SPC 307 - Aerodynamics
Sheet 4 - Solution
Dynamics of an incompressible, inviscid flow field

1. The velocity in a certain flow field is given by the equation

$$\vec{V} = x\,\hat{\imath} + x^2\,\hat{\jmath} + yz\,\hat{k}$$

Determine the expressions for the three rectangular components of acceleration.

From expression for velocity, $u = x$, $v = x^2 z$, $w = yz$

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

then

$$a_x = 0 + (x)(1) + (x^2 z)(0) + (yz)(0)$$

$$= \underline{\underline{x}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

and

$$a_y = 0 + (x)(2xz) + (x^2 z)(0) + (yz)(x^2)$$

$$= \underline{\underline{2x^2 z + x^2 yz}}$$

Also,

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

so that

$$a_z = 0 + (x)(0) + (x^2 z)(z) + (yz)(y)$$

$$= x^2 z^2 + y^2 z$$

2. The flow in the plane two-dimensional channel shown in Fif. 1. Gas x- and y-components of velocity given by

$$u = u_0\left(1 + \frac{x}{l}\right)\left[1 - \left(\frac{y}{Y}\right)^2\right]$$

$$v = u_0\left[\frac{y^3}{lY_0^2}\left(1 + \frac{x}{l}\right)^2 - \frac{y}{l}\right]$$

Calculate the linear acceleration, rotation, vorticity, rate of volumetric strain, and rate of shear deformation of the flow.

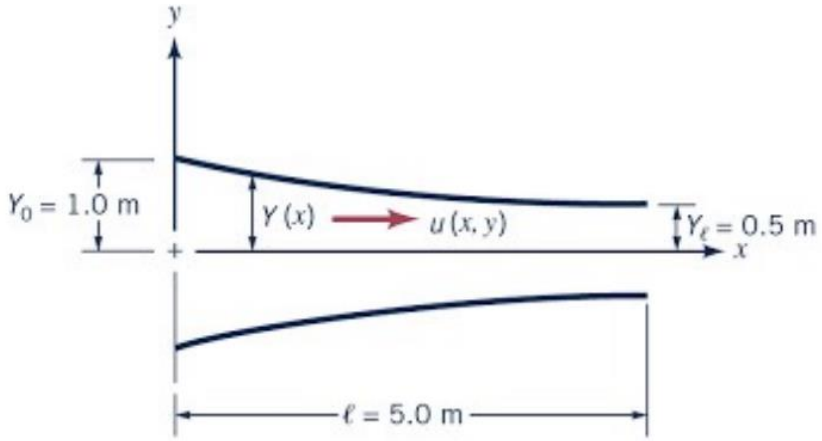


Fig. 1.

Left to the student.

3. If Determine an expression for the vorticity of the flow field described by

$$\vec{V} = -xy^3\,\hat{\imath} + y^4\,\hat{\jmath}$$

Is the flow irrotational?

$\vec{\zeta} = 2\vec{\omega}$ (Eq. 6.17)

From expression for velocity, $u = -xy^3$, $v = y^4$, and $w = 0$, and with

$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$ (Eq. 6.13)

$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$ (Eq. 6.14)

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ (Eq. 6.12)

it follows that

$\omega_x = 0$, $\omega_y = 0$, and $\omega_z = \frac{1}{2}\left[0 - (-3xy^2)\right] = \frac{3}{2}xy^2$

Thus,

$$\vec{\zeta} = 2\left(\omega_x\,\hat{i} + \omega_y\,\hat{j} + \omega_z\,\hat{k}\right)$$

$$= 2\left[(0)\,\hat{i} + (0)\,\hat{j} + \left(\tfrac{3}{2}xy^2\right)\hat{k}\right]$$

$$= \underline{\underline{3xy^2\,\hat{k}}}$$

Since $\vec{\zeta}$ is not zero everywhere the flow is not irrotational. <u>No.</u>

4. The Equation of the x-velocity in fully developed laminar flow between parallel plates is given by

$$u = \frac{1}{2\mu}\left(\frac{\delta p}{\delta x}\right)(y^2 - h^2)$$

The y-velocity is v=0. Determine the volumetric strain rate, the vorticity and the rate of angular deformation. What is the shear stress at the plate surface?

Left to the student.

5. The stream function for an incompressible, two-dimensional flow field is

$$\Psi = 3x^2y + y$$

For this flow field, plot several streamlines.

The equation for a streamline is found by setting ψ = constant in the equation for the stream function. Thus, for the given stream function

