OPTIMUM DESIGN

Dr. / Ahmed Nagib Elmekawy

Lecture 3

Graphical Solution

1. Sketch coordinate system
2. Plot constraints
3. Determine feasible region
4. Plot f(**x**) contours
5. Find opt solution **x*** & opt value f(**x***)

Profit Maximization Problem–Formulation

- Step 1: Project/problem description. A company manufactures two machines, A and B. Using available resources, either 28 A or 14 B can be manufactured daily. The sales department can sell up to 14 A machines or 24 B machines. The shipping facility can handle no more than 16 machines per day. The company makes a profit of $400 on each A machine and $600 on each B machine. How many A and B machines should the company manufacture every day to maximize its profit?

Profit Maximization Problem–Formulation

- Step 2 Data and information collection. Data and information are defined in the project statement. No additional information is needed.
- Step 3: Definition of design variables. The following two design variables are identified in the problem statement:
  x1 = number of A machines manufactured each day
  x2 = number of B machines manufactured each day
- Step 4: Optimization criterion. The objective is to maximize daily profit, which can be expressed in terms of design variables using the data given in step 1 as
  $P = 400\,x1 + 600\,x2$

Profit Maximization Problem–Formulation

- Step 5: Formulation of constraints. Design constraints are placed on manufacturing capacity, on sales personnel, and on the shipping and handling facility. The constraint on the shipping and handling facility is quite straightforward:

$$x_1 + x_2 \leq 16 \text{ (shipping and handling constraint)}$$

Profit Maximization Problem–Formulation

- Constraints on manufacturing and sales facilities are a bit tricky because they are either “this” or “that” type of requirements. First, consider the manufacturing limitation. It is assumed that if the company is manufacturing x1 A machines per day, then the remaining resources and equipment can be proportionately used to manufacture x2 B machines, and vice versa. Therefore, noting that x1 /28 is the fraction of resources used to produce A and x2 /14 is the fraction used to produce B, the constraint is expressed as :

$$\frac{x_1}{28}+\frac{x_2}{14}\leq 1\,(\text{manufacturing constraint})$$

Profit Maximization Problem–Formulation

- Similarly, the constraint on sales department resources is given as

$$\frac{x_1}{14}+\frac{x_2}{24}\leq 1\,(\text{limitation on sale department})$$

- Finally, the design variables must be nonnegative as

$$x_1,\ x_2 \geq 0$$

- Step-by-Step Graphical Solution Procedure

$$x_1 + x_2 \leq 16 \text{ (shipping and handling constraint)}$$

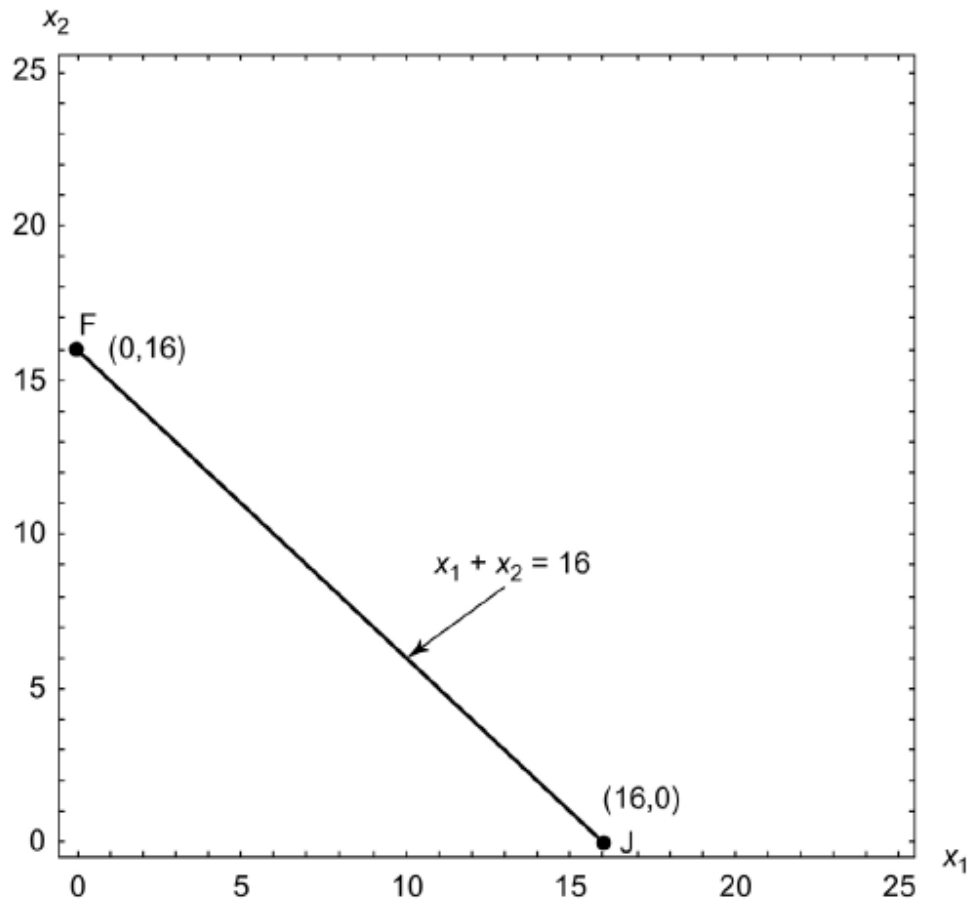


FIGURE 3.1 Constraint boundary for the inequality $x_1 + x_2 \leq 16$ in the profit maximization problem.

