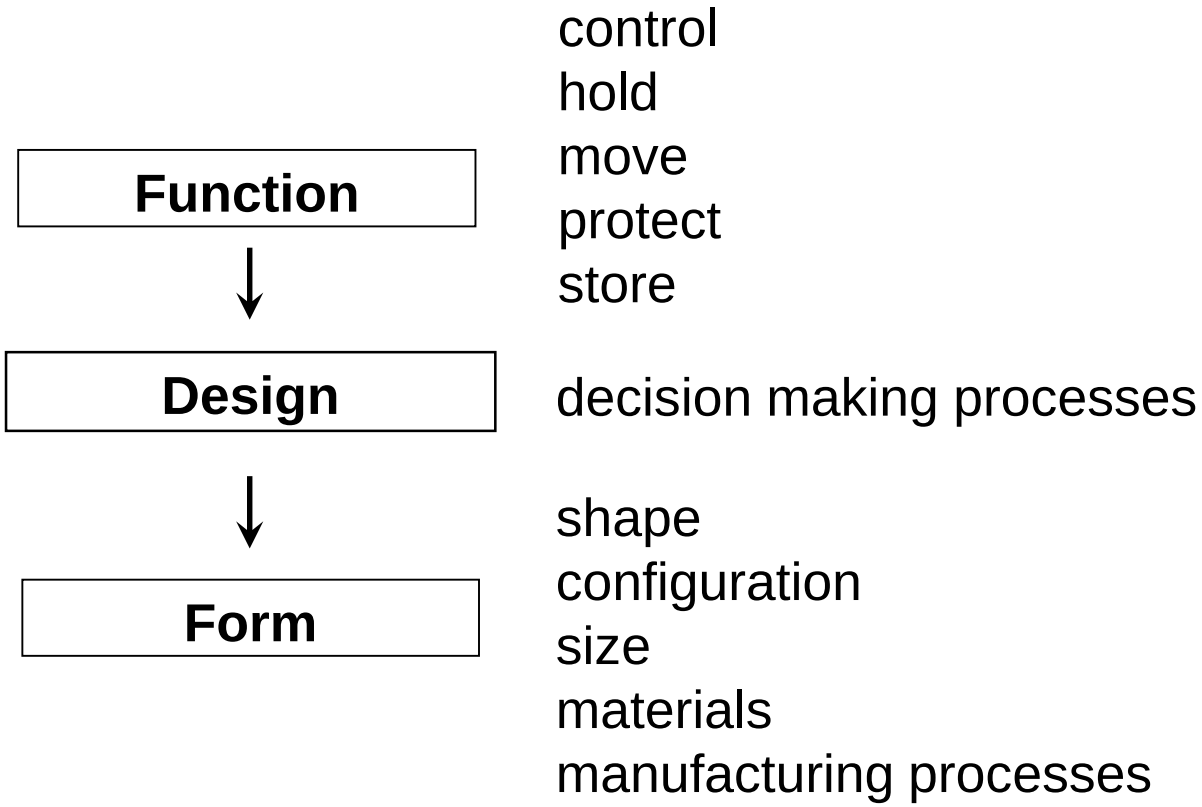
Product Realization Process



Design

Set of *decision making processes and activities* to determine:
the ***form*** of an object,
given the customer's desired ***function***.



Analysis is not Design

Which of the following is design and which is analysis?

A. Given that the customer wishes to fasten together two steel plates, select appropriate sizes for the bolt, nut and washer.

B. Given the cross-section geometry of a new airplane wing we determine the lift it produces by conducting wind tunnel experiments.

Problem Type	Solution
Design	Form (size, shape, materials,, manufacturing)
Analysis	Predicted behavior (performance)

System Evolution (Arora)