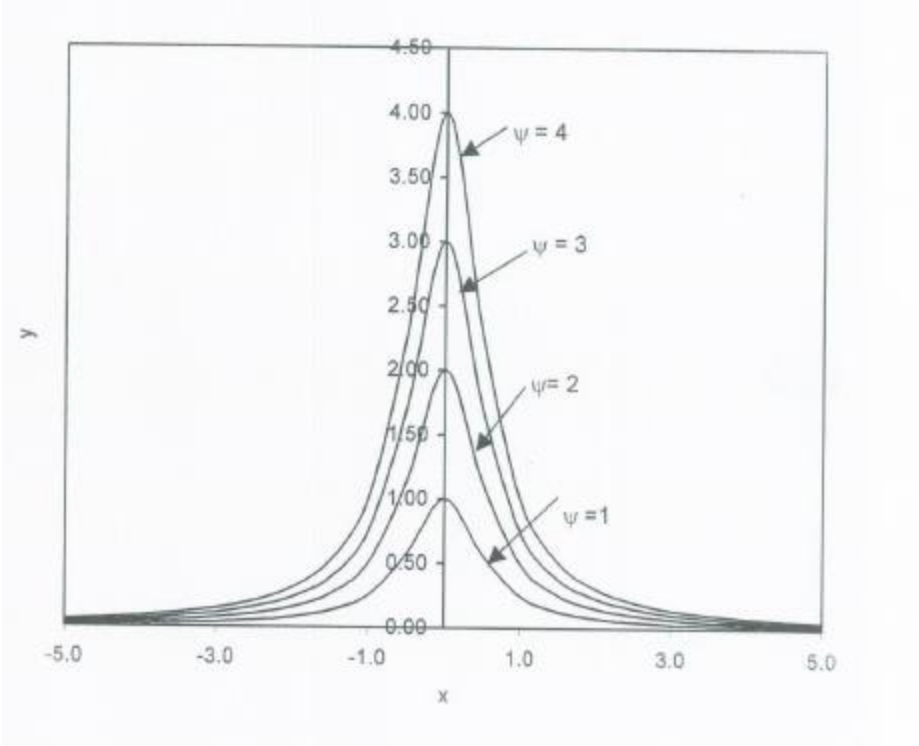
$$\psi = 3x^2y + y$$

it follows that the equation of a streamline is

$$y = \frac{\psi}{1 + 3x^2}$$

where various constant values can be assigned to ψ to obtain a family of streamlines. Tabulated results for $\psi = 1, 2, 3, 4$, and a plot showing the streamlines are given below.

	ψ =1	ψ =2	ψ =3	ψ=4
x	y	y	y	y
-5.0	0.0132	0.0263	0.0395	0.0526
-4.5	0.0162	0.0324	0.0486	0.0648
-4.0	0.0204	0.0408	0.0612	0.0816
-3.5	0.0265	0.0530	0.0795	0.1060
-3.0	0.0357	0.0714	0.1071	0.1429
-2.5	0.0506	0.1013	0.1519	0.2025
-2.0	0.0769	0.1538	0.2308	0.3077
-1.5	0.1290	0.2581	0.3871	0.5161
-1.0	0.2500	0.5000	0.7500	1.0000
-0.5	0.5714	1.1429	1.7143	2.2857
0.0	1.0000	2.0000	3.0000	4.0000
0.5	0.5714	1.1429	1.7143	2.2857
1.0	0.2500	0.5000	0.7500	1.0000
1.5	0.1290	0.2581	0.3871	0.5161
2.0	0.0769	0.1538	0.2308	0.3077
2.5	0.0506	0.1013	0.1519	0.2025
3.0	0.0357	0.0714	0.1071	0.1429
3.5	0.0265	0.0530	0.0795	0.1060
4.0	0.0204	0.0408	0.0612	0.0816
4.5	0.0162	0.0324	0.0486	0.0648
5.0	0.0132	0.0263	0.0395	0.0526



This problem could be plotted by Matlab

6. A The stream function for a given two-dimensional flow field is

$$\phi = x^3 - 3xy^2$$

Show that the continuity equation is satisfied and determine the corresponding stream function.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 3x^2 - 3y^2$$

Integrate with respect to y to obtain

$$\int d\psi = \int (3x^2 - 3y^2)\, dy$$

or

$$\psi = 3\left(x^2 y - \frac{y^3}{3}\right) + f_1(x) \qquad (1)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -6xy$$

and integrating with respect to x yields

$$\int d\psi = \int 6xy\, dx$$

or

$$\psi = 3x^2 y + f_2(y) \qquad (2)$$

To satisfy both Eqs. (1) and (2)

$$\psi = 3x^2 y - y^3 + C$$

where C is an arbitrary constant. Since the streamline $\psi = 0$ passes through the origin $(x = 0, y = 0)$ it follows that $C = 0$ and

$$\psi = 3x^2 y - y^3 \qquad (3)$$

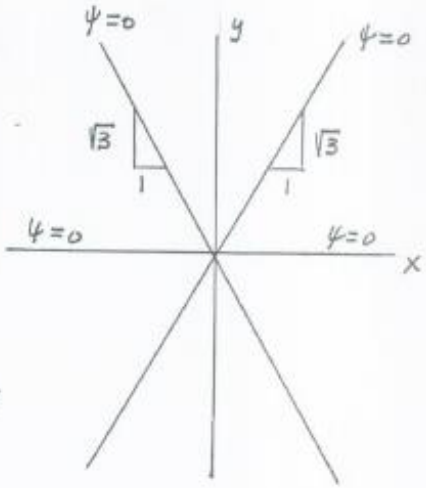
The equation of the streamline passing through the origin is found by setting $\psi = 0$ in Eq. (3) to yield

$$y(3x^2 - y^2) = 0$$

which is satisfied for $y = 0$ and

$$y = \pm\sqrt{3}\, x$$

A sketch of the $\psi = 0$ streamlines are shown in the figure.



7. The stream function for a given two-dimensional flow field is

$$\Psi = 5x^2y - \left(\frac{5}{3}y^3\right)$$

Determine the corresponding velocity potential.

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x} = 5x^2 - 5y^2 \qquad (1)$$

Integrate with respect to x to obtain

$$\int d\phi = \int (5x^2 - 5y^2)\, dx$$

or

$$\phi = \frac{5}{3}x^3 - 5xy^2 + f_1(y) \qquad (2)$$

Similarly,

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y} = -10xy \qquad (3)$$

and

$$\int d\phi = -\int 10xy\, dy$$

or

$$\phi = -5xy^2 + f_2(x) \qquad (4)$$

To satisfy both Eqs. (2) and (4)

$$\phi = \left(\frac{5}{3}\right)x^3 - 5xy^2 + C$$

where C is an arbitrary constant.

