SPC 307 - Aerodynamics
Course Assignment
Due Date Monday 28 May 2018 at 11:30

1. The maximum velocity at which an aircraft can cruise occurs when the thrust available with the engines operating with the afterburner lit ("Max") equals the thrust required, which are represented by the bucket shaped curves. What is the maximum cruise velocity that a 5,000-lbf T-38A can sustain at 26,000 feet?
   As the vehicle slows down, the drag acting on the vehicle (which is equal to the thrust required to cruise at constant velocity and altitude) reaches a minimum (Dmin). The lift-to-drag ratio is, therefore, a maximum $(L/D)_{max}$. What is the maximum value of the lift-to-drag ratio $(L/D)_{max}$. for our 5,000-lbf T-38A cruising at 26,000 ft? What is the velocity at which the vehicle cruises, when the lift-to-drag ratio is a maximum? As the vehicle slows to speeds below that for $(L/D)_{min}$, which is equal to $(L/D)_{max}$, it actually requires more thrust (i. e., more power) to fly slower. You are operating the aircraft in the region of reverse command. More thrust is required to cruise at a slower speed. Eventually, one of two things happens: either the aircraft stalls (which is designated by the term "Buffet Limit" in Fig. P1.1, Berton Book) or the drag acting on the aircraft exceeds the thrust available. What is the minimum velocity at which a 5,000-lbf T-38A can cruise at 26,000 ft? Is this minimum velocity due to stall or is it due to the lack of sufficient power?

2. Consider steady two-dimensional flow about a cylinder of radius R (Fig. 1). Using cylindrical coordinates, we can express the velocity field for steady, inviscid, incompressible flow around the cylinder as

$$\vec{V}(r,\theta) = U_\infty\left(1 - \frac{R^2}{r^2}\right)\cos\theta\,\hat{e}_r - U_\infty\left(1 + \frac{R^2}{r^2}\right)\sin\theta\,\hat{e}_\theta$$

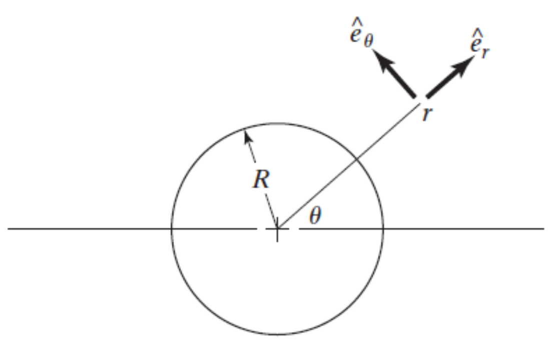

Fig. 1.

where U_∞ is the velocity of the undisturbed stream (and is, therefore, a constant). Derive the expression for the acceleration of a fluid particle at the surface of the cylinder (i.e., at points where r = R). Use equation:

$$\frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

and the definition that

$$\nabla = \hat{e}_r\frac{\partial}{\partial r} + \frac{\hat{e}_\theta}{r}\frac{\partial}{\partial \theta} + \hat{e}_z\frac{\partial}{\partial z}$$

and

$$\vec{V} = v_r\hat{e}_r + v_\theta\hat{e}_\theta + v_z\hat{e}_z$$

3. Consider steady, low-speed flow of a viscous fluid in an infinitely long, two-dimensional channel of height h (i.e., the flow is fully developed; Fig. 2). Since this is a low-speed flow, we will assume that the viscosity and the density are constant. Assume the body forces to be negligible. The upper plate (which is at y = h) moves in the x direction at the speed V_0, while the lower plate (which is at y = 0) is stationary.
(a) Develop expressions for u , v , and w (which satisfy the boundary conditions) as functions of U_0, h, μ, dp/dx, and y .
(b) Write the expression for dp/dx in terms of m, U_0, and h , if u = 0 at y = h/2.