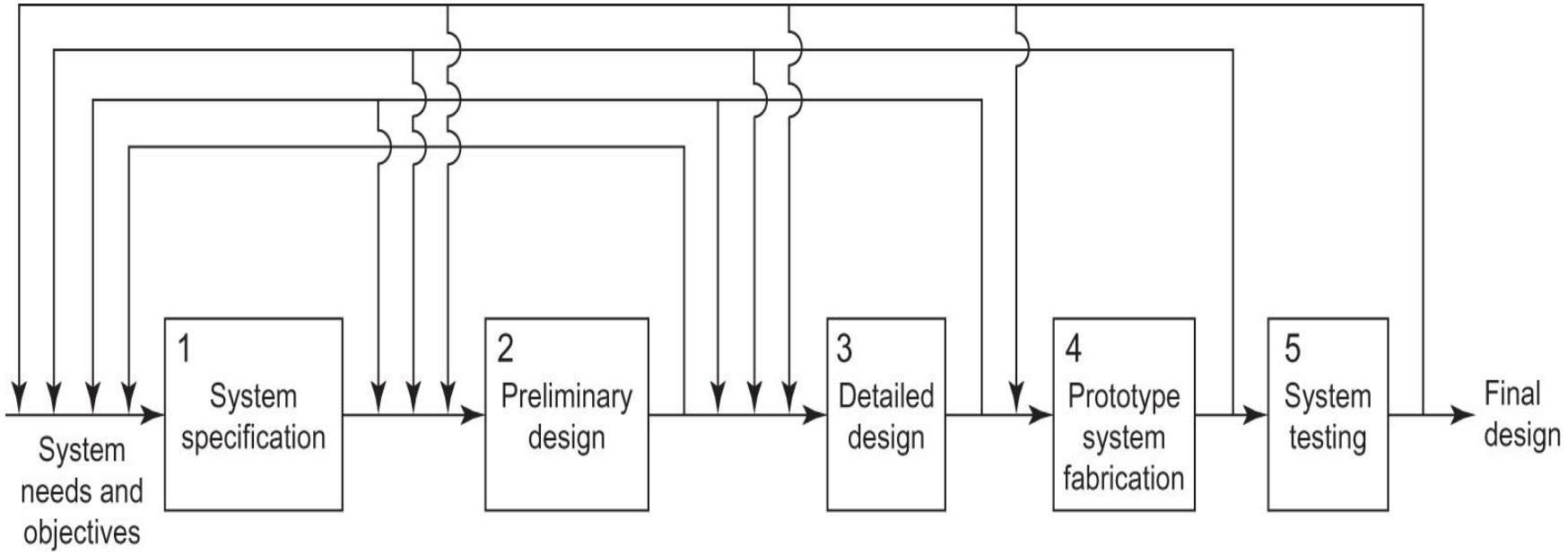
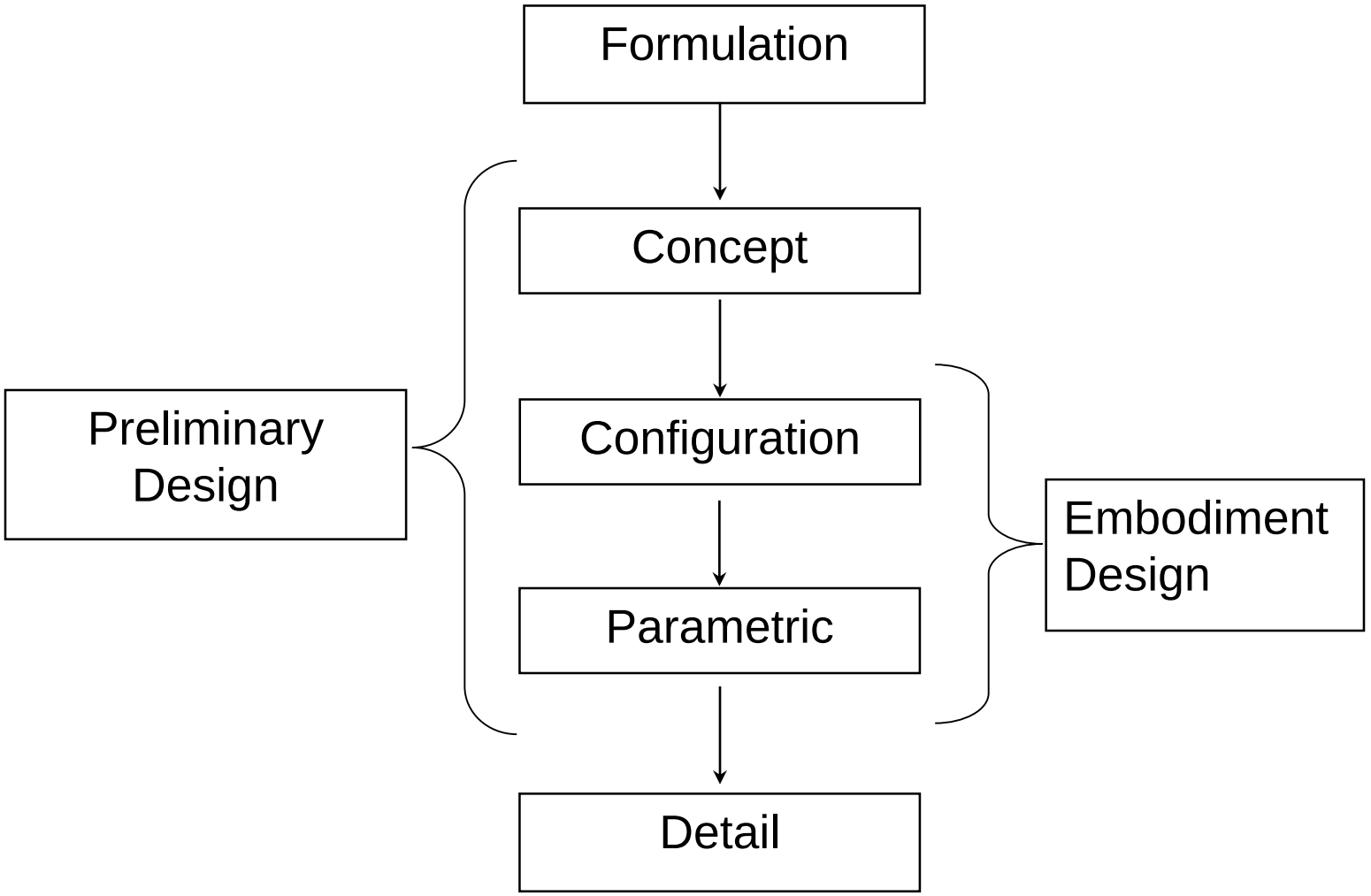


Figure 1.1 System evolution model.