- Step-by-Step Graphical Solution Procedure

$$x_1 + x_2 \leq 16 \text{ (shipping and handling constraint)}$$

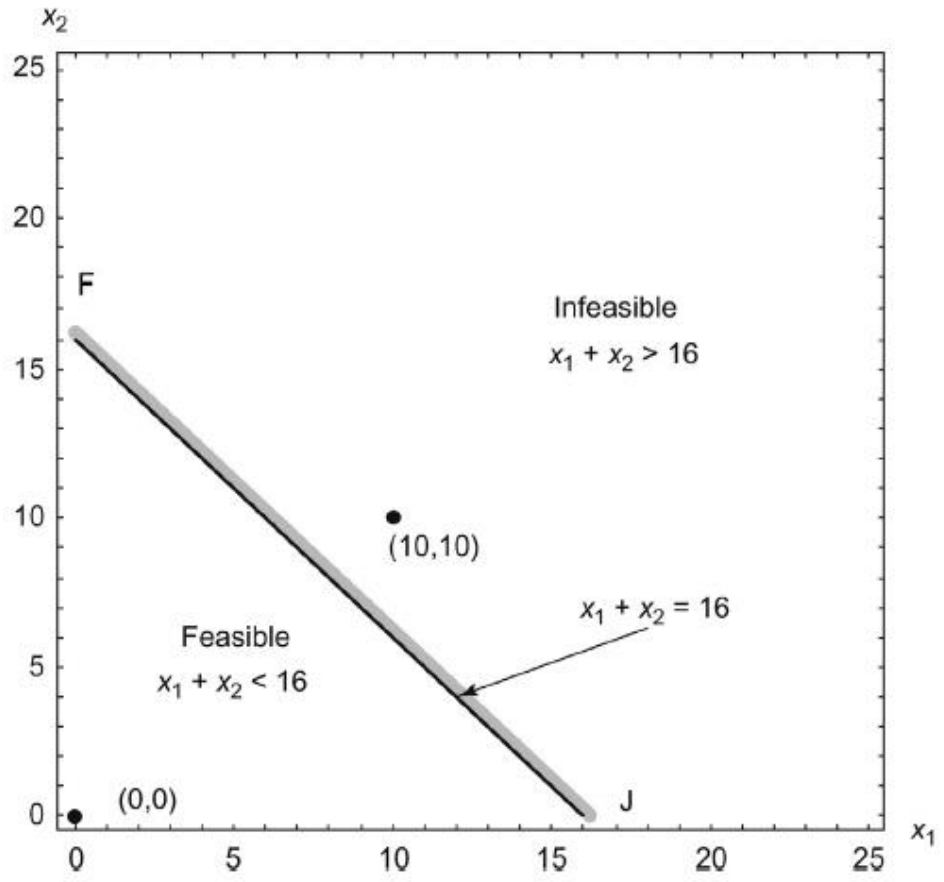


FIGURE 3.2 Feasible/infeasible side for the inequality $x_1 + x_2 \leq 16$ in the profit maximization problem.