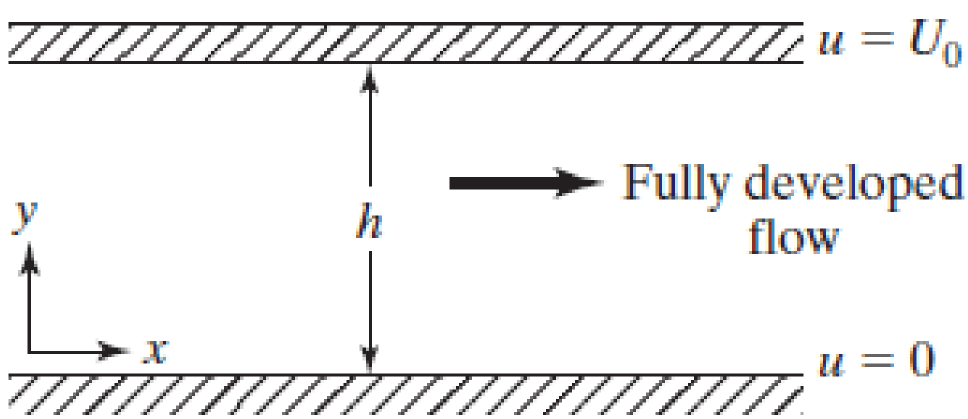

Fig. 2.

4. What are the free-stream Reynolds number and the freestream Mach number for the following flows?
    (a) A golf ball, whose characteristic length (i.e., its diameter) is 4.5 cm, moves through the standard sea level atmosphere at 60 m/s.
    (b) Boeing 747 whose characteristic length is 70.6 m flies at an altitude of 10 km. with a
    (c) speed of 250 m/s.

5. (a) An airplane has a characteristic chord length of 10.4 m. What is the free-stream Reynolds number for the Mach 3 flight at an altitude of 20 km?
   (b) What is the characteristic free-stream Reynolds number of an airplane flying 160 mi/h in a standard sea-level environment? The characteristic chord length is 4.0 ft.

6. A Pitot tube is mounted on the nose of an aircraft to measure the pressure. If the aircraft is flying at an altitude of 4500 m and the reading of Pitot tube is 60000 N>m2 (abs). What is the airspeed? (revise section 3.3 in Bertin Book)

7. Consider a low-speed, steady flow around the thin airfoil shown in Fig. 3. We know the velocity and altitude at which the vehicle is flying. Thus, we know p_∞ (i.e., p_1) and U_∞. We have obtained experimental values of the local static pressure at points 2 through 6. At which of these points can we use Bernoulli's equation to determine the local velocity? If we cannot, why not?
   Point 2: at the stagnation point of airfoil
   Point 3: at a point in the inviscid region just outside the laminar boundary layer
   Point 4: at a point in the laminar boundary layer
   Point 5: at a point in the turbulent boundary layer
   Point 6: at a point in the inviscid region just outside the turbulent boundary layer

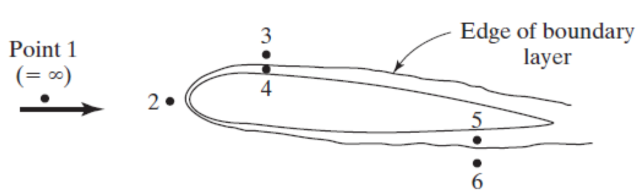

Fig. 3.

8. Assume that the airfoil of problem 7 is moving at 300 km/h at an altitude of 3 km. The experimentally determined pressure coefficients are

Point	2	3	4	5	6
C_p	1.00	−3.00	−3.00	+0.16	+0.16

   (a) What is the Mach number and the Reynolds number for this configuration? Assume that the characteristic dimension for the airfoil is 1.5 m.
   (b) Calculate the local pressure in N>m2 and in lbf>in.2 at all five points. What is the percentage change in the pressure relative to the free-stream value? That is, what is $(p_{local} - p_{\infty}) > p_{\infty}$? Was it reasonable to assume that the pressure changes are sufficiently small that the density is approximately constant?