Design Phases



Design Optimal Design

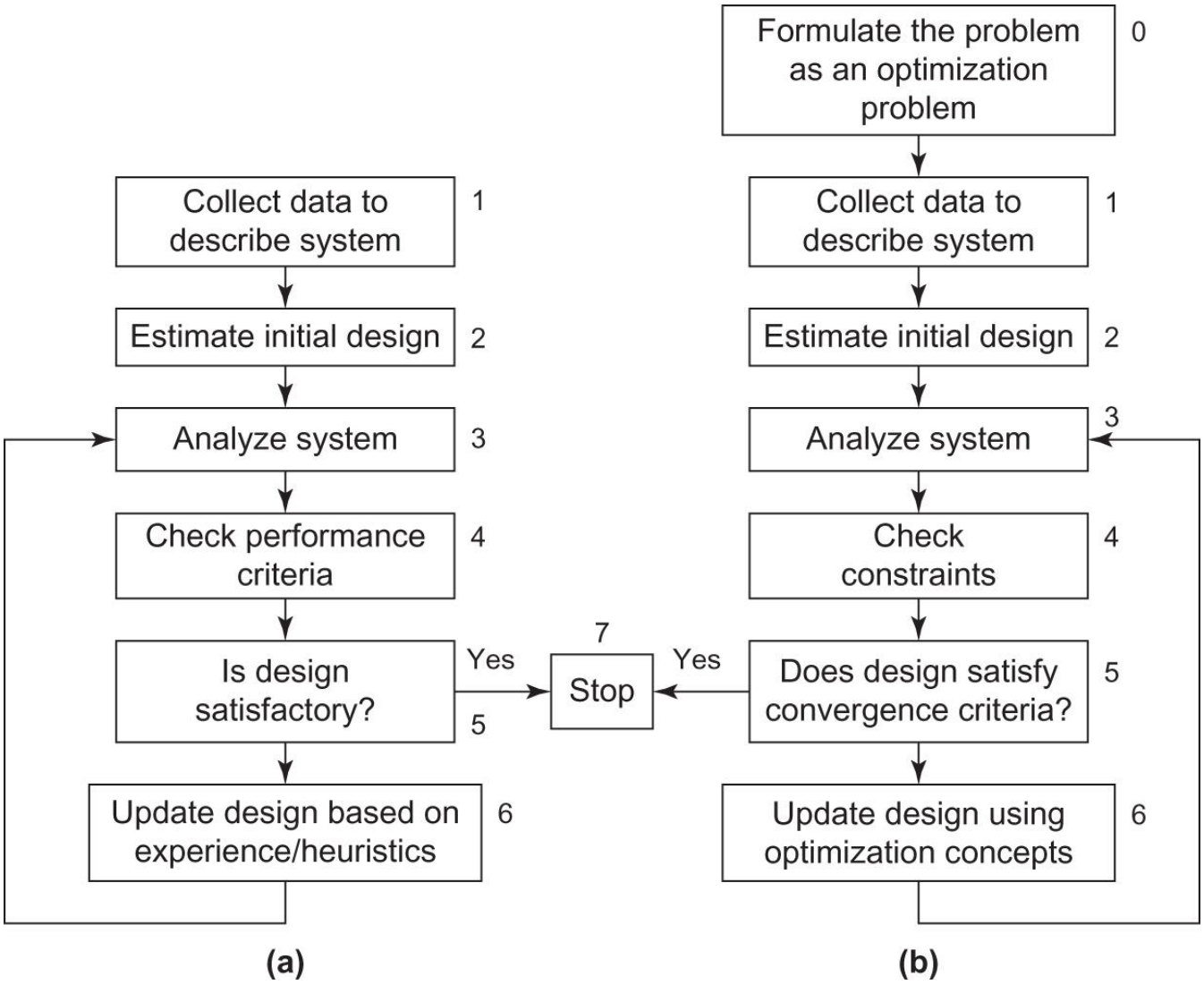
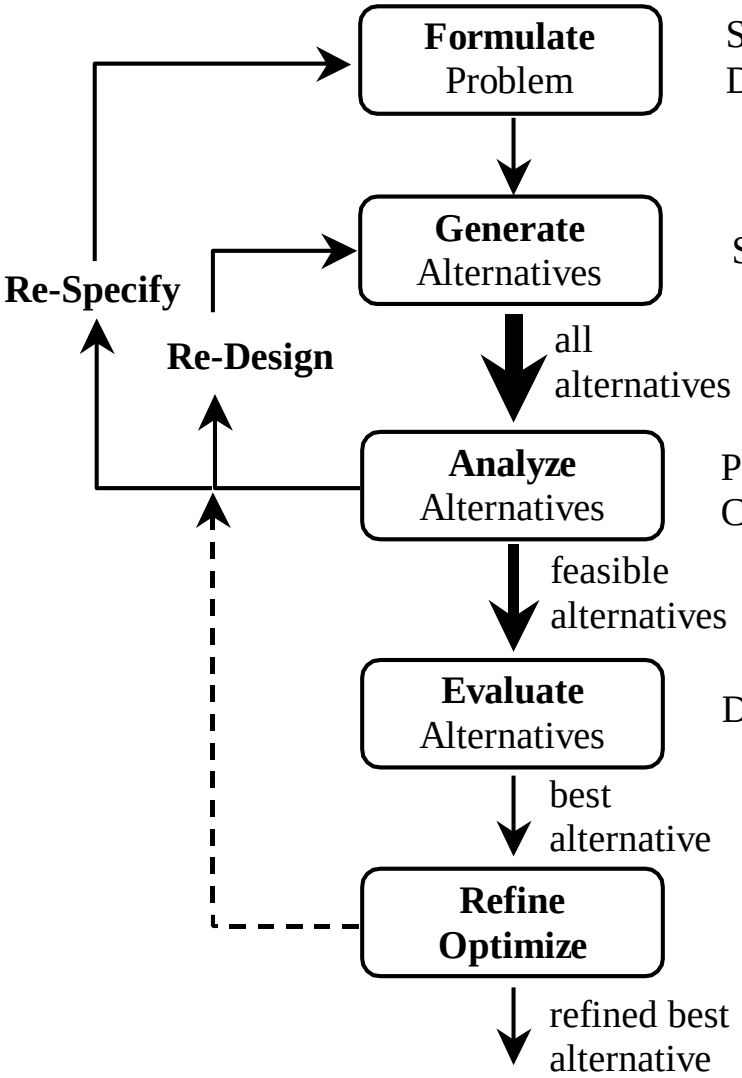


