Machine DESIGN II

Lecture 3

Brakes

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Internal Contracting Shoe Brake Equations :

$$M_N = \left(\frac{p_{max.}brc}{4\sin\theta_{max}}\right)\{2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)\} \quad (1)$$

$$M_f = \left(\frac{fp_{max.}br}{4\sin\theta_{max}}\right)\{4r(\cos\theta_2 - \cos\theta_1) - c(\cos 2\theta_2 - \cos 2\theta_1)\} \quad (2)$$

$$T = \left(\frac{fp_{max.}br^2}{\sin\theta_{max}}\right)\{\cos\theta_1 - \cos\theta_2\} \quad (3)$$

- For self-Energized,

$$F = \frac{M_N - M_f}{a} \quad (4)$$

- For Non self-Energized.,

$$F = \frac{M_N + M_f}{a} \quad (5)$$

INTERNAL CONTRACTING SHOE BRAKE EXAMPLE:

Fig. shows a 400-mm-diameter brake drum with four internally expanding shoes. Each of the hinge pins A and B supports a pair of shoes. The actuating mechanism is to be arranged to produce the same force F on each shoe. The face width of the shoes is 75 mm. The material used permits a coefficient of friction of 0.24 and a maximum pressure of 1000 kPa.

Determine the actuating force.

Estimate the brake capacity.

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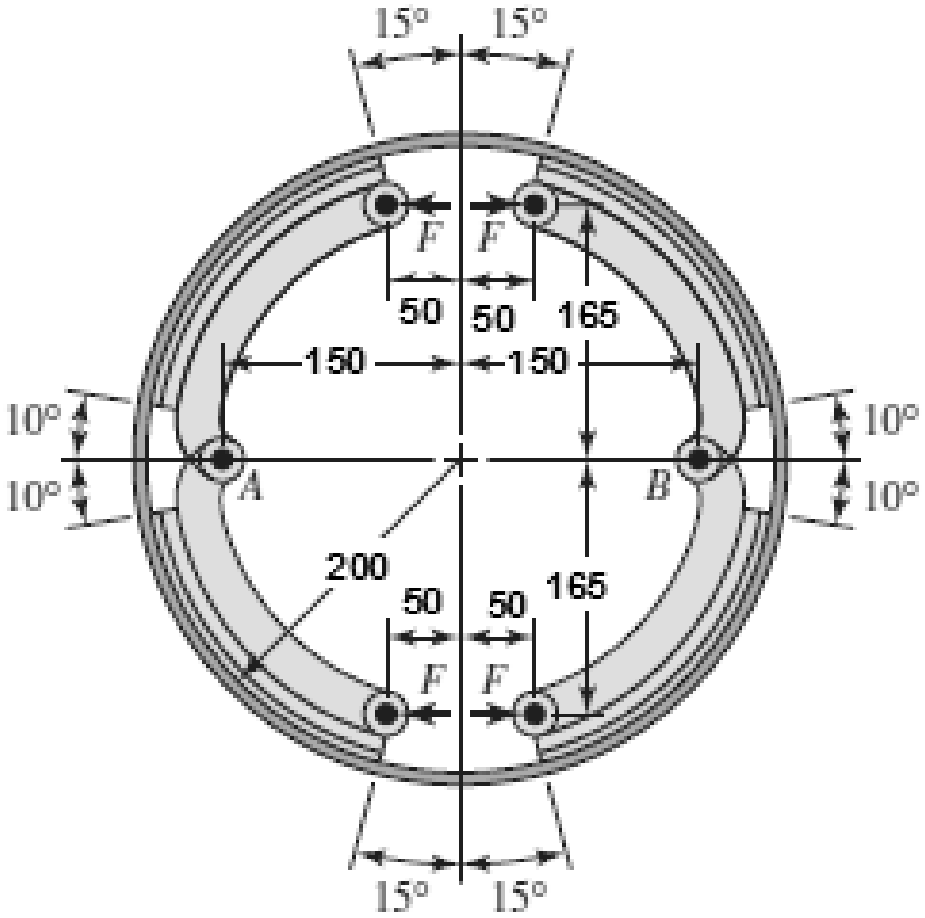

INTERNAL CONTRACTING SHOE BRAKE EXAMPLE:

The brake shown in Fig. 8.7 is 350 mm in diameter and is actuated by a mechanism that exerts two different forces F on each shoe. The shoes are identical and have a face width of 45 mm. The lining is a molded asbestos having a coefficient of friction of 0.35 and a pressure limitation of 0.85 MPa. Estimate the maximum

(a) Actuating forces to have the same braking capacity from both shoes.

(b) Braking capacity.