9. An in-draft wind tunnel (Fig. 4) takes air from the quiescent atmosphere (outside the tunnel) and accelerates it in the converging section, so that the velocity of the air at a point in the test section but far from the model is 60 m/s. What is the static pressure at this point? What is the pressure at the stagnation point on a model in the test section? Use Tables at the end of this assignment to obtain the properties of the ambient air, assuming that the conditions are those for the standard atmosphere at sea level.

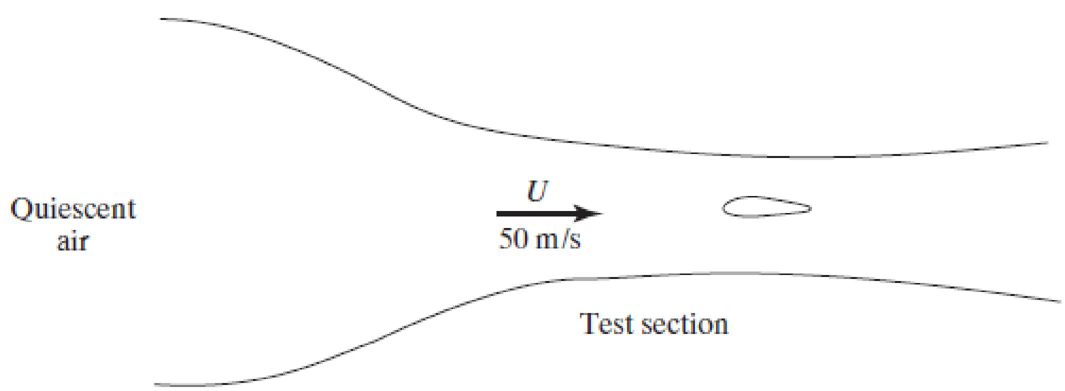

Fig. 4.

10. You are in charge of the pumping unit used to pressurize a large water tank on a fire truck. The fire that you are to extinguish is on the sixth floor of a building, 70 ft higher than the truck hose level, as shown in Fig. 5.

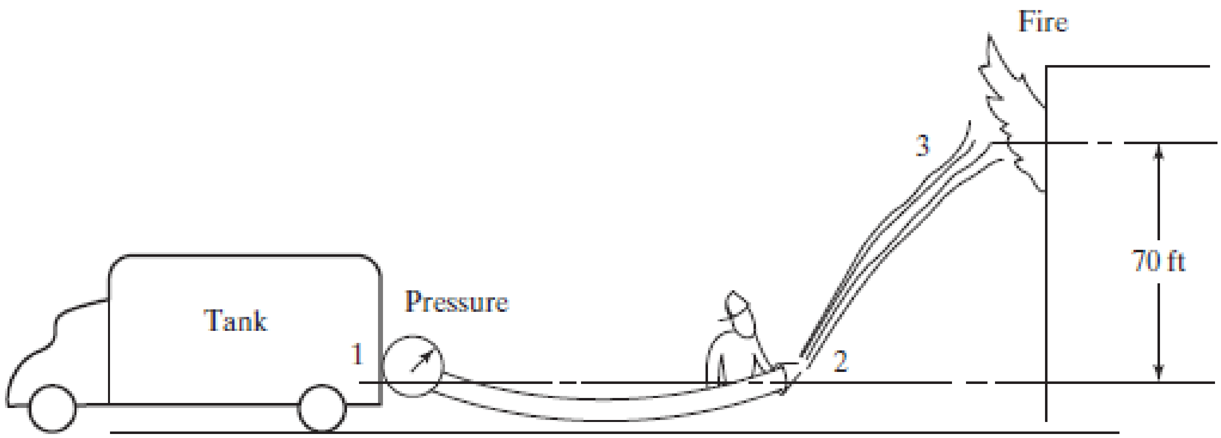

Fig. 5.

(a) What is the minimum pressure in the large tank for the water to reach the fire? Neglect pressure losses in the hose.
(b) What is the velocity of the water as it exits the hose? The diameter of the nozzle is 3.0 in. What is the flow rate in gallons per minute? Note that 1 gal/min equals 0.002228 ft^3/s.