Figure 1.2 Comparison of (a) conventional design method and (b) optimum design method.

Systematic Parametric Design



Engineering Design, Eggert, 2010

Tools used in Optimal Design

- Algebra
- Calculus
- Vector and matrix aritmetic
- Excel (computation & graphing)
- Graphing (hand)
- Computer Programming (any language)
- Engineering principles

Mathematical Notation

Recall from Calculus, a function of many variables:

$$f(\mathrm{x}, \mathrm{y}, \mathrm{z})$$

We shall use vectors for multiple variables:

$$f(\mathbf{x}) \quad \text{note bold x}$$

All vectors are columns

The transpose is used to show a row

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1, x_2 & \cdots & x_n \end{bmatrix}^T$$

Handwritten vectors

The book shows vectors as lower case bolded, for example:

$$f(\mathbf{x}) \quad \text{note bold x}$$

For handwritten homework and tests… we will use lower case hand-printed with an underscore, for example:

$$f(\underline{x}) = \text{ note underscore}$$

$$\underline{x} = \begin{bmatrix} x_1, x_2 & \cdots & x_n \end{bmatrix}^T$$

Points **P** described by $\mathbf{x}^{(1)}$

Superscripts (1),(2)

$$\mathbf{x}^{(1)} = \begin{bmatrix} 1.3 \\ 2.3 \\ 1.2 \end{bmatrix}$$

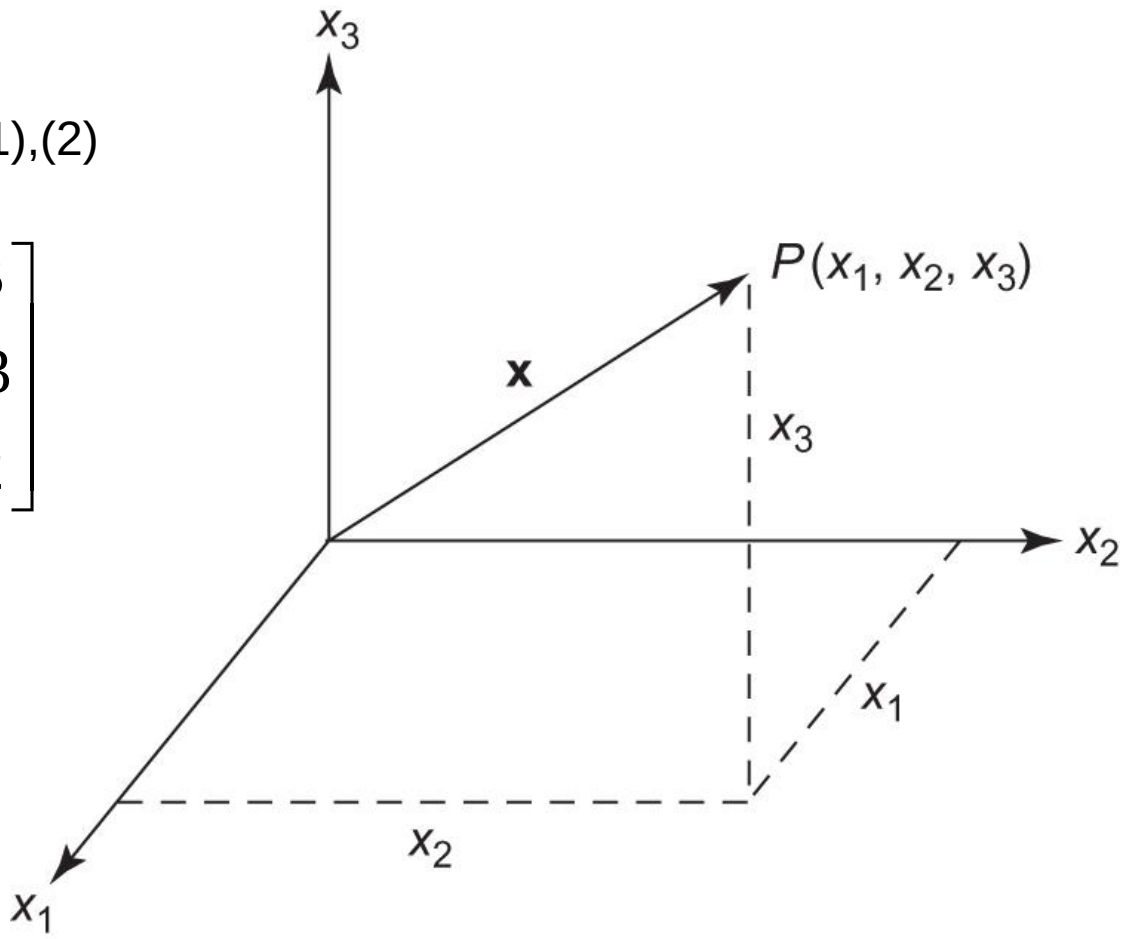


Figure 1.3 Vector representation of a point *P* that is in 3-dimensional space.

Set of Points, *S*

$S = \{ (x_1 - 4)^2 + (x_2 - 4)^2 \leq 9\}.$

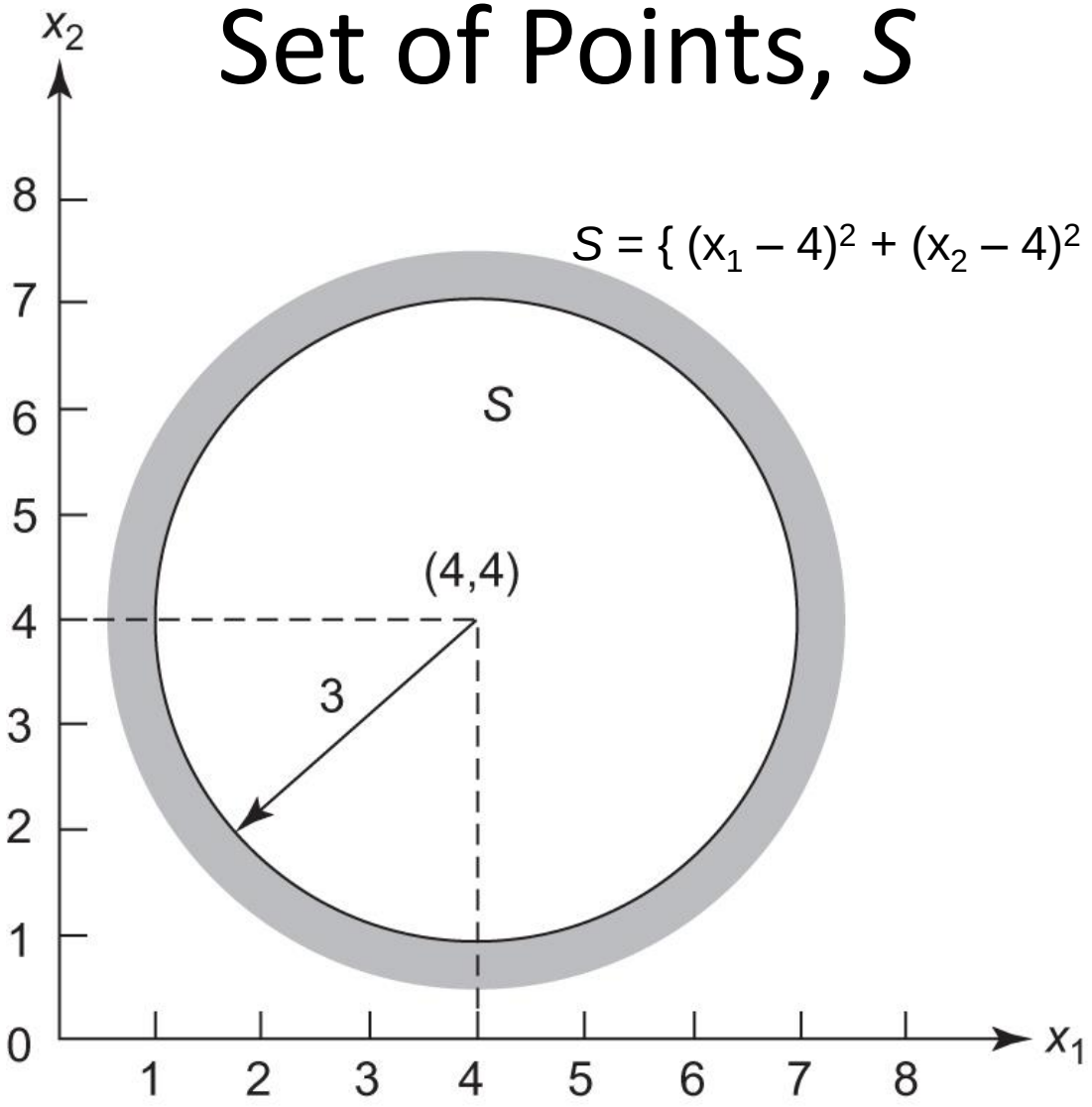


Figure 1.4 Image of a geometrical representation for the set $S = \{x|(x_1 - 4)^2 + (x_2 - 4)^2 \leq 9\}$.