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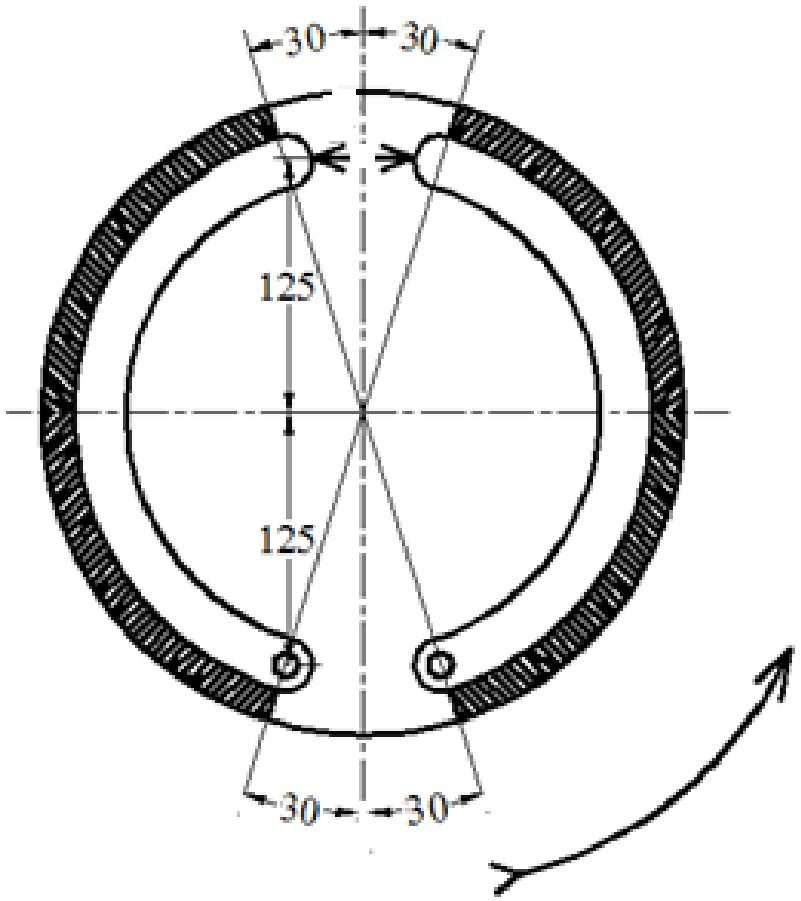
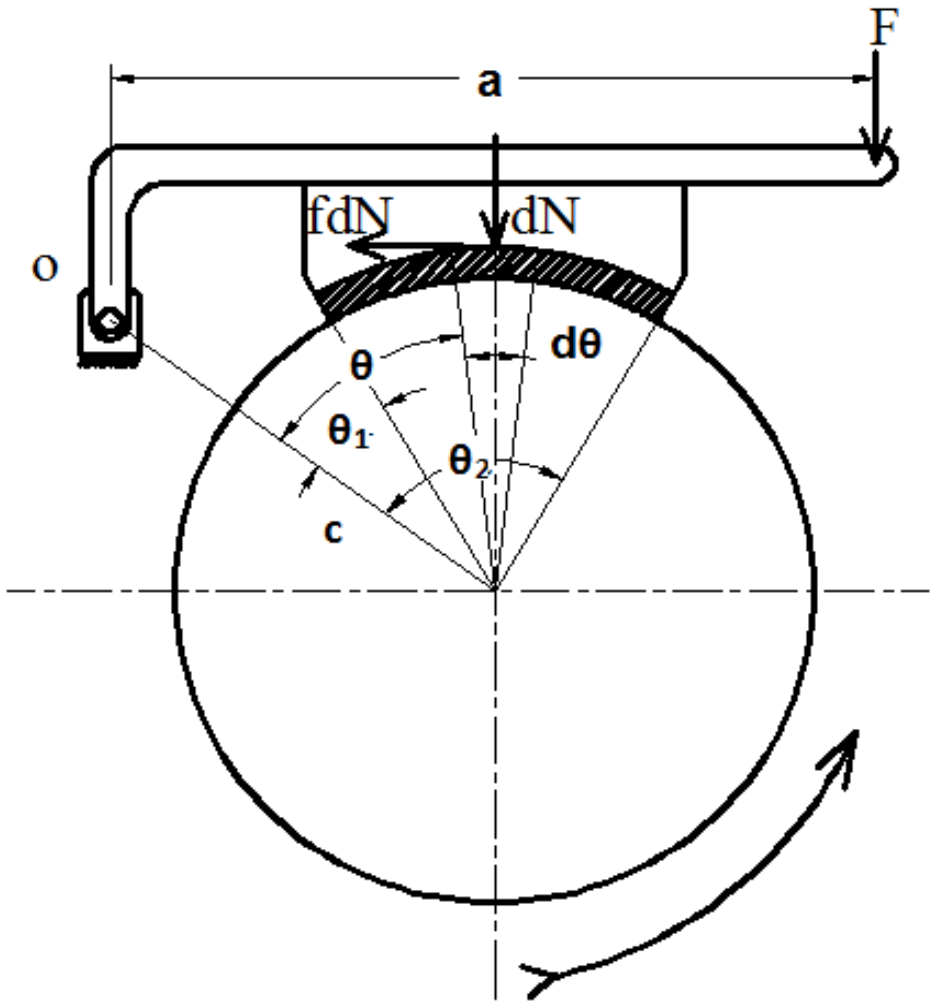


EXTERNAL CONTRACTING SHOE BRAKE:

The same analysis can be extended to a drum brake with external contracting type of shoes, typically used in elevators and cranes.



The moments of the normal force M_N and M_f about the hinge pin are the same as for the internal shoe brake.

$$\mathbf{M_N} = \left(\frac{\mathbf{p_{max.}brc}}{\mathbf{4\sin\theta_{max}}}\right)\{\mathbf{2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)}\})$$

The moment (M_f) of the frictional force(fN) about the hinge pin at A as shown in figure:

$$M_f = \int fdN(c\cos\theta - r) = \left(\frac{fp_{max.}br}{\sin\theta_{max}}\right)\int_{\theta_1}^{\theta_2} \sin\theta(c\cos\theta - r)\, d\theta$$

$$\therefore\ \mathbf{M_f} = \left(\frac{\mathbf{fp_{max.}br}}{4\,\mathbf{\sin\theta_{max}}}\right)\{4\mathbf{r(\cos\theta_1 - \cos\theta_2) + c(\cos 2\theta_2 - \cos 2\theta_1)}\}$$

The torque applied to the drum by the brake shoe is

$$\mathbf{T} = \left(\frac{\mathbf{f p_{max.} b r^2}}{\mathbf{\sin \theta_{max}}}\right)\{\mathbf{\cos\theta_1 - \cos\theta_2}\}$$

The actuating force F is

$$\mathrm{F} = \frac{\mathrm{M_N} \pm \mathrm{M_f}}{\mathrm{a}}$$

The sign depends upon the direction of drum rotation,

(- sign for self-energizing and + sign for non self-energizing shoe)

External Contracting Shoe Brake Equations :

$$M_N = \left(\frac{p_{max.}brc}{4\sin\theta_{max}}\right)\{2(\theta_2 - \theta_1) - (\sin 2\theta_2 - \sin 2\theta_1)\} \quad (1)$$

$$M_f = \left(\frac{fp_{max.}br}{4\sin\theta_{max}}\right)\{4r(\cos\theta_2 - \cos\theta_1) - c(\cos 2\theta_2 - \cos 2\theta_1)\} \quad (2)$$

$$T = \left(\frac{fp_{max.}br^2}{\sin\theta_{max}}\right)\{\cos\theta_1 - \cos\theta_2\} \quad (3)$$