- Step-by-Step Graphical Solution Procedure

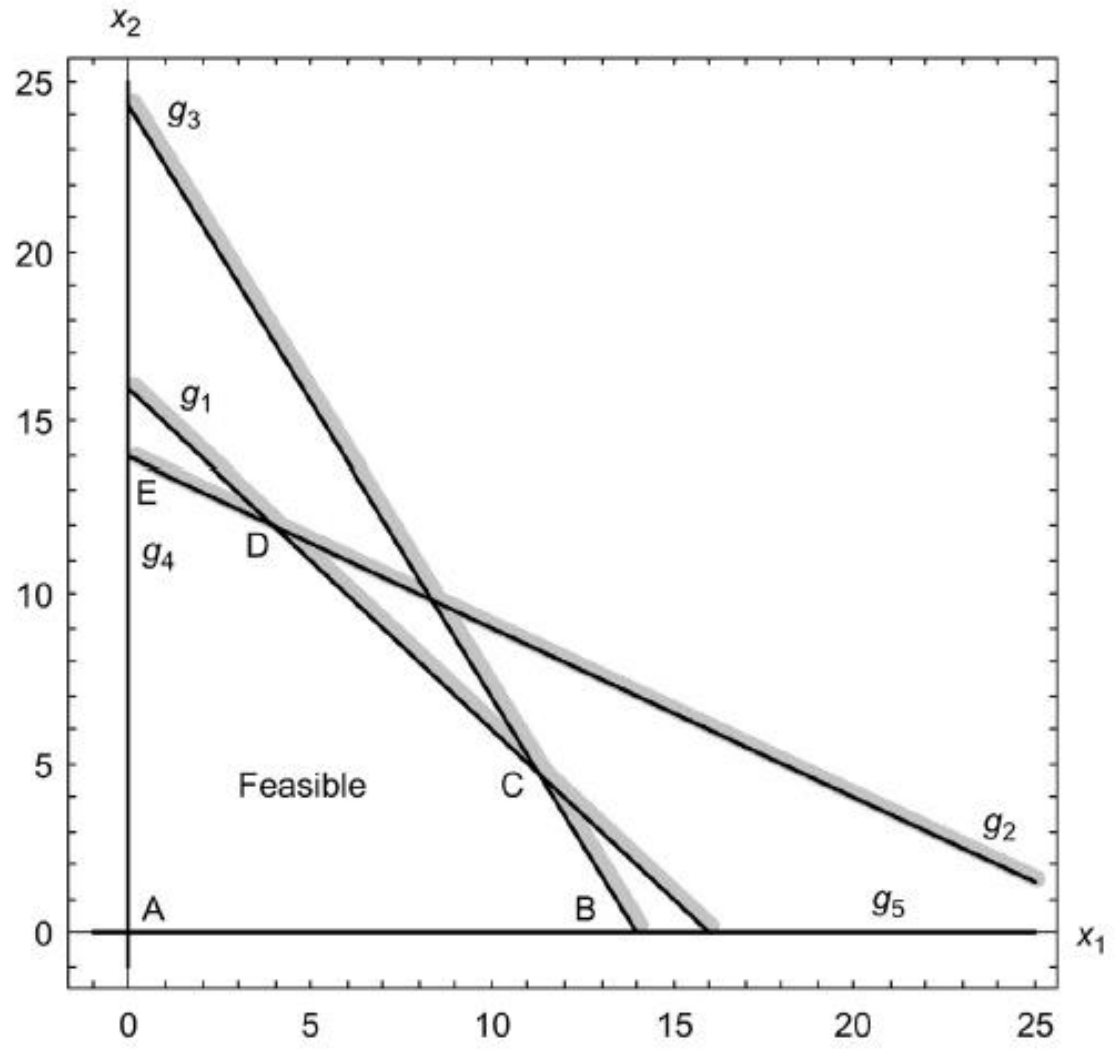


FIGURE 3.3 **Feasible region for the profit maximization problem ABCDE.**

- plot an objective function contour through the feasible region:

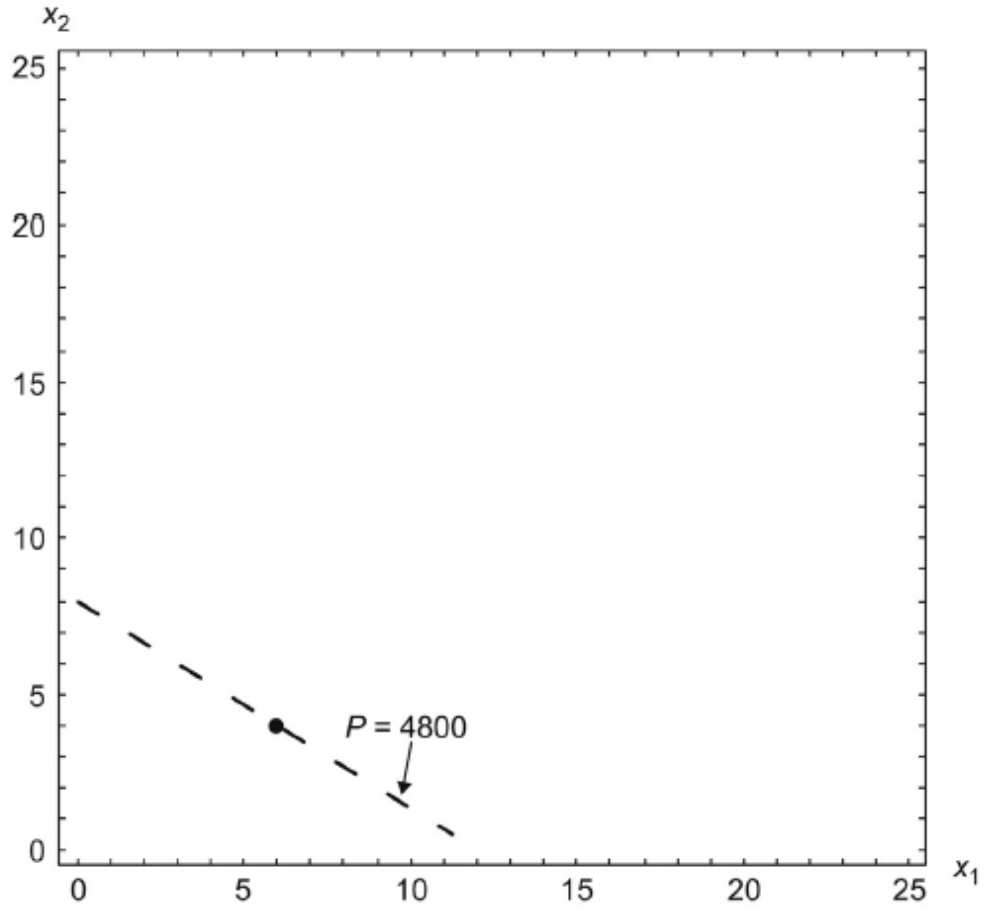


FIGURE 3.4 Plot of $P = 4800$ objective function contour for the profit maximization problem.

5. Find Optimal solution & value

Opt. solution
point D
x*= [4,12]