8. The velocity potential for a certain inviscid flow field is

$$\phi = -(3x^2y - y^3)$$

where ϕ has the units of ft^2/s when x and y are in feet. Determine the pressure difference (in psi) between the points (1,2) and (4,4), where the coordinates are in feet, if the fluid is water and elevation changes are negligible.

Since the flow field is described by a velocity potential the flow is irrotational and the Bernoulli equation can be applied between any two points. Thus,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad (1)$$

Also,

$$u = \frac{\partial \phi}{\partial x} = -6xy \qquad v = \frac{\partial \phi}{\partial y} = -3x^2 + 3y^2$$

At $x = 1\,ft$, $y = 2\,ft$

$$u_1 = -6(1)(2) = -12\,\frac{ft}{s}$$

$$v_1 = -3(1)^2 + 3(2)^2 = 9\,\frac{ft}{s}$$

So that $V_1^2 = u_1^2 + v_1^2 = \left(-12\,\frac{ft}{s}\right)^2 + \left(9\,\frac{ft}{s}\right)^2 = 225\left(\frac{ft}{s}\right)^2$

At $x = 4\,ft$, $y = 4\,ft$

$$u_2 = -6(4)(4) = -96\,\frac{ft}{s}$$

$$v_2 = -3(4)^2 + 3(4)^2 = 0$$

So that $V_2^2 = \left(-96\,\frac{ft}{s}\right)^2$

Thus, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\frac{\gamma}{g}\left[V_2^2 - V_1^2\right]$$

$$= \frac{1}{2}\frac{\left(62.4\,\frac{lb}{ft^3}\right)}{\left(32.2\,\frac{ft}{s^2}\right)}\left[\left(-96\,\frac{ft}{s}\right)^2 - 225\left(\frac{ft}{s}\right)^2\right]$$

$$= 8710\,\frac{lb}{ft^2} = \left(8710\,\frac{lb}{ft^2}\right)\left(\frac{ft^2}{144\,in.^2}\right) = \underline{\underline{60.5\ psi}}$$

9. A The velocity potential for a certain inviscid, incompressible flow field is given by the equation

$$\phi = 2x^2y - \left(\frac{2}{3}\right)y^3$$

where ϕ has the units of m^2/s when x and y are in meters. Determine the pressure at the point x = 2 m, y = 2 m if the pressure at x = 1 m, y = 1 m is 200 kPa. Elevation changes can be neglected, and the fluid is water.

Since the flow is irrotational,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} \qquad (1)$$

with $V^2 = u^2 + v^2$. For the velocity potential given,

$$u = \frac{\partial \phi}{\partial x} = 4xy \qquad v = \frac{\partial \phi}{\partial y} = 2x^2 - 2y^2$$

At point 1 let $x = 1m$ and $y = 1m$ so that

$$u_1 = 4(1)(1) = 4\ \frac{m}{s} \qquad v_1 = 2(1)^2 - 2(1)^2 = 0$$

and

$$V_1^2 = \left(4\ \frac{m}{s}\right)^2 = 16\ \frac{m^2}{s^2}$$

At point 2 $x = 2m$ and $y = 2m$ so that

$$u_2 = 4(2)(2) = 16\ \frac{m}{s} \qquad v_2 = 2(2)^2 - 2(2)^2 = 0$$

and

$$V_2^2 = \left(16\ \frac{m}{s}\right)^2 = 256\ \frac{m^2}{s^2}$$

Thus, from Eq. (1)

$$p_2 = p_1 + \frac{\gamma}{2g}\left(V_1^2 - V_2^2\right)$$

$$= 200 \times 10^3\ \frac{N}{m^2} + \frac{\left(9.80 \times 10^3\ \frac{N}{m^3}\right)}{2\left(9.81\ \frac{m}{s^2}\right)}\left(16\ \frac{m^2}{s^2} - 256\ \frac{m^2}{s^2}\right)$$

$$= 80.1\ kPa$$

10. Consider the two-dimensional flow air flow around the corner shown in in Fig. 2. The x- and y-direction velocities are

$$u = \frac{v_0}{l} \sin h\left(\frac{x}{l}\right) \cos h\left(\frac{y}{l}\right)$$

and

$$u = -\frac{v_0}{l} \cos h\left(\frac{x}{l}\right) \sin h\left(\frac{y}{l}\right)$$

respectively. Assume constant density, steady flow, negligible gravity and inviscid flow. Find $p(x,y)$.

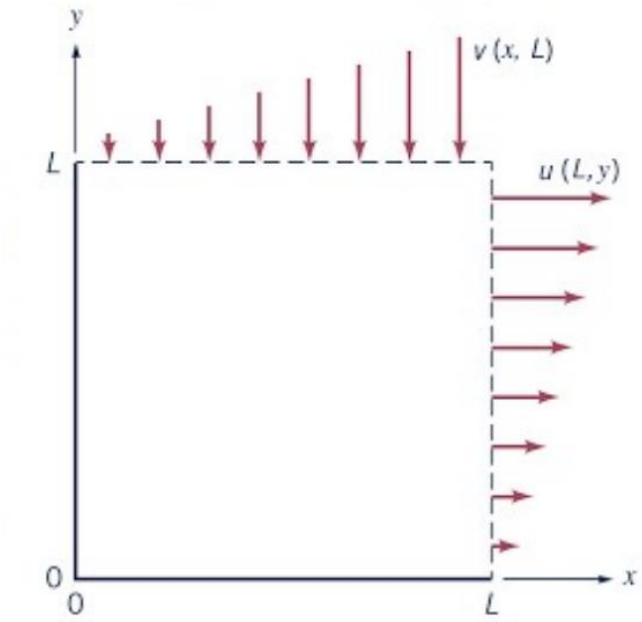


Fig. 2.

Left to the student.