11. (a)What conditions are necessary before you can use a stream function to solve for the flow field?
(b) What conditions are necessary before you can use a potential function to solve for the flow field?
(c) What conditions are necessary before you can apply Bernoulli's equation to relate two points in a flow field?
(d) Under what conditions does the circulation around a closed fluid line remain constant with respect to time?

12. Consider the incompressible, irrotational two-dimensional flow where the potential function is

$$\phi = K \ln\sqrt{x^2 + y^2}$$

where K is an arbitrary constant.
(a) What is the velocity field for this flow? Verify that the flow is irrotational. What is the magnitude and direction of the velocity at (2, 0), at (12, 12), and at (0, 2)?
(b) What is the stream function for this flow? Sketch the streamline pattern.
(c) Sketch the lines of constant potential. How do the lines of equipotential relate to the streamlines?

13. A two-dimensional free vortex is located near an infinite plane at a distance h above the plane (Fig. 6). The pressure at infinity is p_∞ and the velocity at infinity is U_∞ parallel to the plane. Find the total force (per unit depth normal to the paper) on the plane if the pressure on the underside of the plane is p_∞. The strength of the vortex is Γ. The fluid is incompressible and perfect. To what expression does the force simplify if h becomes very large?

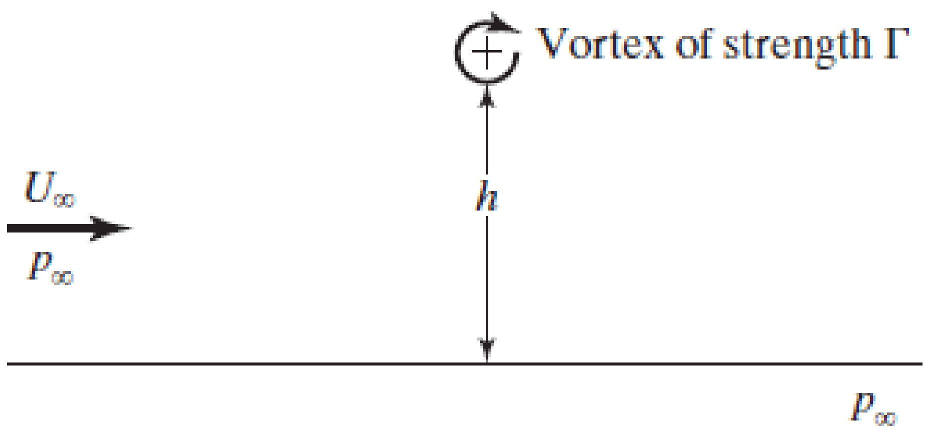

Fig. 6.

14. An infinite-span cylinder (two-dimensional) serves as a plug between the two airstreams, as shown in Fig. 7. Both air flows may be considered to be steady, inviscid, and incompressible. Neglecting the body forces in the air and the weight of the cylinder, in which direction does the plug move (i.e., due to the airflow)?

Note that the pressure variation on a cylinder in a free stream flow is given by:

$$p = p_\infty + \rho_\infty U_\infty^2/2 - 2\rho_\infty U_\infty^2 \sin^2\theta$$