Opt. Value
P=4(400)+12(600)
P=8800
f(**x***)=8800

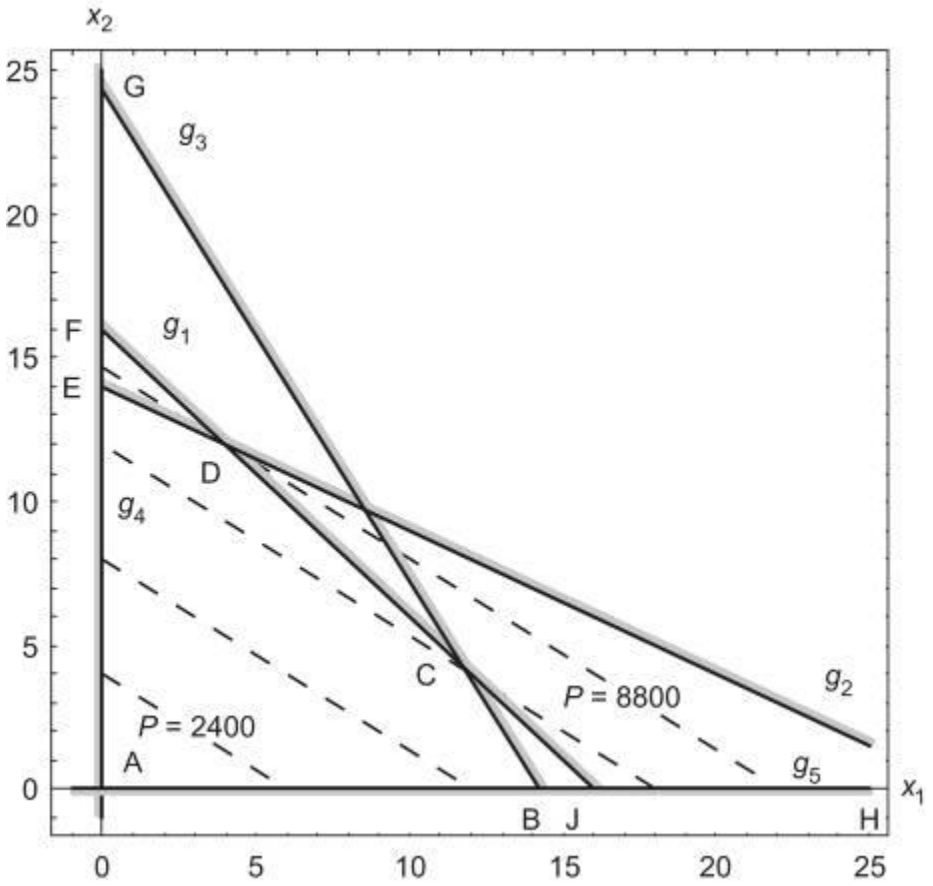


Figure 3.5 Graphical solution to the profit maximization problem: optimum point D = (4, 12); maximum profit, P = 8800.

Infinite/multiple solutions

When $f(\mathbf{x})$ is parallel to a binding constraint

$$f(\mathbf{x}) = -x_1 - 0.5x_2$$

$$g_1: \quad 2x_1 + 3x_2 \le 12$$

$$g_2: \quad 2x_1 + x_2 \le 8$$

$$g_3: \; -x_1 \le 0$$

$$g_4: \; -x_2 \le 0$$

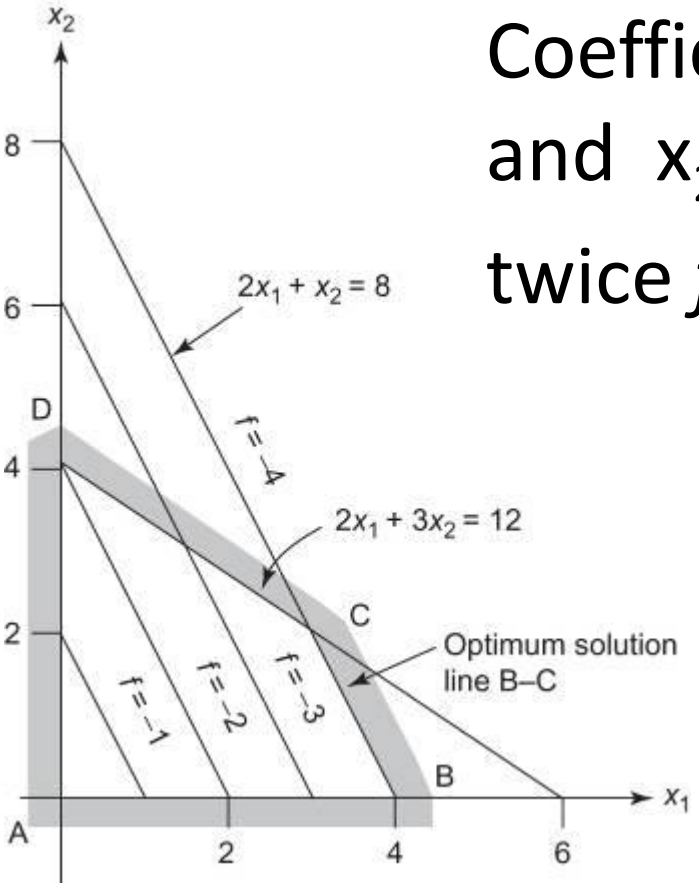


Figure 3.7 Example problem with multiple solutions.

Coefficient of x_1 and x_2 in g_2 are twice f(x)

Unbound Solution

Open region

On R.H.S.