11. The velocity potential

$$\phi = -k(x^2 - y^2) \quad (k = \text{constant})$$

may be used to represent the flow against an infinite plane boundary, as illustrated in Fig. 3. For flow in the vicinity of a stagnation point, it is frequently assumed that the pressure gradient along the surface is of the form

$$\frac{\delta p}{\delta x} = Ax$$

where A is a constant. Use the given velocity potential to show that this is true.

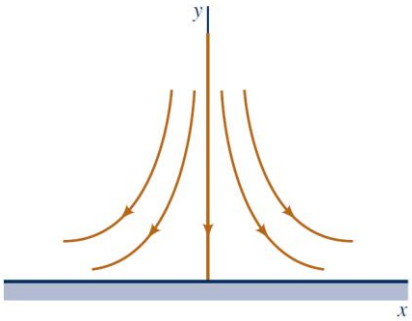


Fig. 3.

For the velocity potential given

$$u = \frac{\partial \phi}{\partial x} = -2kx \qquad (1)$$

$$v = \frac{\partial \phi}{\partial y} = -2ky \qquad (2)$$

and the stagnation point occurs at the origin.

For this steady, two-dimensional flow

$$-\frac{\partial p}{\partial x} = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \qquad (\text{Eq. 6.5a})$$

and along the surface ($y=0$) $v=0$ so that

$$\frac{\partial p}{\partial x} = \rho u \frac{\partial u}{\partial x} \qquad (3)$$

From Eq. (1) $u = -2kx$ and therefore

$$\frac{\partial u}{\partial x} = -2k$$

and Eq. (3) becomes

$$\frac{\partial p}{\partial x} = \rho(-2kx)(-2k) = 4k^2 x$$

or

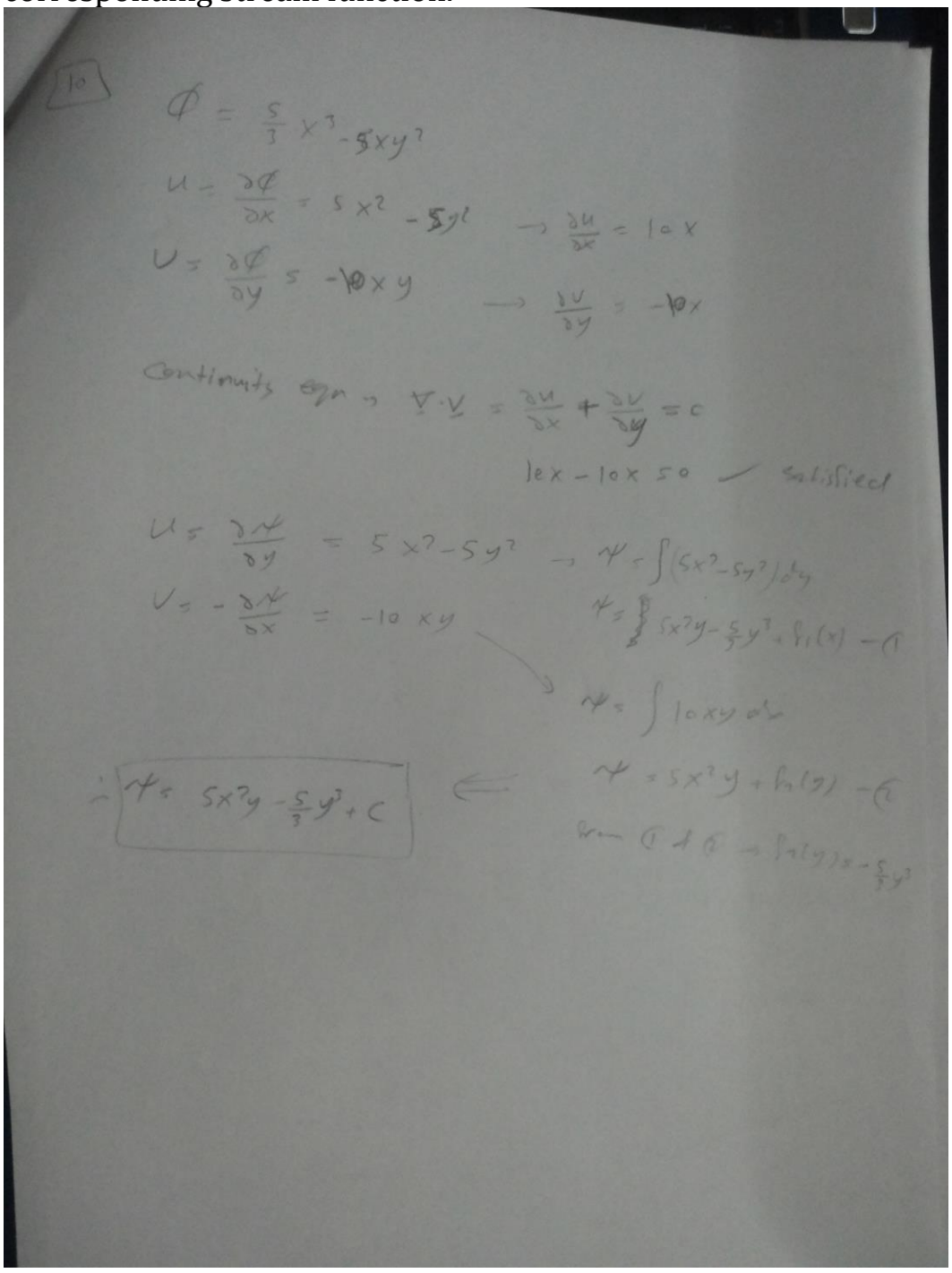
$$\frac{\partial p}{\partial x} = A x$$

where $A = 4k^2$.