Dot Product

From Engineering Statics:

$$\mathbf{x} \bullet \mathbf{y} = |\mathbf{x}||\mathbf{y}|\cos(\theta)$$

$$\mathbf{x} \bullet \mathbf{y} = \sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3$$

How do we know if two vectors are orthogonal (normal) ?

In optimal Design:

$$\mathbf{x} \bullet \mathbf{y} = \mathbf{x}^{\mathbf{T}} \mathbf{y} = \sum_{i=1}^{n} x_i y_i$$

$$\mathbf{x}^{\mathbf{T}} \mathbf{y} = \left[x_1, x_2, x_3\right]^T \left[y_1, y_2, y_3\right]^T$$

$$\sum_{i=1}^{n} x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3$$

Vector or Scalar?

Is a dot product of two vectors a vector or scalar quantity?

Product of vector and matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & -3 & -1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (x_1 + x_2 + x_3) \\ (5x_1 - 3x_2 - x_3) \\ (2x_1 + x_2 + x_3) \end{bmatrix}$$

$$\mathbf{A}_{3x3}\mathbf{x}_{3x1} = \mathbf{y}_{3x1}$$

Is the product a scalar or vector?

Triple Product

$$\mathbf{c} = \mathbf{x}^{\mathbf{T}}\mathbf{A}\mathbf{x} = \left(\mathbf{x} \bullet \mathbf{A}\mathbf{x}\right)$$

$$\begin{bmatrix}3 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 5 & -3 & -1\\ 2 & 1 & 1\end{bmatrix}\begin{bmatrix}3\\ 2\\ 1\end{bmatrix} = \begin{bmatrix}3 & 2 & 1\end{bmatrix}\begin{bmatrix}(1)(3)+(1)(2)+(1)(1)\\ (5)(3)+(-3)(2)+(-1)(1)\\ (2)(3)+(1)(2)+(1)(1)\end{bmatrix}$$

$$(3x3)(3x1) = (3x1) = (i.e.\ 3\ rows\ x\ 1\ column)$$

$$\begin{bmatrix}3 & 2 & 1\end{bmatrix}\begin{bmatrix}6\\ 8\\ 9\end{bmatrix} = 3(6)+2(8)+1(9) = 43$$

Rusty? …. Review appendix A, pgs 785-822

Function continuity

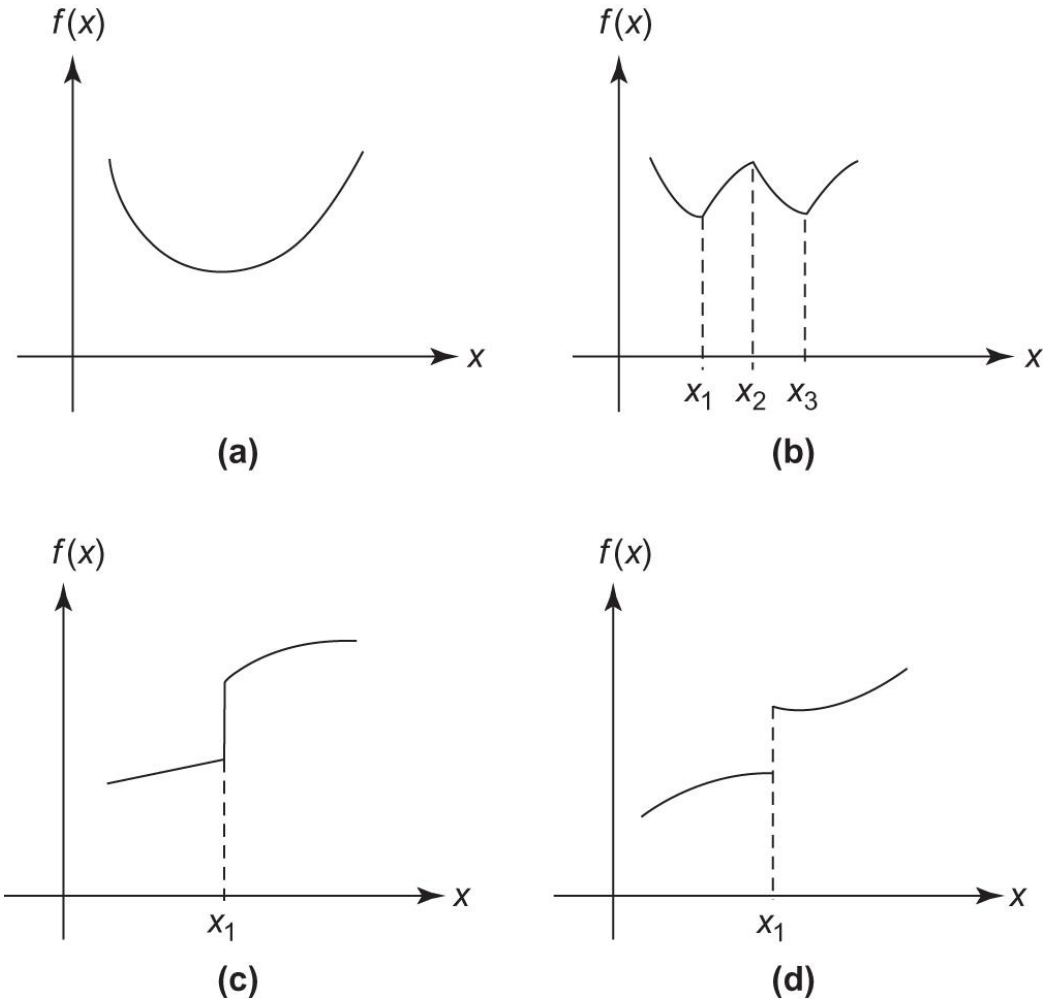


Figure 1.5 Continuous and discontinuous functions: (a) and (b) continuous functions; (c) not a function; (d) discontinuous function.