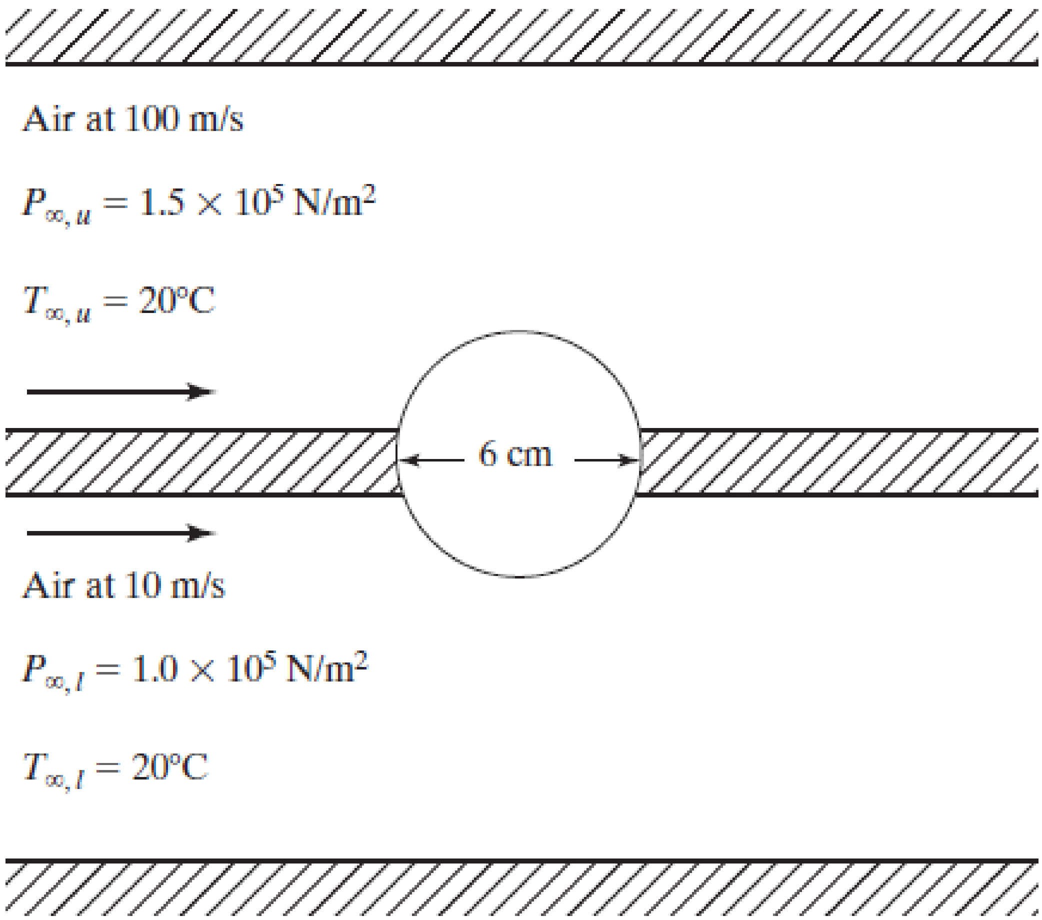

Fig. 7.

15. Consider the flow around the quonset hut shown in Fig. 8 to be represented by superimposing a uniform flow and a doublet. Assume steady, incompressible, potential flow. The ground plane is represented by the plane of symmetry and the hut by the upper half of the cylinder. The free-stream velocity is 175 km/h; the radius R_0 of the hut is 6 m. The door is not well sealed, and the static pressure inside the hut is equal to that on the outer surface of the hut, where the door is located.

(a) If the door to the hut is located at ground level (i.e., at the stagnation point), what is the net lift acting on the hut? What is the lift coefficient?

(b) Where should the door be located (i.e., at what angle θ_0 relative to the ground) so that the net force on the hut will vanish?

For both parts of the problem, the opening is very small compared to the radius R_0. Thus, the pressure on the door is essentially constant and equal to the value of the angle θ_0 at which the door is located. Assume that the wall is negligibly thin.

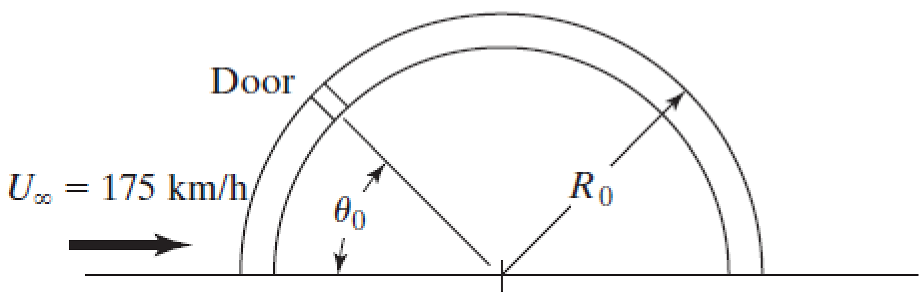

Fig. 8.