- For self-Energized,

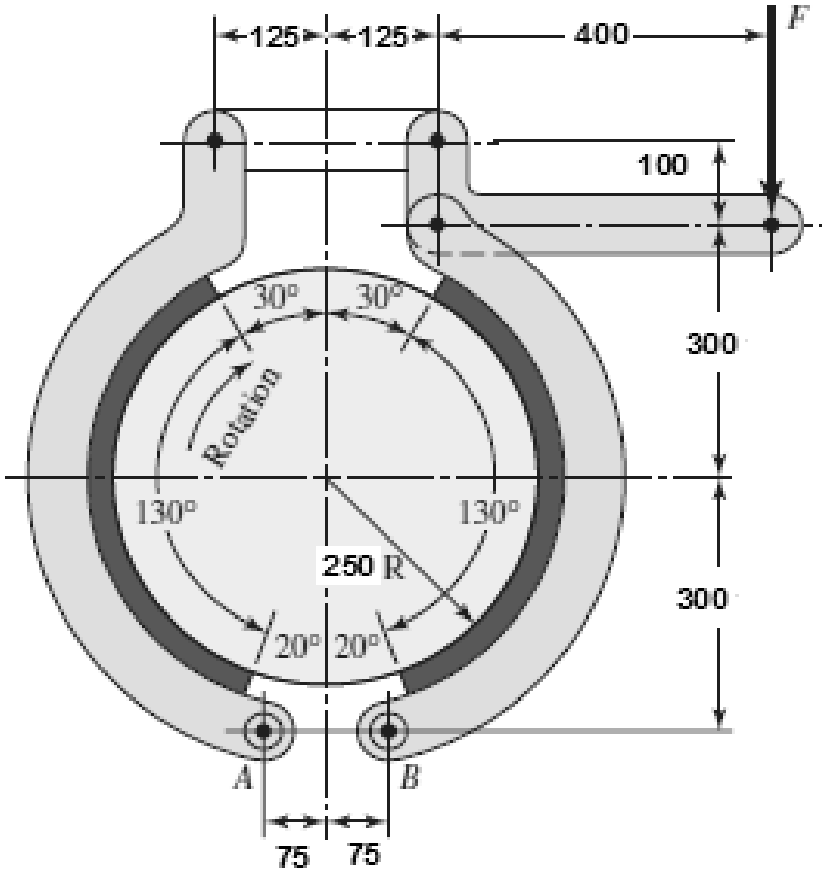
$$F = \frac{M_N - M_f}{a} \quad (4)$$

- For Non self-Energized.,

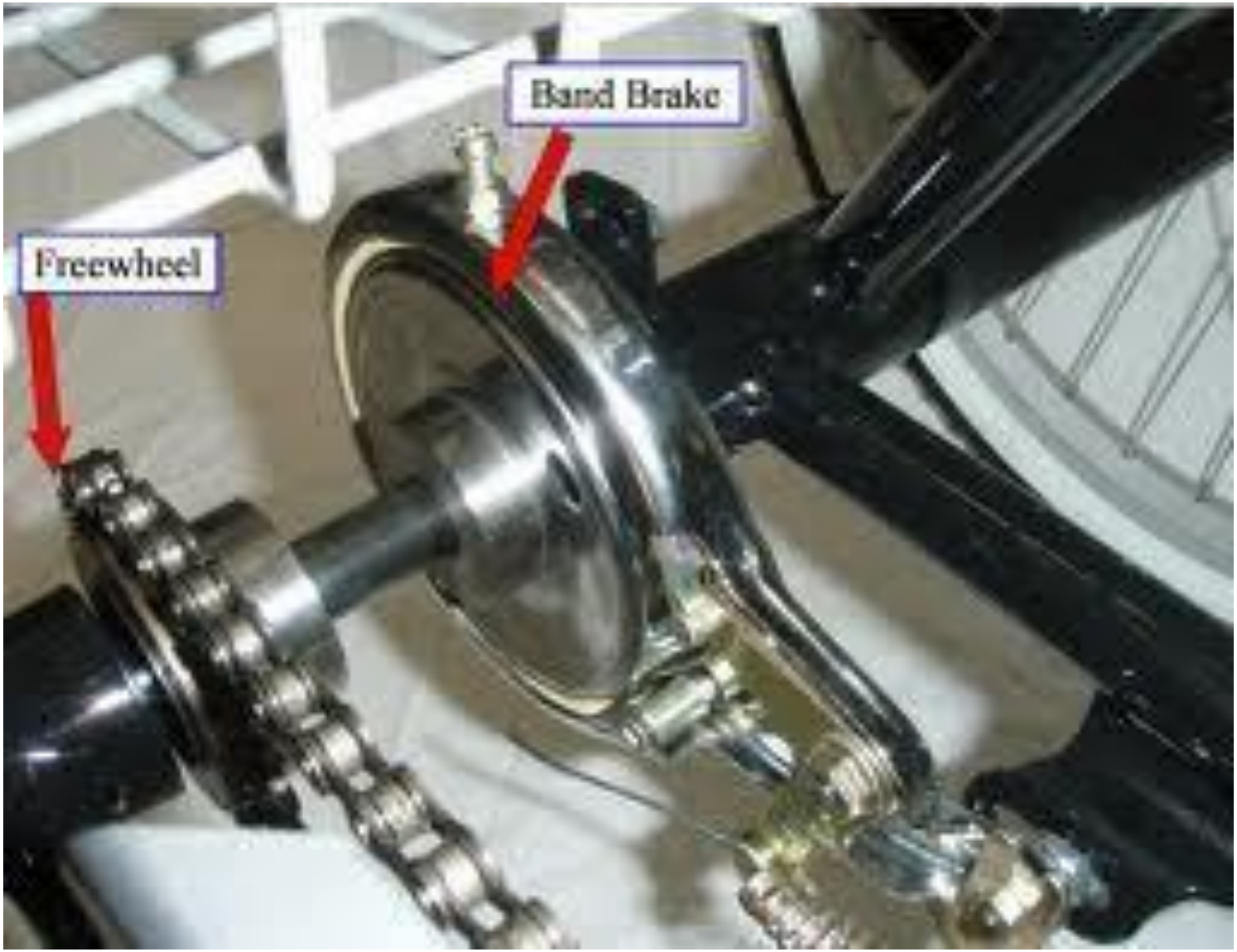
$$F = \frac{M_N + M_f}{a} \quad (5)$$

EXTERNAL CONTRACTING SHOE BRAKE EXAMPLE 1:

The brake shown in Fig. has a coefficient of friction of 0.30, a face width of 50 mm, and a limiting shoe lining pressure of 1000 KPa. Find the limiting actuating force F and the torque capacity.