12. The velocity potential for a given two-dimensional flow field is

$$\phi = \left(\frac{5}{3}\right)x^3 - 2xy^2$$

Show that the continuity equation is satisfied and determine the corresponding stream function.



13. One end of a pond has a shoreline that resembles a half-body as shown in Fig. 2. A vertical porous pipe is located near the end of the pond so that water can be pumped out. When water is pumped at the rate of 0.02 m^2/s (flow rate per unit length), what will be the velocity at point A in Fig. 2? Hint: Consider the flow inside a half-body. This flow is similar to the flow shown in Fig. 4.

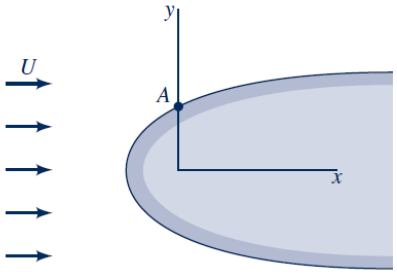


Fig. 4.

For a half-body,

$$\psi = U r \sin\theta + \frac{m}{2\pi}\theta \qquad \text{(Eq. 6.97)}$$

so that

$$v_\theta = -\frac{\partial \psi}{\partial r} = U \sin\theta$$

and

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U\cos\theta + \frac{m}{2\pi r}$$

Thus, at point A, $\theta = 0$, $r = 15\,m$ and

$$v_\theta = 0$$

$$v_r = V_A = U + \frac{m}{2\pi(15)} \qquad (1)$$

For a flowrate of $0.06\ \frac{m^3}{s}$ in a 3-m long pipe, the source strength is $\frac{0.06}{3}\ \frac{m^2}{s}$. Since

$$b = \frac{m}{2\pi U} \qquad \text{(Eq. 6.99)}$$

then with $b = 5m$

$$U = \frac{m}{2\pi b} = \frac{\left(\frac{0.06}{3}\ \frac{m^2}{s}\right)}{2\pi(5m)} = 6.37 \times 10^{-4}\ \frac{m}{s}$$

From Eq. (1)

$$V_A = 6.37 \times 10^{-4}\ \frac{m}{s} + \frac{\left(\frac{0.06}{3}\ \frac{m^2}{s}\right)}{2\pi(15m)}$$

$$= 8.49 \times 10^{-4}\ \frac{m}{s}$$

14. Water flows over a flat surface at 4 ft/s, as shown in Fig. 5. A pump draws off water through a narrow slit at a volume rate of 0.1 ft3/s per foot length of the slit. Assume that the fluid is incompressible and inviscid and can be represented by the combination of a uniform flow and a sink. Locate the stagnation point on the wall (point A) and determine the equation for the stagnation streamline. How far above the surface, H, must the fluid be so that it does not get sucked into the slit?

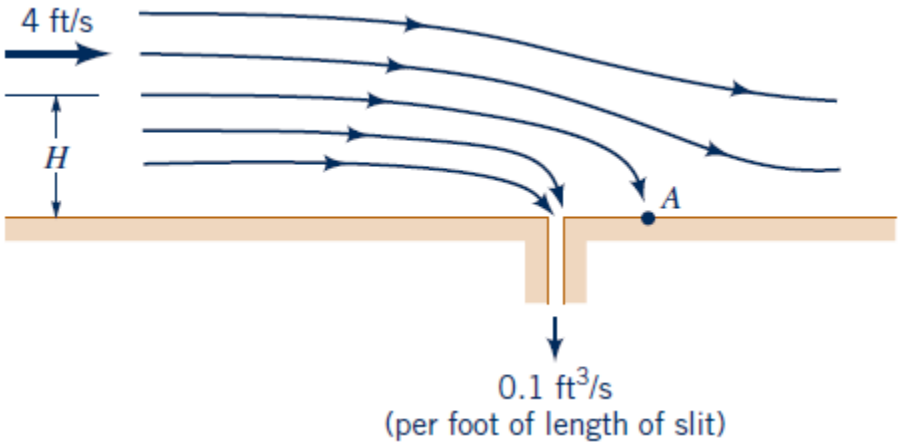
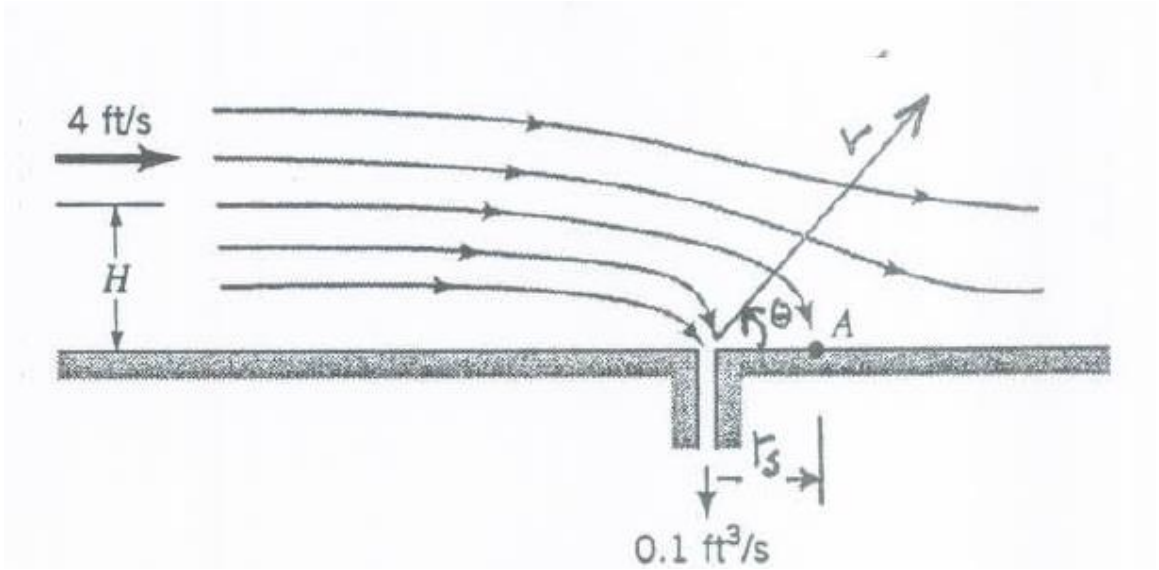