First Partial Derivatives of a function

Gradient vector

$$\nabla f(\mathbf{x}^*) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}_{\mathbf{x}^*} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix}^T_{\mathbf{x}^*}$$

We'll se a lot of these in chapter 4.

Second Partial Deriivatives of a function…

Hessian Matrix

$$\nabla^2 f = \mathbf{H}(\mathbf{x}^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1^2}\frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_1^2}\frac{\partial^2 f}{\partial x_n^2} \\ \frac{\partial^2 f}{\partial x_2^2}\frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2^2}\frac{\partial^2 f}{\partial x_n^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n^2}\frac{\partial^2 f}{\partial x_1^2} & & & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}_{\mathbf{x}^*}$$

What does the **x*** mean?

Opt. Design Problem Formulation

Develop a mathematical model

Include mathematical relations for :

1. A performance criterion or “cost function” (measures “goodness” of the candidate’s design)
2. Necessary behaviors (must do or have)

(obey laws of man, or nature, i.e. safety, physics, chemistry etc)

Standard Design Optimzation Model

Find $\mathbf{x}^*$ such that

MINIMIZE : $f(\mathbf{x})$

Subject To :

$$h_j(\mathbf{x}) = 0 \qquad j = 1 \ldots p$$

$$g_i(\mathbf{x}) \leq 0 \qquad i = 1 \ldots m$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)} \qquad i = 1 \ldots n$$

Design Problem Formulation (Arora)

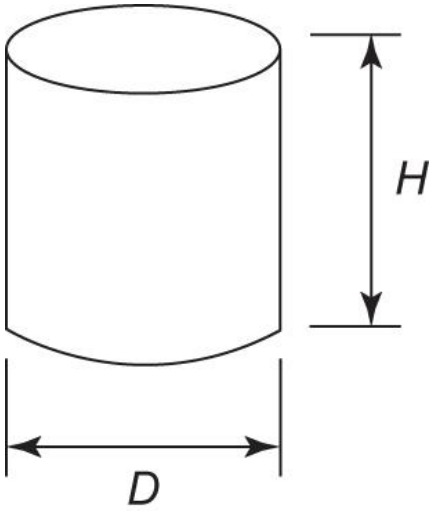
Step 1. Project/problem description
Step 2. Data and information collection
Step 3. Definition of design variables
Step 4. Optimization criterion
Step 5. Formulation of constraints

Let's reword these as actions to perform…

Design of a can

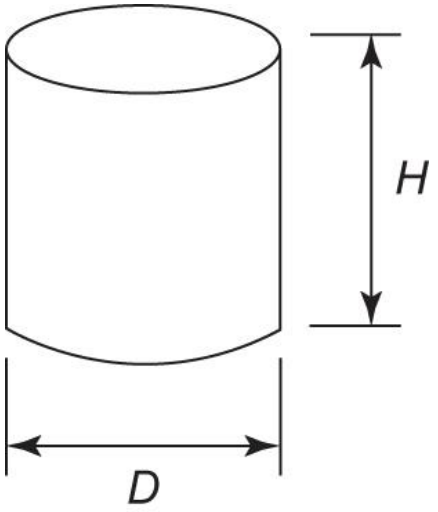
Step 1. Describe problem (restate w/bullets)

- Must hold at least 400 ml
- Min manufacturing cost which is proportional to surface area
- Diameter no more than 8 cm
- Diameter no less than 3.5
- Height no more than 18 cm
- Height no less than 8 cm

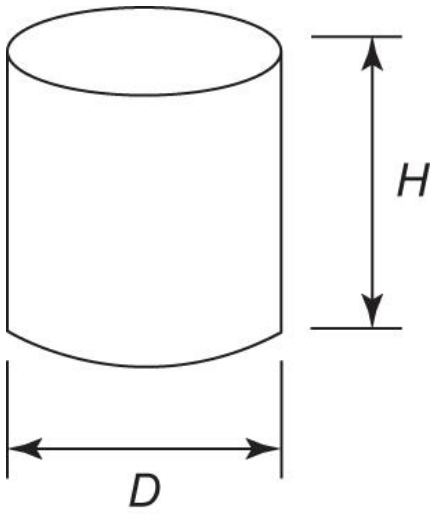


Design of a can

Step 1. Describe problem
Step 2. Collect info
Step 3. Define Design variables (DVs)
Step 4. Determine objective function
Step 5. Formulate constraints



Design of a can



Step 2. Collect info

- Draw diagram
- Relation for volume
- Relation for surface area
- Other?