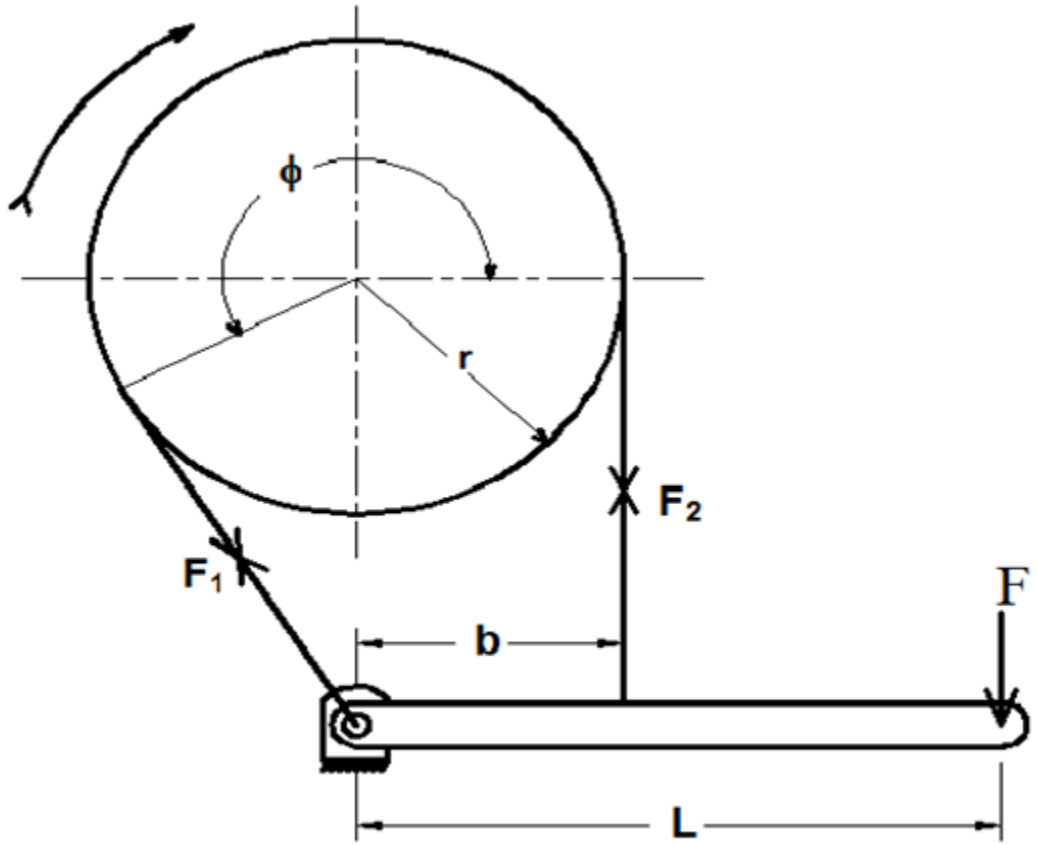
BAND BRAKES


Band Brake
Freewheel


01/20/2008

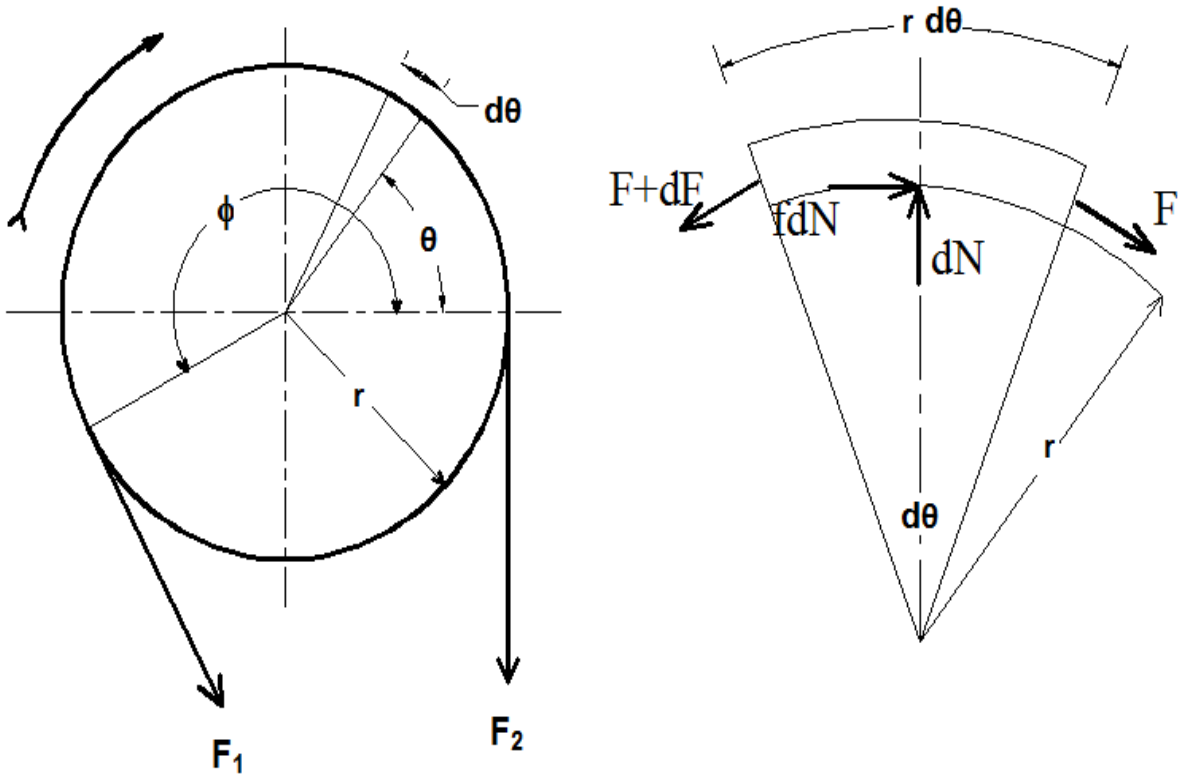
BAND BRAKES

This is a very simple type of brake using the principle that a band is wrapped part round a rotating drum. Tension can be applied to the band using a lever. The analysis follows the notation of figure.



BAND BRAKES

This is a very simple type of brake using the principle that a band is wrapped part round a rotating drum. Tension can be applied to the band using a lever. The analysis follows the notation of figure.



The tension force F_2 is less than the tension F_1 because of friction and the rotation of the drum. Any element of the band, of angular length $d\theta$, will be in equilibrium under the action of the forces shown in the figure. Summing these forces in the vertical direction,

$$(F + dF)\sin\frac{d\theta}{2} + F\sin\frac{d\theta}{2} - dN = 0$$

Ignoring $(d\theta . dF)$ term & for small angle $\sin\frac{d\theta}{2} \rightarrow \frac{d\theta}{2}$

Therefore,

$$dN = Fd\theta$$

Summing the forces in the horizontal direction gives

$$(F + dF)\cos\frac{d\theta}{2} - F\cos\frac{d\theta}{2} - fdN = 0$$

For small angles, $\cos\frac{d\theta}{2} \rightarrow 1$

$$dF = fdN$$

Substituting the value of dN and integrating give

$$\int_{F_2}^{F_1} \frac{dF}{F} = f \int_0^{\theta} d\theta$$

Or,

$$\frac{F_1}{F_2} = e^{f\phi}$$

Where,

F_1 = Tension in the tight side

F_2= Tension in the slack side

f = Coefficient of Friction.

ϕ = Band lap angle (rad)

The braking capacity T (torque) can be obtained from the equation,

$$\mathbf{T} = (\mathbf{F_1} - \mathbf{F_2}) * \mathbf{r}$$

r : is the radius of the drum.

Since,

The normal force acting on an element band is,

$$dN = P * \text{area} = P\, w\, r\, d\theta$$

where,

P = Contact pressure between band and drum

$$\therefore P\, w\, r\, d\theta = F\, d\theta$$

Or,

$$F = P\, w\, r$$

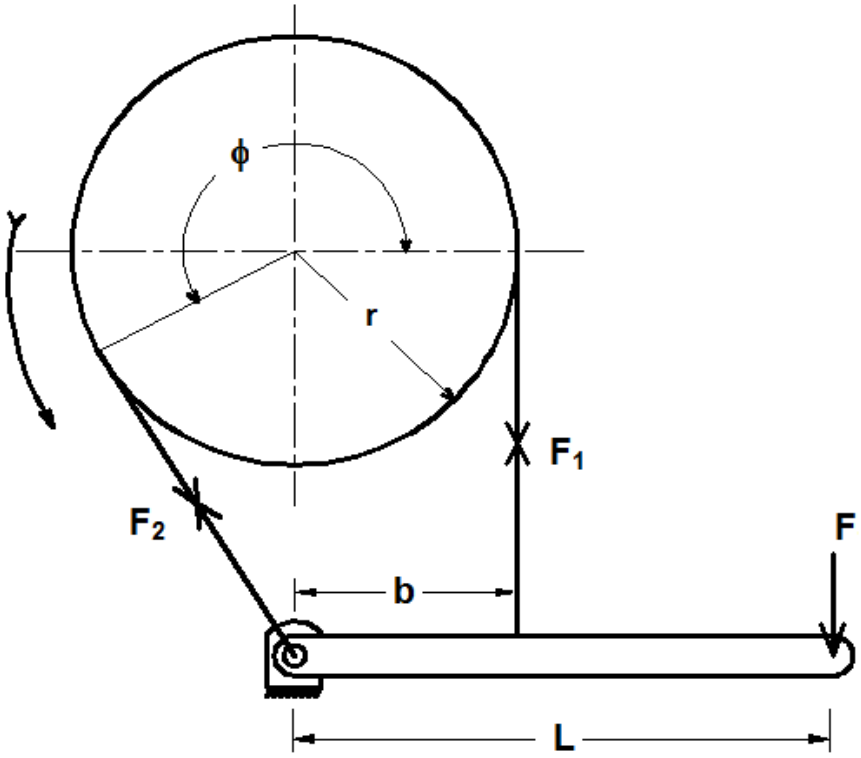
So, the maximum tension in a band is provided by,