Fig. 5.



$$\psi = \psi_{\substack{\text{uniform}\\ \text{flow}}} + \psi_{sink} = \bar{U} r \sin\theta - \frac{m}{2\pi}\theta \qquad (1)$$

Thus,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \bar{U}\cos\theta - \frac{m}{2\pi r} \qquad (2)$$

and

$$v_\theta = -\frac{\partial \psi}{\partial r} = -\bar{U}\sin\theta$$

Along the wall $v_\theta = 0$, and the stagnation point occurs where $v_r = 0$, so that from Eq. (2)

$$0 = \bar{U}\cos(0^\circ) - \frac{m}{2\pi r_s}$$

and therefore

$$r_s = \frac{m}{2\pi \bar{U}}$$

For $\bar{U} = 4\ \frac{ft}{s}$ and $m = 0.2\ \frac{ft^2}{s}$ (note that a source strength of $0.2\ \frac{ft^2}{s}$ must be used to obtain $0.1\ \frac{ft^2}{s}$ through slit which is only one half of a "full" sink). Thus,

$$r_s = \frac{0.2\ \frac{ft^2}{s}}{2\pi\ (4\ \frac{ft}{s})} = 0.00796\ ft$$

and the stagnation point is on the wall <u>0.00796 ft to the right of slit</u>.

The value of ψ at the stagnation point ($r = 0.00796$ ft, $\theta = 0°$) is zero (Eq. 1) so that the equation of the stagnation streamline is

$$0 = U r \sin\theta - \frac{m}{2\pi}\theta$$

or

$$r\sin\theta = \frac{m}{2\pi U}\theta$$

Since $y = r\sin\theta$ the equation of the stagnation streamline can be written as

$$\underline{\underline{y = \frac{m}{2\pi U}\theta}}$$

Fluid above the stagnation streamline will not be sucked into slit. The maximum distance, H, for the stagnation streamline occurs as $\theta \to \pi$ so that

$$H = \frac{m\pi}{2\pi U} = \frac{0.2\ \frac{ft^2}{s}}{2\left(4\ \frac{ft}{s}\right)} = \underline{\underline{0.0250\ ft}}$$

(Note: All the fluid below the stagnation streamline must pass through the slit. Thus, from conservation of mass

$$HU = \text{flow into slit}$$

or $$H = \frac{0.1\ \frac{ft^2}{s}}{4\ \frac{ft}{s}} = 0.0250\ ft$$

which checks with the answer above.)

15. A body having the general shape of a half-body is placed in a stream of fluid. At a great distance upstream the velocity is U as shown in Fig. 6. Show how a measurement of the differential pressure between the stagnation point and point A can be used to predict the free-stream velocity, U. Express the pressure differential in terms of U and fluid density. Neglect body forces and assume that the fluid is nonviscous and incompressible.

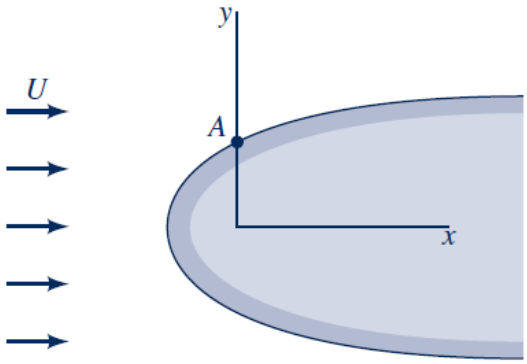


Fig. 6.

Write Bernoulli equation between stagnation point and point A to obtain

$$p_{stag} = p_A + \frac{1}{2}\rho V_A^2 \qquad (1)$$

Also, $$V_A^2 = U^2\left(1 + 2\frac{b}{r_A}\cos\theta + \frac{b^2}{r_A^2}\right) \qquad (Eq.\ 6.101)$$

and $$r = \frac{b(\pi-\theta)}{\sin\theta} \qquad (Eq.\ 6.100)$$

At point A $\theta = \frac{\pi}{2}$ so that

$$r_A = \frac{b\left(\pi - \frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = \frac{\pi b}{2}$$

or $$\frac{b}{r_A} = \frac{2}{\pi} \qquad (2)$$

Substitution of Eq. (2) into Eq. 6.101 yields

$$V_A^2 = U^2\left(1 + 0 + \frac{4}{\pi^2}\right)$$

and therefore from Eq. (1)

$$p_{stag} = p_A + \frac{1}{2}\rho U^2\left(1 + \frac{4}{\pi^2}\right) = p_A + 0.703\rho U^2$$

Thus,

$$\underline{p_{stag} - p_A = 0.703\rho U^2}$$

16. Water flows around a 6-ft-diameter bridge pier with a velocity of 12 ft/s. Estimate the force (per unit length) that the water exerts on the pier. Assume that the flow can be approximated as an ideal fluid flow around the front half of the cylinder, but due to flow separation (see Video V6.8), the average pressure on the rear half is constant and approximately equal to 1/2 the pressure at point A (see Fig. 7).