Minimize

$$f(x) = -x_1 + 2x_2$$

subject to

$$-2x_1 + x_2 \le 0,$$

$$-2x_1 + 3x_2 \le 6,$$

$$-x_1 \le 0, -x_2 \le 0$$

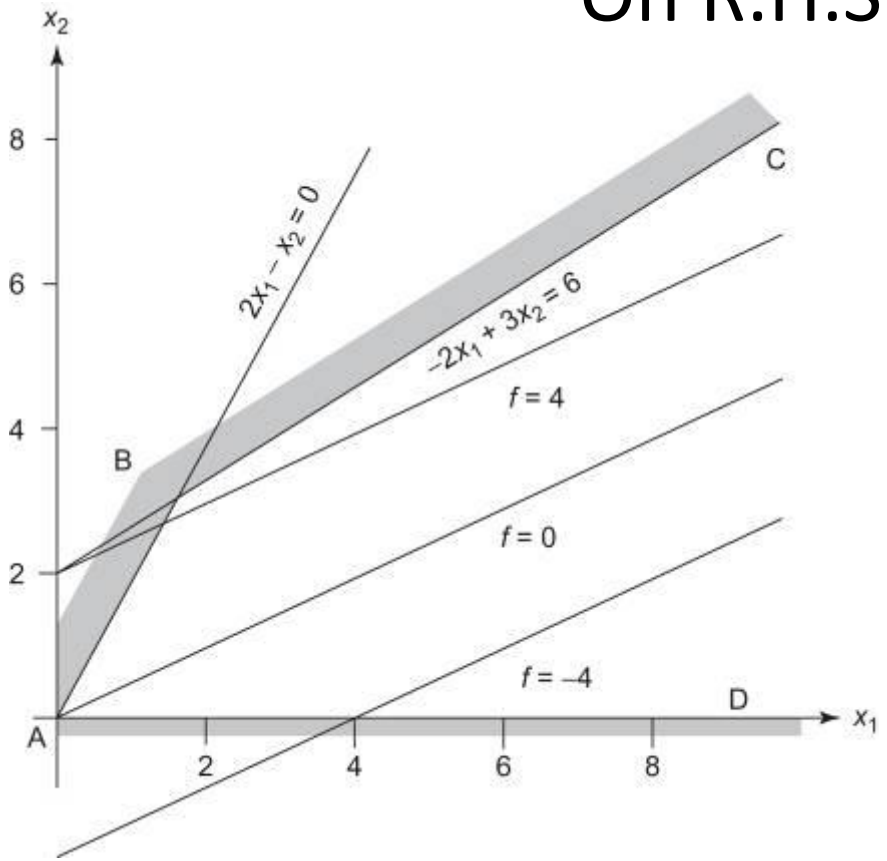
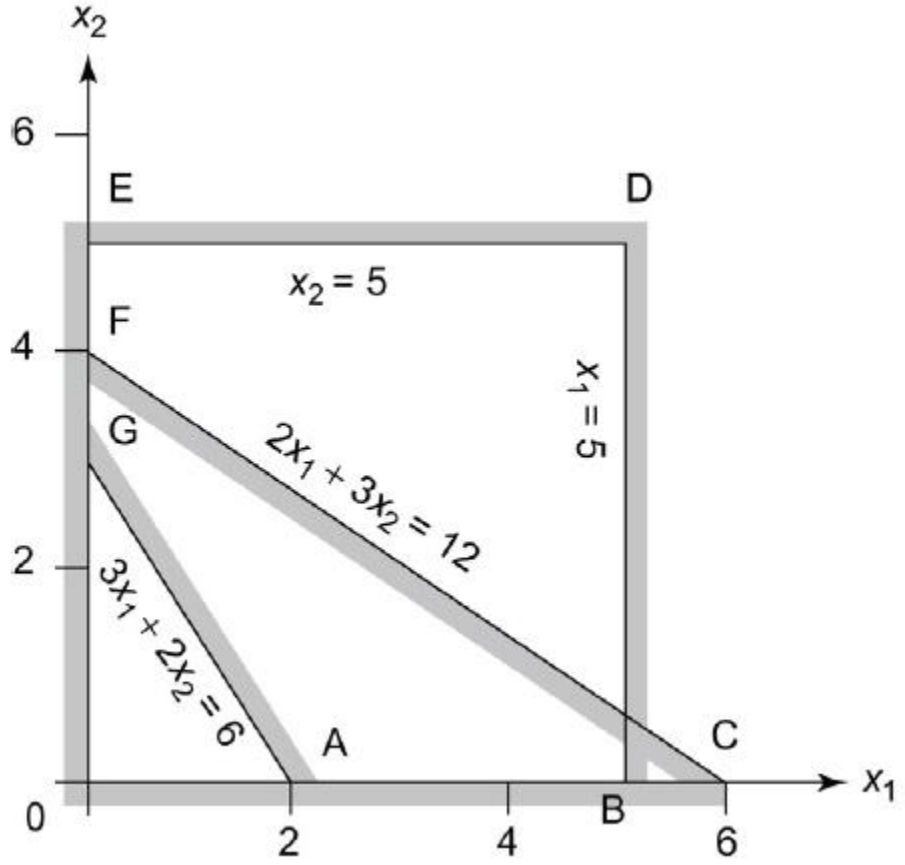


Figure 3.8 Example problem with an unbounded solution.