$$\mathbf{F_{max.}} = \mathbf{P_{max.}}\, \mathbf{w}\, \mathbf{r}$$

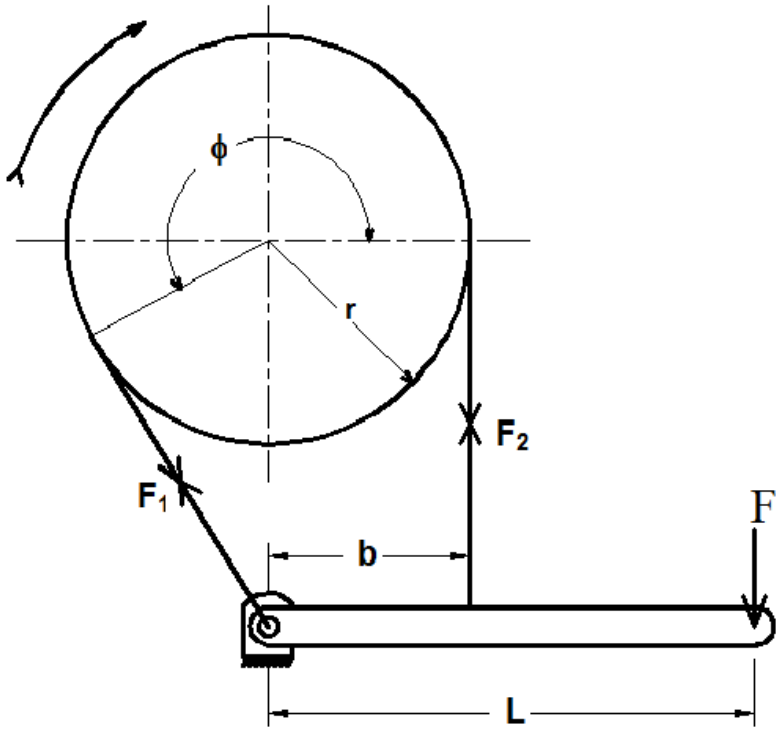
Simple band brake:

Figure shows the arrangement of the simple band brake.

In this arrangement, one end of the band is attached to the pivot and the other to the brake lever



(a)



(b)

For anticlockwise rotation of the brake drum, Fig. a, the braking lever applies pull to the tight side of the band and the equilibrium equation, taking the

moments of forces about the pivot, is

$$F * L = F_1 * b$$

and the braking torque is

$$T = (F_1 - F_2)r = F_1\left(1 - \frac{F_2}{F_1}\right)r$$

$$\therefore T = \frac{F * L * r}{b}\left(\frac{e^{f\emptyset} - 1}{e^{f\emptyset}}\right)$$

For clockwise rotation, Fig. b, the equilibrium equation, lever applies tension to the slack side, is

$$\mathrm{F} * \mathrm{L} = \mathrm{F}_2 * \mathrm{b}$$

Similarly, the braking torque is

$$T = \frac{FLr}{b}\left(e^{f\emptyset} - 1\right)$$

Comparing, it can be seen that when the lever pulls the slack side of the band, the braking torque is $\mathbf{e}^{\mathbf{f}\emptyset}$ times as great as when it is so arranged that the brake lever applies the pull to the tension side of the band.

Band Brake Equations :

$$\frac{F_1}{F_2} = e^{f\emptyset}$$

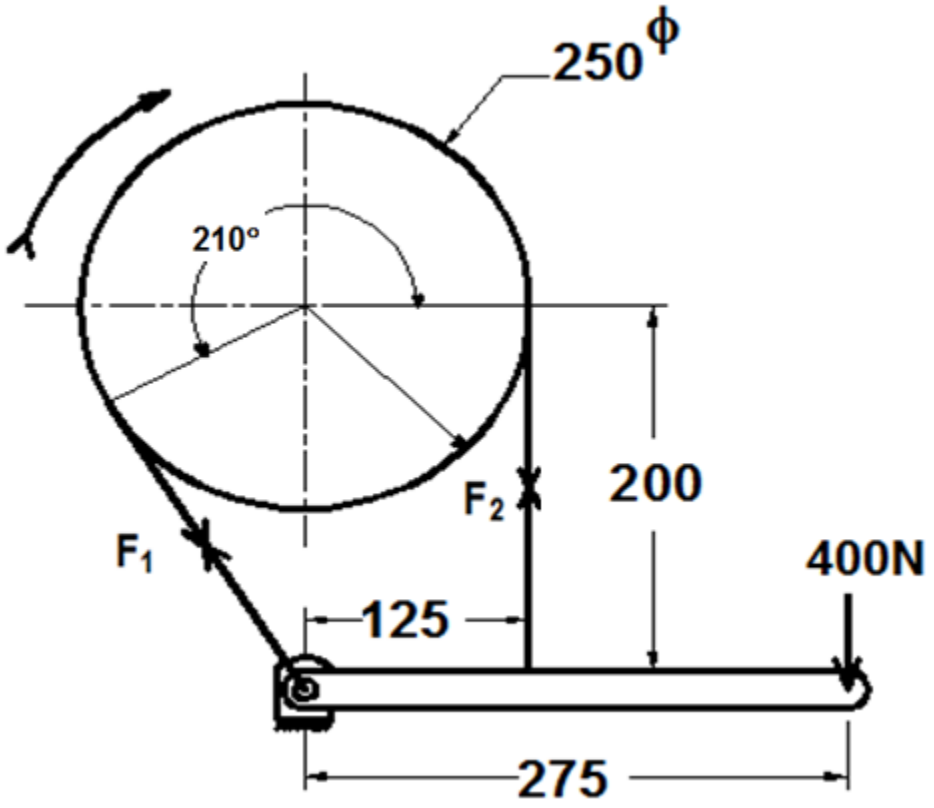
$$T = (F_1 - F_2) \times r$$

$$\sum M_{Hinge} = zero$$

$$F_{max} = F_1 = P_{max} \times b \times r$$

Band Brake Example 1:

The brake shown in the Fig. has a coefficient of friction of 0.30. If the band width is 50 mm, find the band tensions and the braking torque.