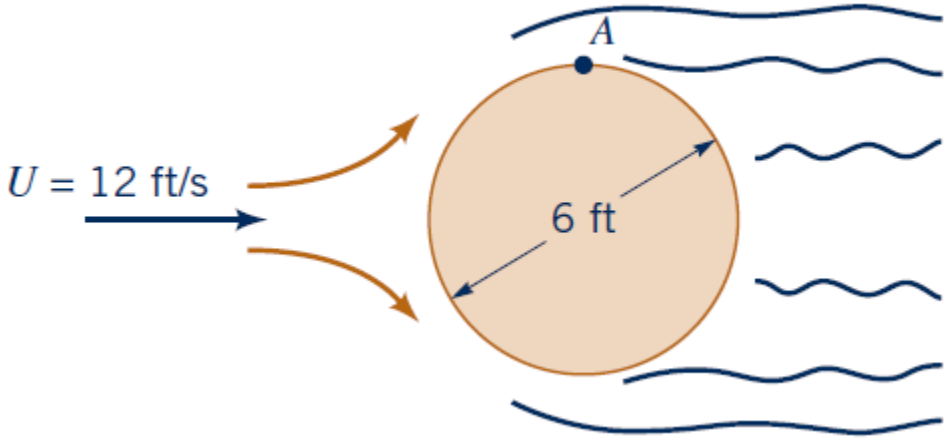
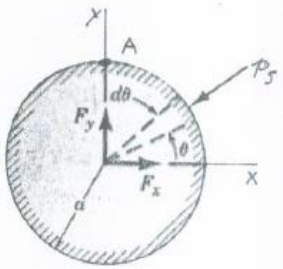


Fig. 7.



From Fig. 6.28 it follows that the drag on a section (between $\theta = 0$ and $\theta = \alpha$) of a circular cylinder is given by the equation

$$\text{Drag} = F_x = -\int_0^{\alpha} p_s \cos\theta \, a \, d\theta$$

For the force on the front half of the cylinder (per unit length)

$$F_{x_1} = -2\int_{\pi/2}^{\pi} p_s \cos\theta \, a \, d\theta \qquad (1)$$

and due to symmetry $F_y = 0$. From Eq. 6.116

$$p_s = p_0 + \frac{1}{2}\rho V^2 (1 - 4\sin^2\theta) \qquad (Eq.\ 6.116)$$

and since we are only interested in the force due to the flowing fluid we will let $p_0 = 0$. Thus, from Eq. (1)

$$F_{x_1} = -2\int_{\pi/2}^{\pi} \frac{1}{2}\rho V^2 (1 - 4\sin^2\theta)\cos\theta\, a\, d\theta \qquad (2)$$

Since $$\int_{\pi/2}^{\pi} \cos\theta\, d\theta = \sin\theta\Big]_{\pi/2}^{\pi} = -1$$

and $$\int_{\pi/2}^{\pi} \sin^2\theta\cos\theta\, d\theta = \frac{\sin^3\theta}{3}\Big]_{\pi/2}^{\pi} = -\frac{1}{3}$$

It follows from Eq. (2) that

$$F_{x_1} = -\frac{\rho V^2 a}{3}$$

Note that the negative sign indicates that the water is actually "pulling" on the cylinder (front half) in the upstream direction. However, when the effect of the rear half of the cylinder is taken into account (in a real fluid) there will be a net drag in the direction of flow.

The pressure at the top of the cylinder (point A) is given by

$$p_s = p_0 + \frac{1}{2}\rho V^2 (1 - 4\sin^2\theta) \qquad (Eq.\ 6.116)$$

and with $\theta = \pi/2$

$$p_A = p_0 - \frac{3}{2}\rho V^2$$

Since $p_0 = 0$

$$p_A = -\frac{3}{2}\rho V^2$$

Note that the negative pressure will give a positive F_x and

$$F_{x_2} = -\frac{p_A}{2} \times \text{projected area} = -\frac{p_A}{2} \times 2a(1)$$

So that

$$F_{x_2} = \frac{3}{4}\rho V^2 (2a)(1) = \frac{3}{2}\rho V^2 a$$

Thus,

$$F_x = F_{x_1} + F_{x_2}$$

$$= -\frac{\rho V^2 a}{3} + \frac{3\rho V^2 a}{2}$$

$$= \frac{7}{6}\rho V^2 a$$

With the data given,

$$F_x = \frac{7}{6}\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)\left(12 \frac{\text{ft}}{\text{s}}\right)^2 (3\ \text{ft}) = \underline{\underline{978\ \frac{\text{lb}}{\text{ft}}}}$$

17. A long porous pipe runs parallel to a horizontal plane surface as shown in Fig. 8. The longitudinal axis of the pipe is perpendicular to the plane of the paper. Water flows radially from the pipe at a rate of 0.5 p ft^3/s per foot of pipe. Determine the difference in pressure (in lb/ft^2) between point B and point A. The flow from the pipe may be approximated by a two-dimensional source. Hint: To develop the stream function or velocity potential for this type of flow, place (symmetrically) another equal source on the other side of the wall. With this combination there is no flow across the x axis, and this axis can be replaced with a solid boundary. This technique is called the method of images.

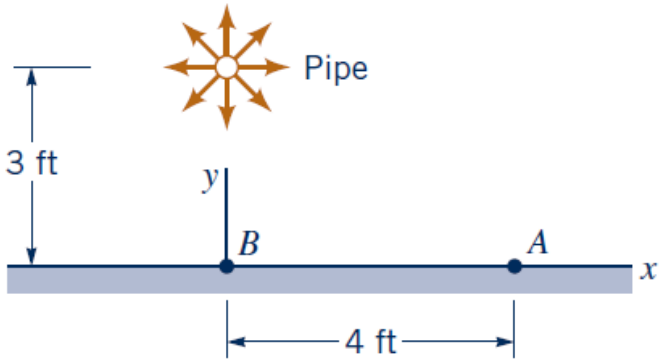


Fig. 8.