Infeasible design



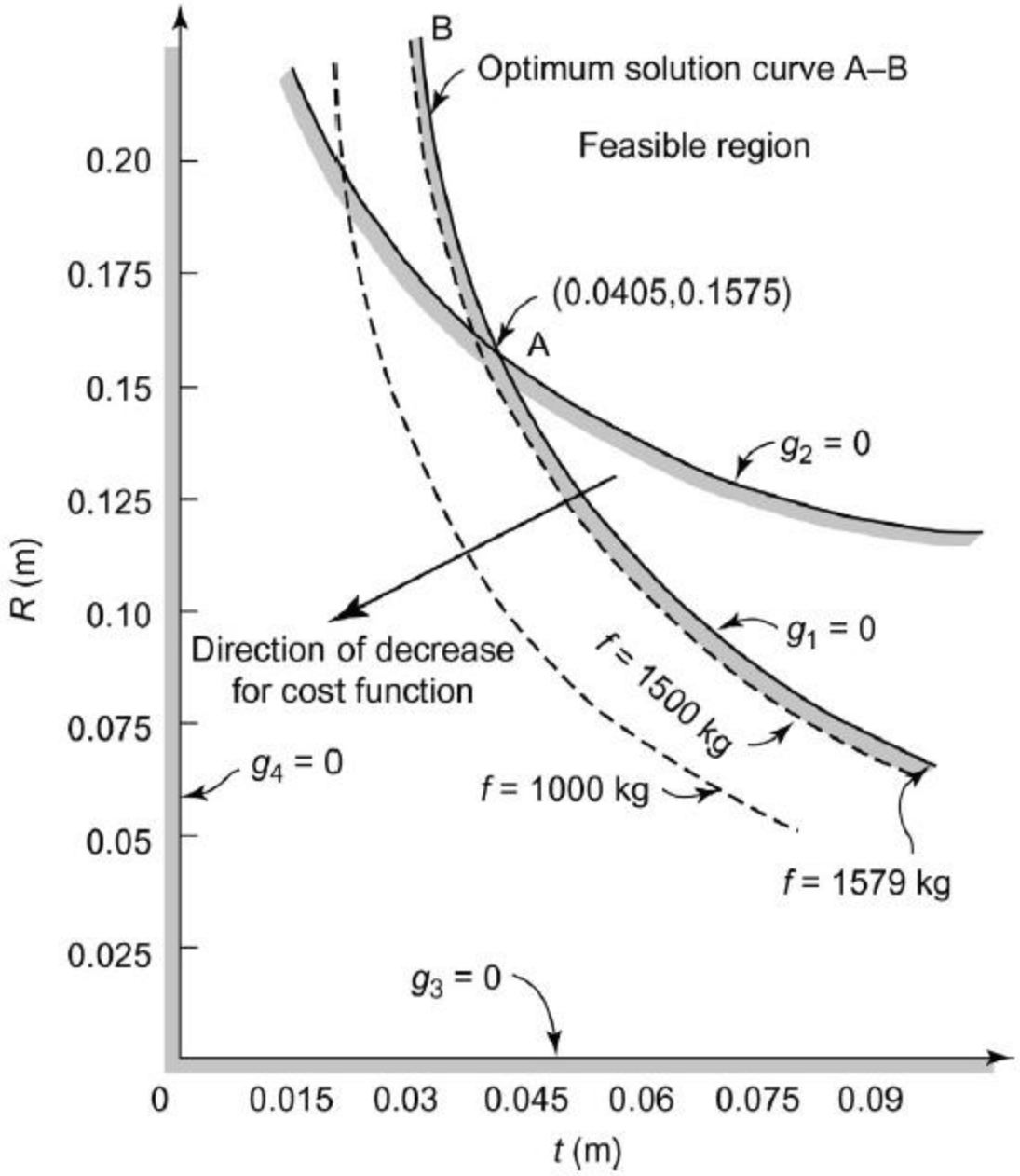
The design problem formulated in Section 2.7 will now be solved by the graphical method using the following data: $P = 10\,\text{MN}, E = 207\,\text{GPa}, \rho = 7833\,\text{kg/m}^3, l = 5.0\,\text{m}$, and $\sigma_a = 248\,\text{MPa}$. Using these data, formulation 1 for the problem is defined as, "find thickness t (m) and mean radius R (m) to minimize the mass function."

$$f(R,t) = 2\rho l\pi Rt = 2(7833)(5)\pi Rt = 2.4608\times 10^5 Rt,\ \text{kg} \tag{a}$$

subject to the four inequality constraints

$$g_1(R,t) = \frac{P}{2\pi Rt} - \sigma_a = \frac{10\times 10^6}{2\pi Rt} - 248\times 10^6 \le 0\ \text{(stress constraint)} \tag{b}$$

$$g_2(R,t) = P - \frac{\pi^3 ER^3 t}{4l^2} = 10\times 10^6 - \frac{\pi^3 (207\times 10^9) R^3 t}{4(5)(5)} \le 0\ \text{(buckling load constraint)} \tag{c}$$


B
Optimum solution curve A–B
Feasible region
0.20
0.175
0.15
0.125
0.10
0.075
0.05
0.025
R (m)
(0.0405,0.1575)
A
$g_2 = 0$
$g_1 = 0$
Direction of decrease
for cost function
f = 1500 kg
$g_4 = 0$
f = 1000 kg
f = 1579 kg
$g_3 = 0$
0
0.015
0.03
0.045
0.06
0.075
0.09
t (m)

Infeasible Problem

Constraints are:

inconsistent

conflicting

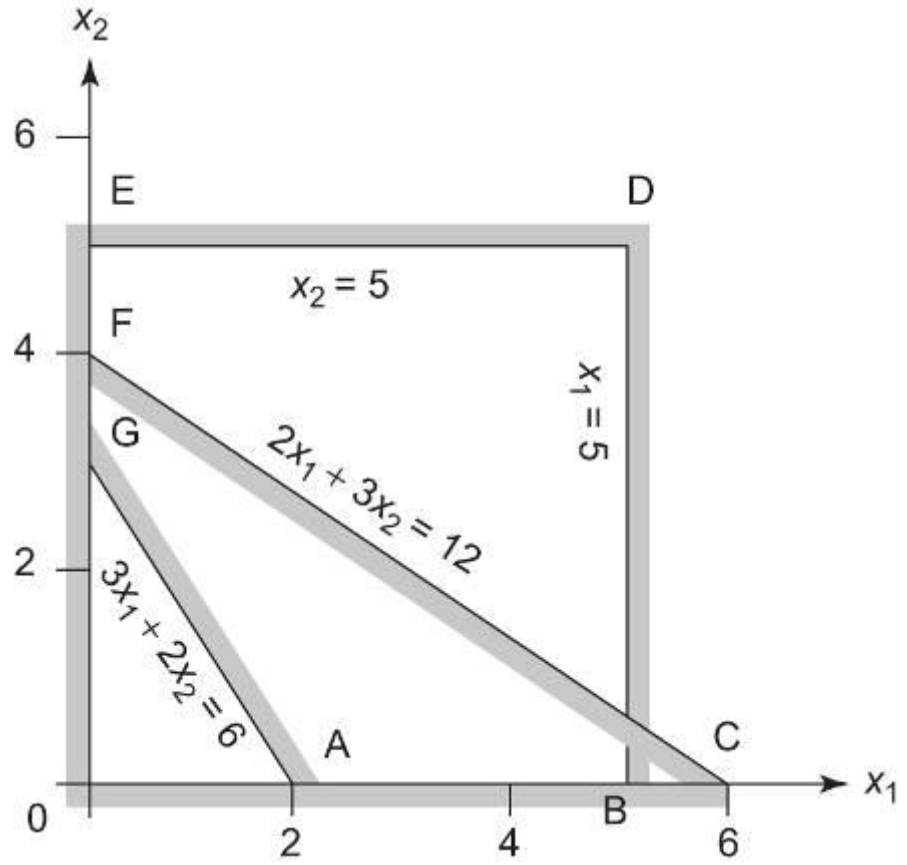


Figure 3.9 Infeasible design optimization problem.

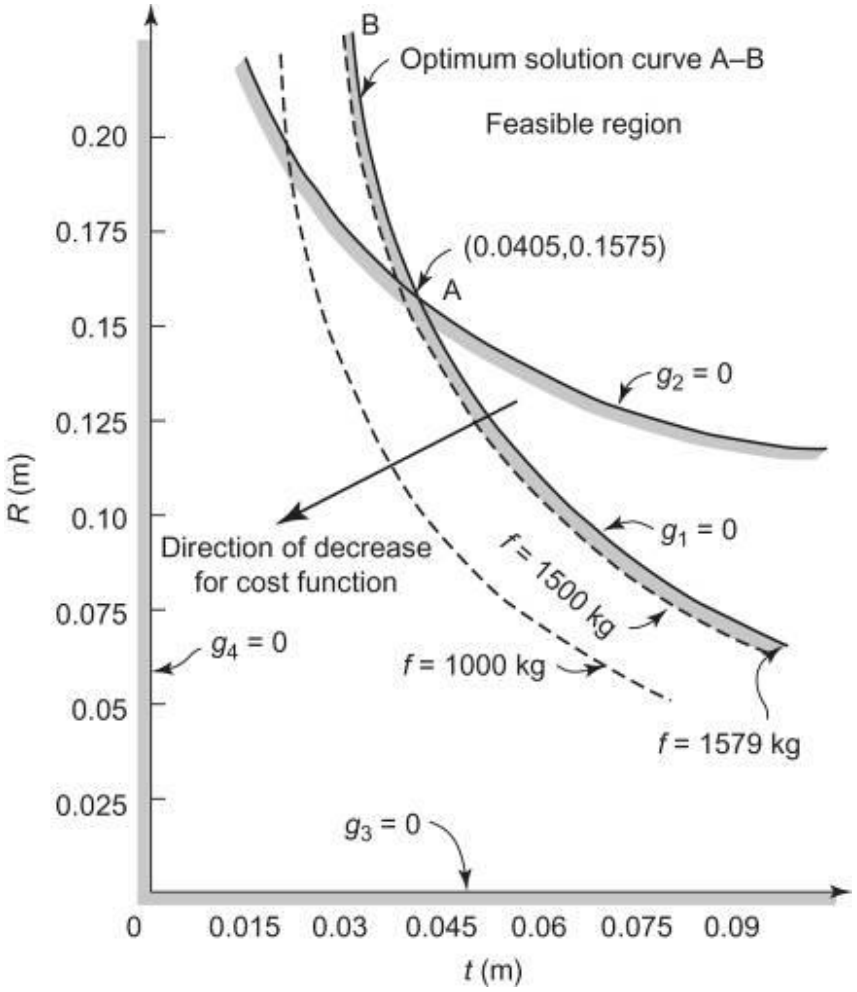
How many inequality constraints can we have?

How many active inequality constraints?

Non-linear constraints & Inf. Solns

Figure 3.10 A graphical solution to the problem of designing a minimum-weight tubular column.

Which constraint(s) are active?



Graphical Solution

1. Sketch coordinate system
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4. Plot f(**x**) contours (2 or 3)
5. Find opt solution **x*** & opt value f(**x***)

Global/local optima

Global Maximum?
Local Maximum?
Global Minimum?
Local Minimum?