Solution:

f = 0.3, b = 50 mm, ∅=210*π/180 = 3.66

$$\frac{F_1}{F_2} = e^{f\emptyset} = e^{0.3*3.66} = 2.98$$

$$\sum M_{Hinge} = zero$$

$$F_2 \times 125 = 400 \times 275$$

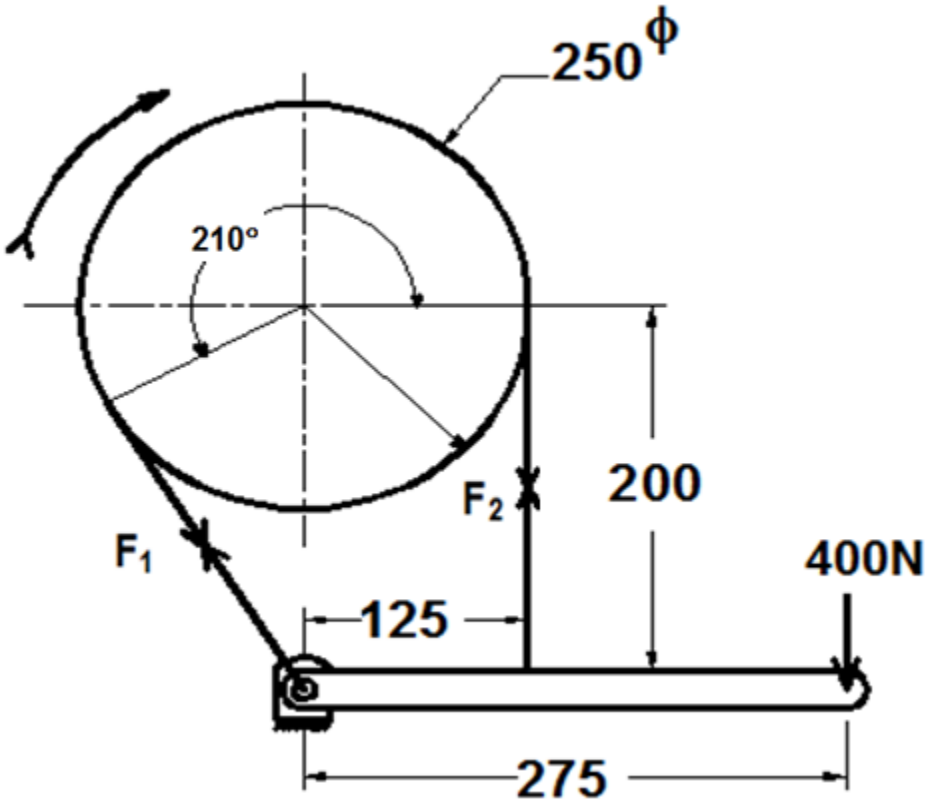
$$F_2 = 880\ N$$

$$\therefore F_1 = 2.98 \times 880 = 2622.4\ N$$

$$T = (F_1 - F_2) \times r$$

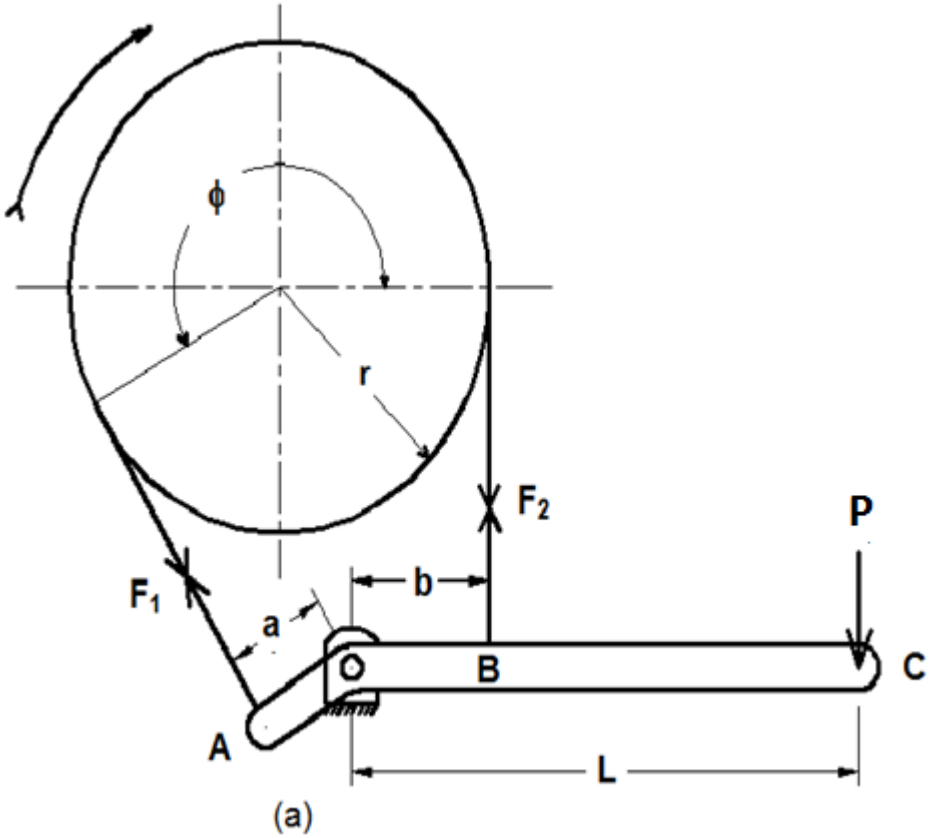
$$T = (2622.4 - 880) \times 125$$

$$T = 217.8\ Nm$$

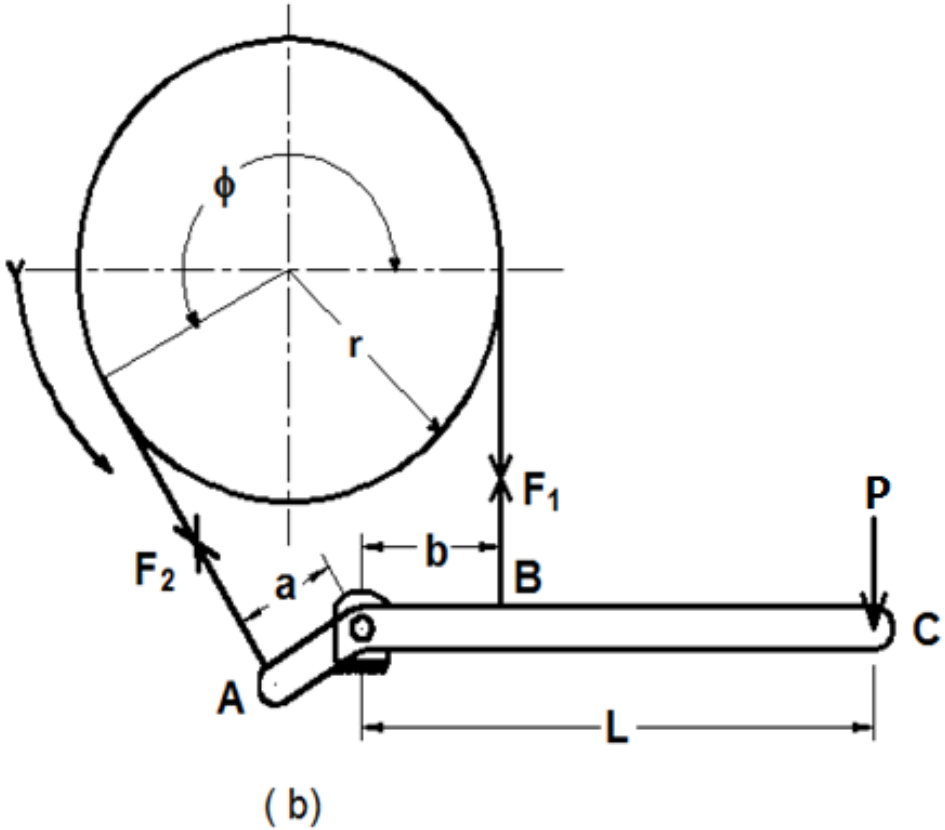


Differential band brake:

The ends of the band are attached to the brake lever, as shown in fig., such that the movement of the lever tightens the band around the drum.