16. A very thin, "flat-plate" wing of a model airplane moves through the air at standard sea level conditions at a velocity of 10 m/s. The dimensions of the plate are such that its chord (stream wise dimension) is 0.25 m and its span (length perpendicular to the flow direction) is 4 m. What is the Reynolds number at the trailing edge (x = 0.25 m)? Assume that the boundary layer is laminar in answering the remaining questions. What are the boundary layer thickness and the displacement thickness at the trailing edge? What are the local shear at the wall and the skin-friction coefficient at x = 0.25 m. Calculate the total drag on the wing (both sides). Prepare a graph of u_x as a function of y, where u_x designates the x component of velocity relative to a point on the ground, at x = 0.25.

17. A flat plate at zero angle of attack is mounted in a wind tunnel where

$$p_\infty = 1.01325 * 105\ \mathrm{N/m^2} \qquad U_\infty = 100\ \mathrm{m/s}$$

$$\mu_\infty = 1.7894 * 10^{-5}\ \mathrm{kg/m.s} \qquad \rho_\infty = 1.2250\ \mathrm{kg/m^3}$$

A Pitot probe is to be used to determine the velocity profile at a station 1.0 m from the leading edge (Fig. 9).

(a) Using a transition criterion that $Re_{x,tr}$ = 500, 000, where does transition occur?

(b) Use equation

$$\frac{\delta}{x} = \frac{0.3747}{(\mathrm{Re}_x)^{0.2}} \qquad \frac{\delta^*}{x} = \frac{0.0468}{(\mathrm{Re}_x)^{0.2}} \qquad \frac{\theta}{x} = \frac{0.0364}{(\mathrm{Re}_x)^{0.2}}$$

to calculate the thickness of the turbulent boundary layer at a point 1.00 m from the leading edge.

(c) If the streamwise velocity varies as the 1/7th power law [i.e., $u/ue = (y/\delta)^{1/7}$], calculate the pressure you should expect to measure with the Pitot probe $p_t(y)$ as a function of y . Present the predicted values as

(1) The difference between that sensed by the Pitot probe and that sensed by the static port in the wall [i.e., y versus $p_t(y) - p_{static}$]

(2) The pressure coefficient

$$y \text{ versus } C_p(y) = \frac{p_t(y) - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

Note that for part (c) we can use Bernoulli's equation to relate the static pressure and the velocity on the streamline just ahead of the probe and the stagnation pressure sensed by the probe. Even though this is in the boundary layer, we can use Bernoulli's equation, since we relate properties on a streamline and since we calculate these properties at "point." Thus, the flow slows down isentropically to zero velocity over a very short distance at the mouth of the probe.

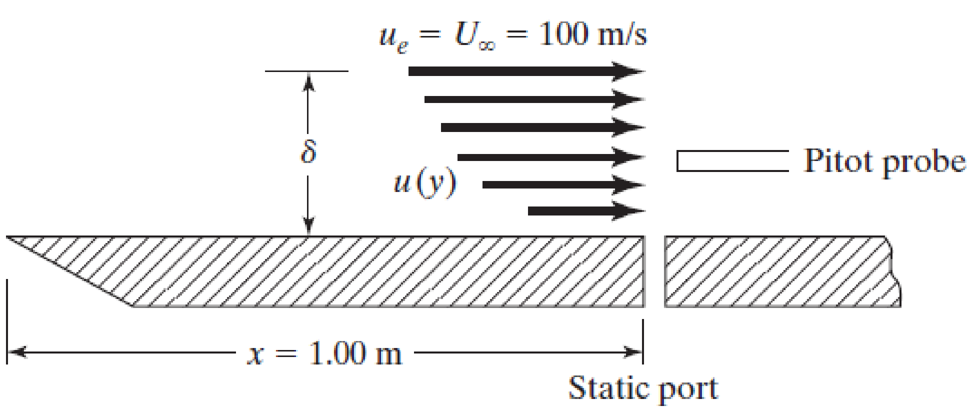

Fig. 9.

