Optimum Design - Sheet 4
Optimality Conditions

1. Write the Taylor's expansion for the following functions up to quadratic terms.

   $\cos x$ about the point $x^* = \pi/4$
   $\cos x$ about the point $x^* = \pi/3$
   $\sin x$ about the point $x^* = \pi/6$
   $\sin x$ about the point $x^* = \pi/4$
   e^x about the point $x^* = 0$
   e^x about the point $x^* = 2$
   $f(x_1, x_2) = 10x_1^4 - 20x_1^2x_2 + 10x_2^2 + x_1^2 - 2x_1 + 5$ about the point (1, 1). Compare approximate and exact values of the function at the point (1.2, 0.8).

2. Determine the nature of the following quadratic forms.

   $F(\mathrm{x}) = x_1^2 + 4x_1x_2 + 2x_1x_3 - 7x_2^2 - 6x_2x_3 + 5x_3^2$
   $F(\mathrm{x}) = 2x_1^2 + 2x_2^2 - 5x_1x_2$
   $F(\mathrm{x}) = x_1^2 + x_2^2 + 3x_1x_2$
   $F(\mathrm{x}) = 3x_1^2 + x_2^2 - x_1x_2$
   $F(\mathrm{x}) = x_1^2 - x_2^2 + 4x_1x_2$
   $F(\mathrm{x}) = x_1^2 - x_2^2 + x_3^2 - 2x_2x_3$
   $F(\mathrm{x}) = x_1^2 - 2x_1x_2 + 2x_2^2$
   $F(\mathrm{x}) = x_1^2 - x_1x_2 - x_2^2$
   $F(\mathrm{x}) = x_1^2 + 2x_1x_3 - 2x_2^2 + 4x_3^2 - 2x_2x_3$
   $F(\mathrm{x}) = 2x_1^2 + x_1x_2 + 2x_2^2 + 3x_3^2 - 2x_1x_3$
   $F(\mathrm{x}) = x_1^2 + 2x_2x_3 + x_2^2 + 4x_3^2$
   $F(\mathrm{x}) = 4x_1^2 + 2x_1x_3 - x_2^2 + 4x_3^2$

3. Write optimality conditions and find stationary points for the following functions (use MATLAB, or Mathematica, if needed to solve the optimality conditions). Also determine the local minimum, local maximum, and inflection points for the functions (inflection points are those stationary points that are neither minimum nor maximum).

   $f(x_1, x_2) = 3x_1^2 + 2x_1x_2 + 2x_2^2 + 7$
   $f(x_1, x_2) = x_1^2 + 4x_1x_2 + x_2^2 + 3$
   $f(x_1, x_2) = x_1^3 + 12x_1x_2^2 + 2x_2^2 + 5x_1^2 + 3x_2$
   $f(x_2, x_2) = 5x_1 - \dfrac{1}{16}x_1^2x_2 + \dfrac{1}{4x_1}x_2^2$
   $f(x) = \cos x$
   $f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2$
   $f(x) = x^2e^{-x}$
   $f(x_1, x_2) = x_1 + \dfrac{10}{x_1x_2} + 5x_2$
   $f(x_1, x_2) = x_1^2 - 2x_1 + 4x_2^2 - 8x_2 + 6$
   $f(x_1, x_2) = 3x_1^2 - 2x_1x_2 + 5x_2^2 + 8x_2$

$$f(x_1, x_2) = x_1^2 + 2x_2^2 - 4x_1 - 2x_1x_2$$
$$f(x_1, x_2) = 12x_1^2 + 22x_2^2 - 1.5x_1 - x_2$$
$$f(x_1, x_2) = 7x_1^2 + 12x_2^2 - x_1$$
$$f(x_1, x_2) = 12x_1^2 + 21x_2^2 - x_2$$
$$f(x_1, x_2) = 25x_1^2 + 20x_2^2 - 2x_1 - x_2$$
$$f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3$$
$$f(x_1, x_2) = 8x_1^2 + 8x_2^2 - 80\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} - 80\sqrt{x_1^2 + x_2^2 + 20x_2 + 100} - 5x_1 - 5x_2$$
$$f(x_1, x_2) = 9x_1^2 + 9x_2^2 - 100\sqrt{x_1^2 + x_2^2 - 20x_2 + 100} - 64\sqrt{x_1^2 + x_2^2 + 16x_2 + 64} - 5x_1 - 41x$$
$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$
$$f(x_1, x_2, x_3, x_4) = (x_1 - 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4$$