(a)



(b)

For $\mathbf{a = b}$, there will be no braking, as the displacement of both C and B points will be equal.

For $\mathbf{a > b}$, the effort F will have to be applied upwards.

For $\mathbf{a < b}$, the effort F will have to be applied downwards.

If the drum is rotating anticlockwise, with $\mathrm{a} < b$, as shown in fig.a, the moments about the pivot is

$$\mathrm{P} * \mathrm{L} = \mathrm{F}_2 * \mathrm{b} - \mathrm{F}_1 * \mathrm{a}$$

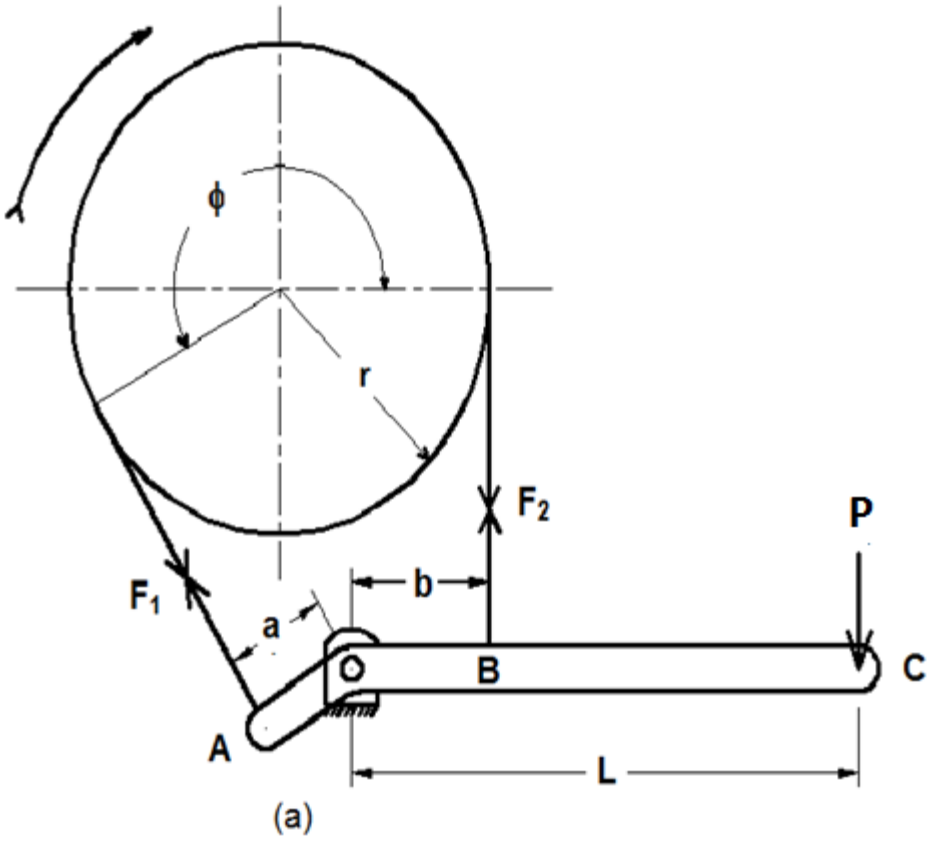
In this equation, if $(\mathrm{F}_1 * \mathrm{a} \geq \mathrm{F}_2 * \mathrm{b})$, the equation gives negative value of F and the brake is said to be **self-locking.**

For the clockwise rotation of drum, with $\mathrm{a} < b$, as shown in Fig.b. Taking moments about the pivot we get

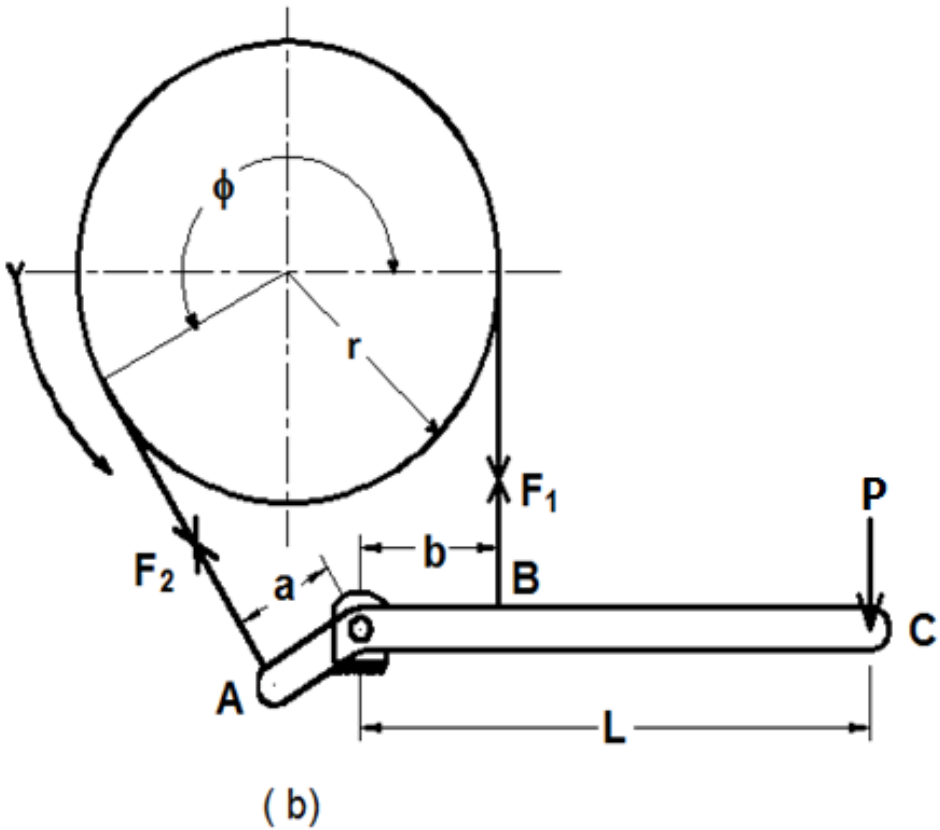
$$\mathrm{P} * \mathrm{L} = \mathrm{F}_1 * \mathrm{b} - \mathrm{F}_2 * \mathrm{a}$$

As b is greater than a and F_1 is always greater than F_2, the brake cannot be self-locking in this direction or rotation.