18. The NACA 25012 wing section airfoil has a mean camber line given by

$z/c = 0.5383(x/c)^3 - 0.6315(x/c)^2 + 0.2147(x/c)$ for the region $0.0 \le x/c \le 0.391$

$z/c = 0.0322(1 - x/c)$ for the region $0.391 \le x/c \le 1.0$

Using thin airfoil theory, when the angle of attack $\alpha = zero\ °$, calculate the zero-lift angle of attack, α_o, and the pitching moment coefficient at the leading edge, $(C_M)_{LE}$.

When the angle of attack $\alpha = 10°$, find the lift coefficient C_L, the pitching moment coefficient at the leading edge $(C_M)_{LE}$

Geometric Altitude (km)	Pressure (N/m^2)	Temperature (K)	Density (kg/m^3)	Viscosity ($kg/m \cdot s$)	Speed of Sound (m/s)
0	1.0133 E + 05	288.150	1.2250 E + 00	1.7894 E − 05	340.29
1	8.9875 E + 04	281.651	1.1117 E + 00	1.7579 E − 05	336.43
2	7.9501 E + 04	275.154	1.0066 E + 00	1.7260 E − 05	332.53
3	7.0121 E + 04	268.659	9.0926 E − 01	1.6938 E − 05	328.58
4	6.1669 E + 04	262.166	8.1934 E − 01	1.6612 E − 05	324.59
5	5.4048 E + 04	255.676	7.3643 E − 01	1.7885 E − 05	320.55
6	4.7217 E + 04	249.187	6.6012 E − 01	1.5949 E − 05	316.45
7	4.1105 E + 04	242.700	5.9002 E − 01	1.5612 E − 05	312.31
8	3.5651 E + 04	236.215	5.2578 E − 01	1.5271 E − 05	308.11
9	3.0800 E + 04	229.733	4.6707 E − 01	1.4926 E − 05	303.85
10	2.6500 E + 04	223.252	4.1351 E − 01	1.4577 E − 05	299.53
11	2.2700 E + 04	216.774	3.6481 E − 01	1.4223 E − 05	295.15
12	1.9399 E + 04	216.650	3.1193 E − 01	1.4216 E − 05	295.07
13	1.6579 E + 04	216.650	2.6660 E − 01	1.4216 E − 05	295.07
14	1.4170 E + 04	216.650	2.2786 E − 01	1.4216 E − 05	295.07
15	1.2111 E + 04	216.650	1.9475 E − 01	1.4216 E − 05	295.07
16	1.0352 E + 04	216.650	1.6647 E − 01	1.4216 E − 05	295.07
17	8.8497 E + 03	216.650	1.4230 E − 01	1.4216 E − 05	295.07
18	7.5652 E + 03	216.650	1.2165 E − 01	1.4216 E − 05	295.07
19	6.4675 E + 03	216.650	1.0400 E − 01	1.4216 E − 05	295.07
20	5.5293 E + 03	216.650	8.8911 E − 02	1.4216 E − 05	295.07
21	4.7289 E + 03	217.581	7.5715 E − 02	1.4267 E − 05	295.70
22	4.0474 E + 03	218.574	6.4510 E − 02	1.4322 E − 05	296.38
23	3.4668 E + 03	219.567	5.5006 E − 02	1.4376 E − 05	297.05
24	2.9717 E + 03	220.560	4.6938 E − 02	1.4430 E − 05	297.72
25	2.5491 E + 03	221.552	4.0084 E − 02	1.4484 E − 05	298.39
26	2.1883 E + 03	222.544	3.4257 E − 02	1.4538 E − 05	299.06
27	1.8799 E + 03	223.536	2.9298 E − 02	1.4592 E − 05	299.72
28	1.6161 E + 03	224.527	2.5076 E − 02	1.4646 E − 05	300.39
29	1.3904 E + 03	225.518	2.1478 E − 02	1.4699 E − 05	301.05
30	1.1970 E + 03	226.509	1.8411 E − 02	1.4753 E − 05	301.71