Global = absolute
Local = relative

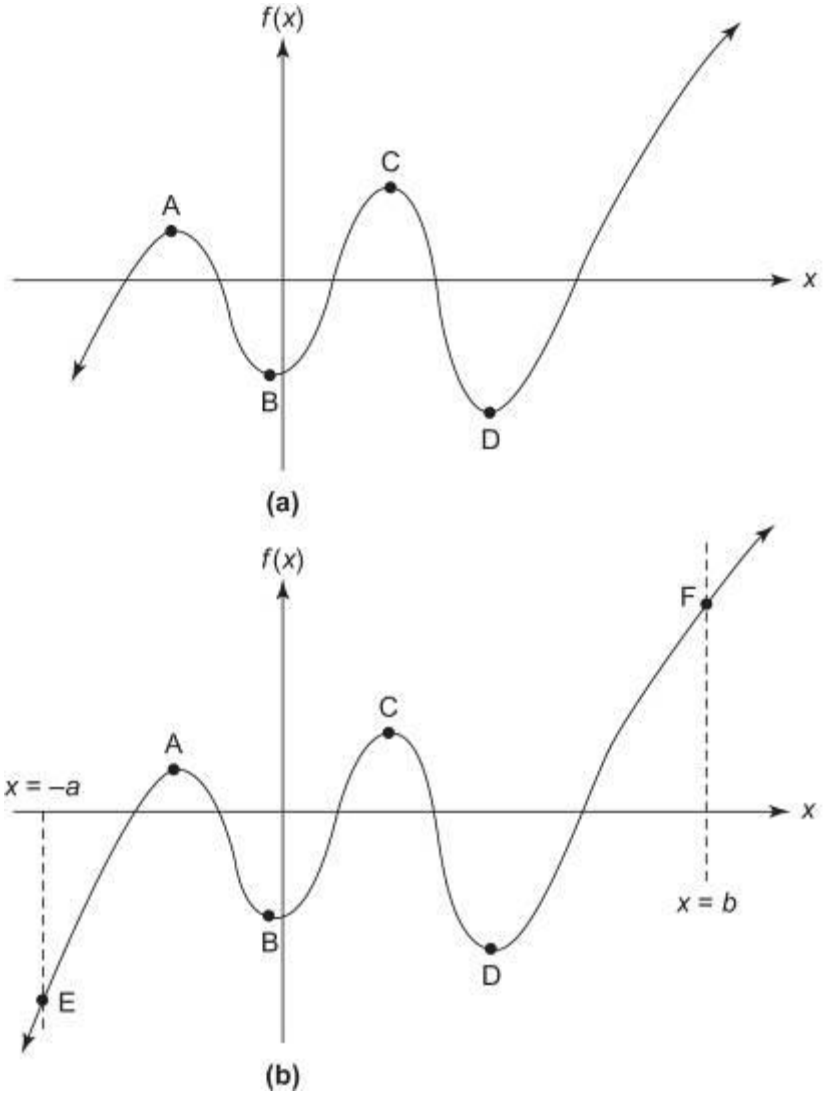

Figure 4.2 Representation of optimum points. (a) The unbounded domain and function (no global optimum). (b) The bounded domain and function (global minimum and maximum exist).

Closed & Bounded

Global/local optima

Global Maximum?
$f(\mathbf{x}^*) \leq f(\mathbf{x})$
Anywhere in S

Local Maximum?
$f(\mathbf{x}^*) \leq f(\mathbf{x})$
In small neighborhood N

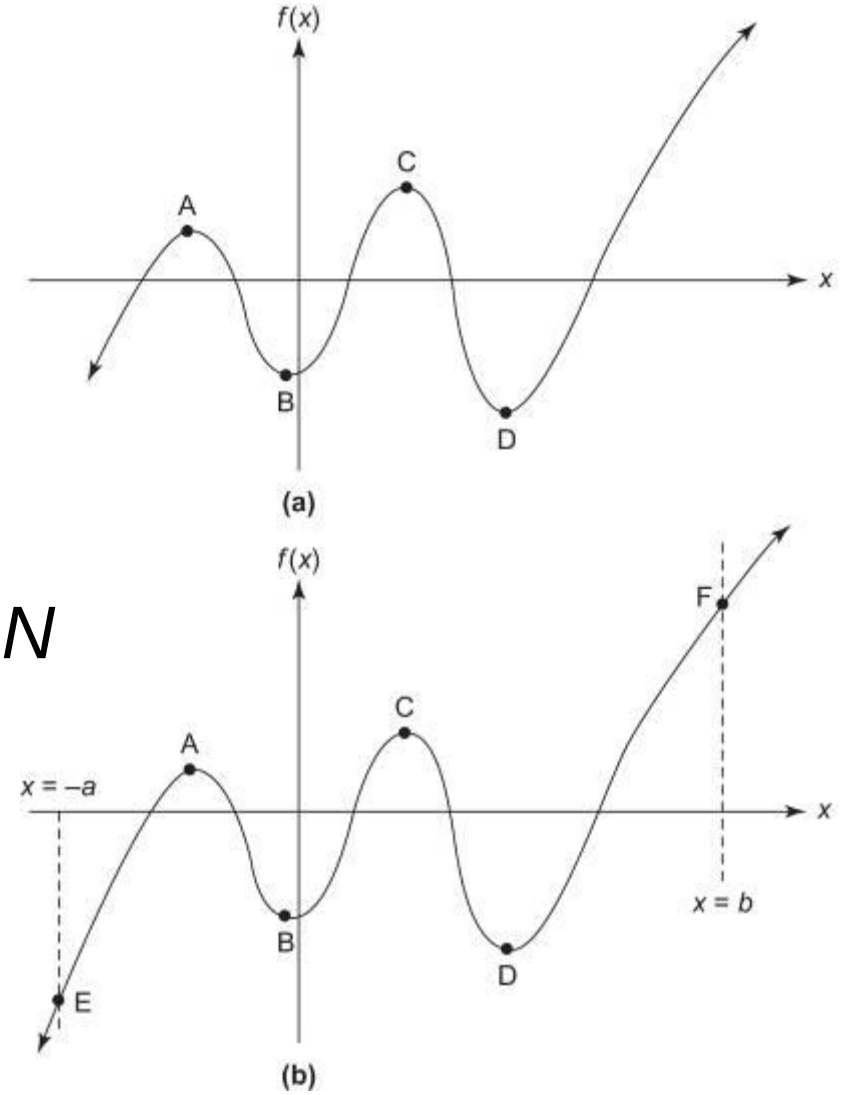

Figure 4.2 Representation of optimum points. (a) The unbounded domain and function (no global optimum). (b) The bounded domain and function (global minimum and maximum exist).

Closed & Bounded

Summary

- Graphical solution – 5 step process
- Feasible region may not exist resulting in an infeasible problem
- When obj function is ll to active/binding g_i an infinite number of solutions exist
- Feasible region may be unbounded
- An unbounded region may result in an unbounded solution