Volume = Area x height = $(\pi/4 * D^2) * H$

Surface area = top + bottom + side

- Area top, bottom = $\pi D^2 /4$
- Area side = πDH

Total area = $\pi D^2 /4 + \pi D^2/4 + \pi DH$ (cm^3)

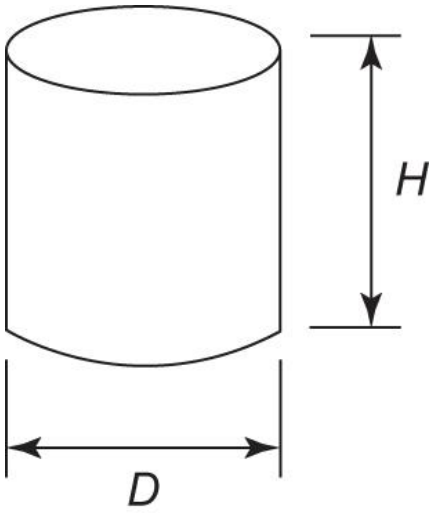
Design of a can

Step 3. Define Design Variables (DVs)

Diameter, D, (cm)

Height, H, (cm)

$\mathbf{x}=[x_1, x_2] = [D, H]$

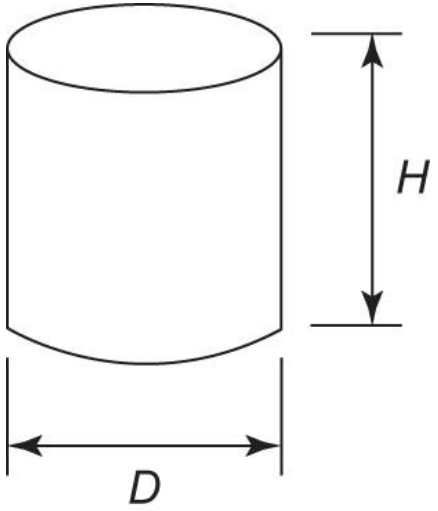


Note: volume and area are functions of the DVs

Design of a can

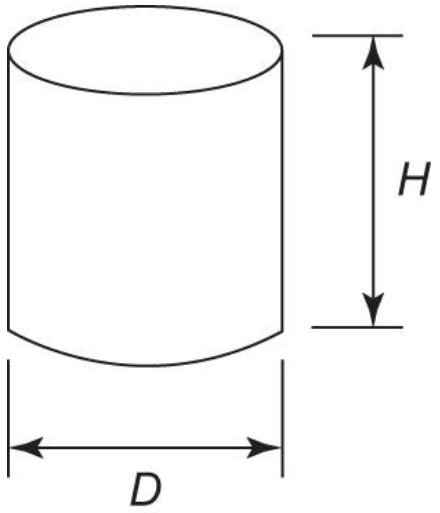
Step 4. Determine objective function

Min $f(\mathbf{x}) = \pi D^2/2 + \pi DH$ (cm^2)



Design of a can

Step 5. Formulate constraints



Volume ≥ 400 ml (cm^3), or
$(\pi D^2/4)H \geq 400\ (cm^3)$

Size limits

$3.5 \leq D \leq 8$
$8 \leq H \leq 18$

$3.5 \leq D$
$D \leq 8$
$8 \leq H$
$H \leq 18$

Design of a can - Summary

Min $f(D,H) = \pi D^2/2 + \pi DH \ \ (cm^2)$

Subject to:

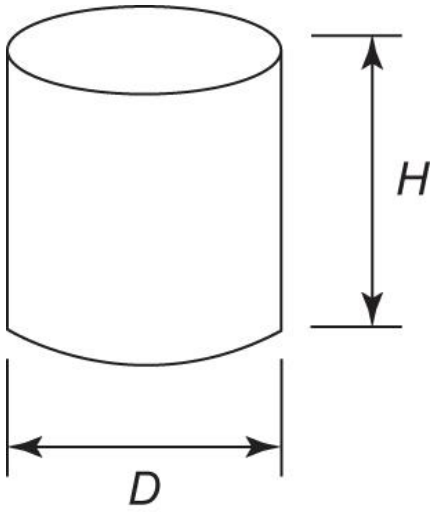
$(\pi D^2/4)H \geq 400 \ (cm^3)$

$3.5 \leq D$

$D \leq 8$

$8 \leq H$

$H \leq 18$



More on design variables (DVs)

Parameters that:

1. can be arbitrarily selected by the design engineer, AND that
2. influence the behavior of the product (or process) to be designed

DV Name	height
Symbol	H
Units	(cm)
Upper bound	18 cm
Lower bound	8 cm

For discrete variables... determine set of permissible values

Likely DV's – think FORM

Sizes
- L, W, H, D, t

Shapes
- square, circular, cylindrical, slender, short

Materials
- metals, polymers, ceramics

Manufacturing processes
- machined, stamped, molded

More on Constraints

How will the product “fail” to function/perform?

- Legally
- Mechanically
- Electrically
- Chemically
- Other?

Mechanical failure modes

Tensile/compressive failure, plastic, brittle
Buckle
Corrosion
Excess deflection
Excessive friction
Thermal (melts, combusts…)
Wear
Vibration
Unsatisfactory motion (i.e. 4-bar, x,v,a)
Other?

Electrical failure modes

Short circuit
Open circuit
Excessive power, heat
Poor filtering
EM interference
Other?

Legal failure modes

Violates codes/standards
Causes unforeseen property damage
Causes unforeseen injury
Infringes existing patent
Other?