For a source,

$$\phi = \frac{m}{2\pi} \ln r = \frac{m}{4\pi} \ln r^2$$

where r is measured from the source. With the coordinate system shown in figure

$$r^2 = x^2 + (y-3)^2 \quad \text{(for upper source)}$$

and

$$r^2 = x^2 + (y+3)^2 \quad \text{(for lower source)}$$

so that for the combined sources

$$\phi = \frac{m}{4\pi} \left\{ \ln \left[x^2 + (y-3)^2 \right] + \ln \left[x^2 + (y+3)^2 \right] \right\}$$

Since,

$$u = \frac{\partial \phi}{\partial x}$$

and

$$\frac{\partial}{\partial x} \ln \left[x^2 + (y-3)^2 \right] = \frac{2x}{x^2 + (y-3)^2}$$

$$\frac{\partial}{\partial x} \ln \left[x^2 + (y+3)^2 \right] = \frac{2x}{x^2 + (y+3)^2}$$

it follows that

$$u = \frac{m}{4\pi} \left[\frac{2x}{x^2 + (y-3)^2} + \frac{2x}{x^2 + (y+3)^2} \right]$$

Along the wall, $y = 0$, $v = 0$ and therefore

$$V_w = u = \frac{m}{4\pi} \left(\frac{4x}{x^2 + 9} \right)$$

At point A, $x = 4\,ft$, and with $m = 0.5\pi \frac{ft^2}{s}$,

$$V_{wA} = \frac{0.5\pi \frac{ft^2}{s}}{4\pi} \left[\frac{4\,(4\,ft)}{(4\,ft)^2 + 9\,ft^2} \right] = \frac{2}{25} \frac{ft}{s}$$

At point B, $x = 0$, and

$$V_{wB} = 0$$

Thus, from the Bernoulli equation

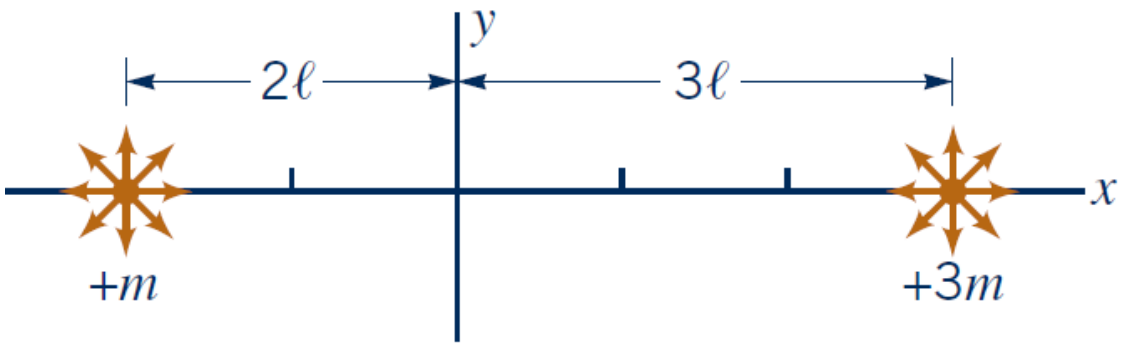
$$\frac{p_B}{\gamma} + \frac{V_{wB}^2}{2g} = \frac{p_A}{\gamma} + \frac{V_{wA}^2}{2g}$$

or

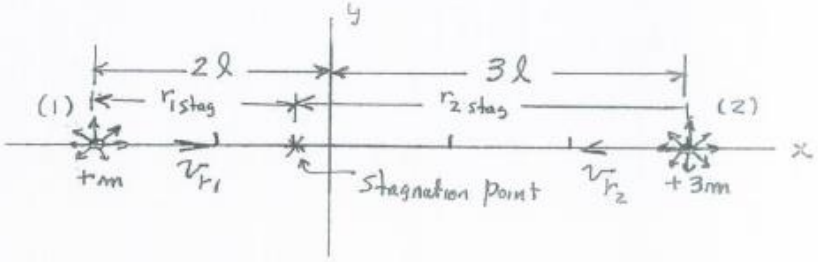
$$p_B - p_A = \frac{1}{2} \frac{\gamma}{g} V_{wA}^2$$

$$= \frac{(62.4 \frac{lb}{ft^3})}{2\,(32.2 \frac{ft}{s^2})} \left(\frac{2}{25} \frac{ft}{s} \right)^2 = \underline{0.00620\ psf}$$

18. Two sources, one of strength m and the other with strength 3m, are located on the x axis as shown in Fig. P6.57. Determine the location of the stagnation point in the flow produced by these sources.



Since the flow from each source is in the radial direction, it is only along the x-axis that the two radial components can cancel and create a stagnation point.



For source (1)

$$v_{r_1} = \frac{m}{2\pi r_1}$$

and for source (2)

$$v_{r_2} = \frac{3m}{2\pi r_2}$$

The stagnation point occurs where $v_{r_1} = v_{r_2}$ so that

$$\frac{m}{2\pi r_{1\,stag}} = \frac{3m}{2\pi r_{2\,stag}}$$

and

$$\frac{r_{2\,stag}}{r_{1\,stag}} = 3$$

Also,

$$r_{1\,stag} + r_{2\,stag} = 2\ell + 3\ell = 5\ell$$

so that

$$r_{1\,stag} + 3r_{1\,stag} = 5\ell$$

$$r_{1\,stag} = \frac{5}{4}\ell$$

Thus,

$$x_{stag} = -\left(2\ell - \frac{5}{4}\ell\right) = -0.75\ell$$