OB must be greater than OA:

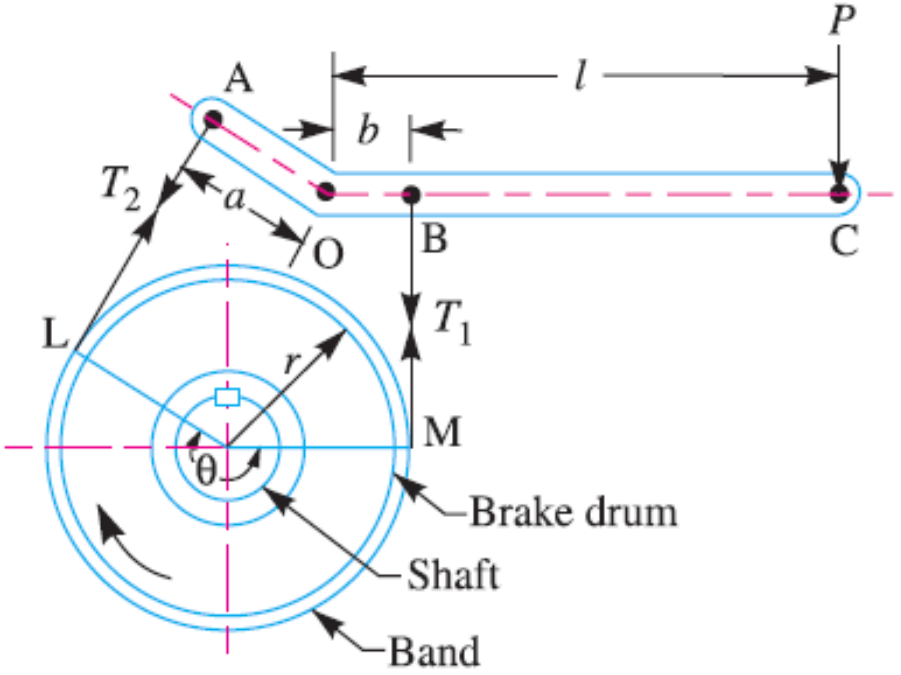


(a)

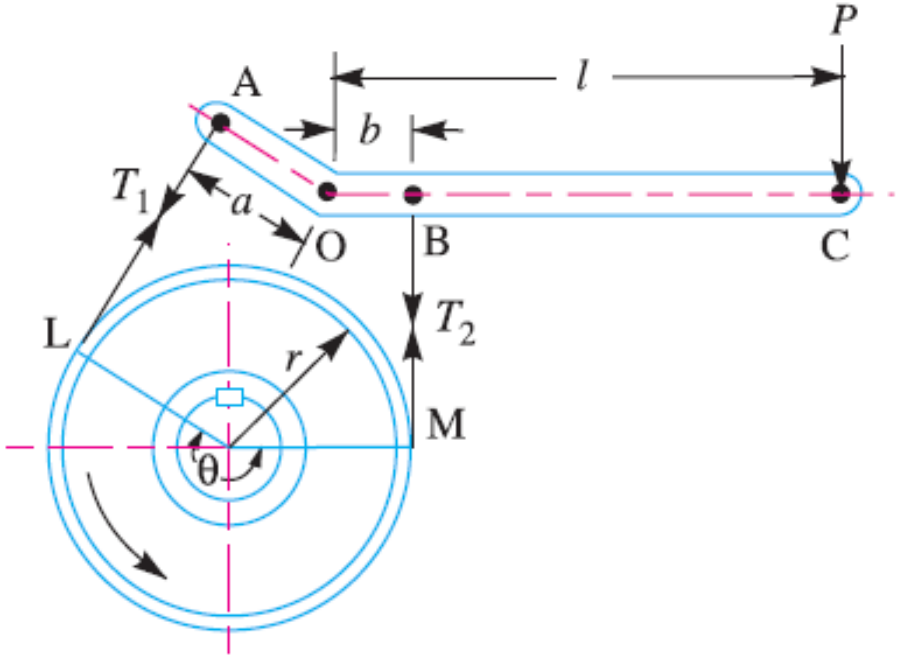


(b)

OA must be greater than OB:



(*a*) Clockwise rotation of the drum.



(*b*) Anticlockwise rotation of the drum.

$$P.l + T_1.b = T_2.a$$

$$P.l = T_2.a - T_1.b$$

$$P.l + T_2.b = T_1.a$$

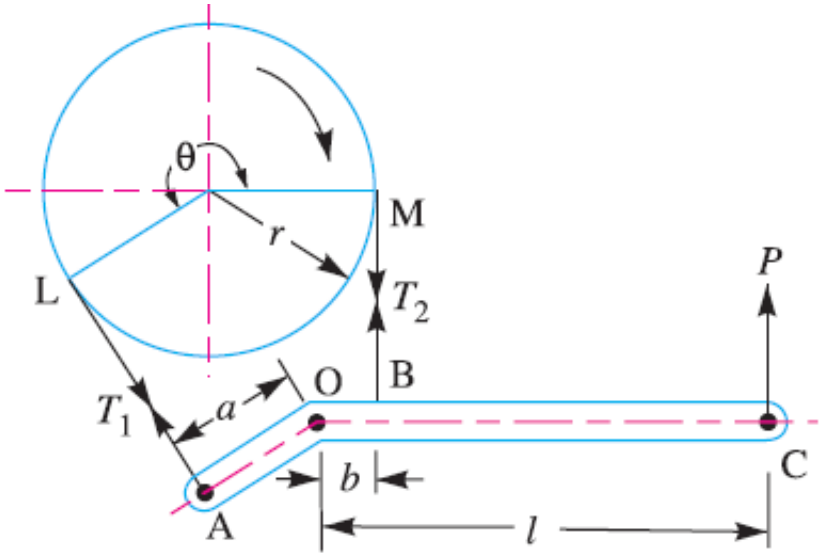
$$P.l = T_1.a - T_2.b$$

Self- Locking Condition

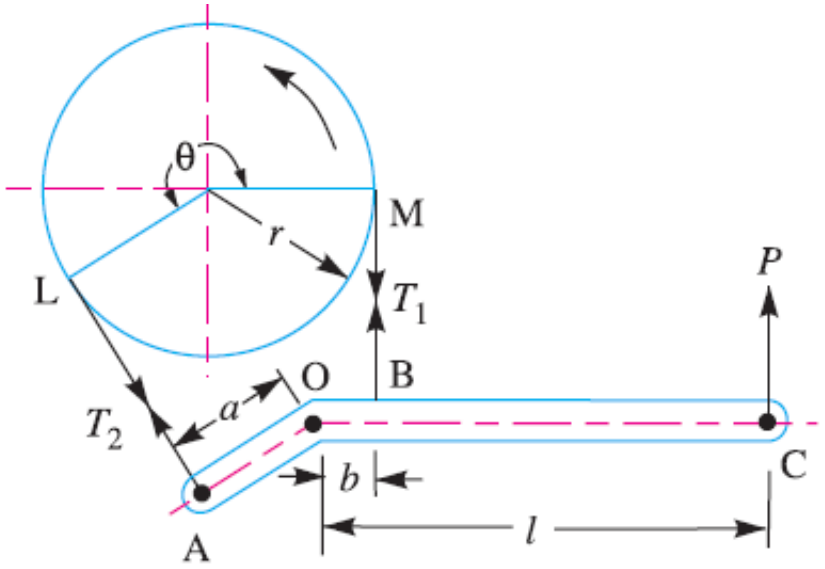
$T_1.b \geq T_2.a$ or $T_1/T_2 \geq a/b$

$T_2.b \geq T_1.a$ or $T_2/T_1 \geq a/b$

OA must be greater than OB:



(*a*) Clockwise rotation of the drum.

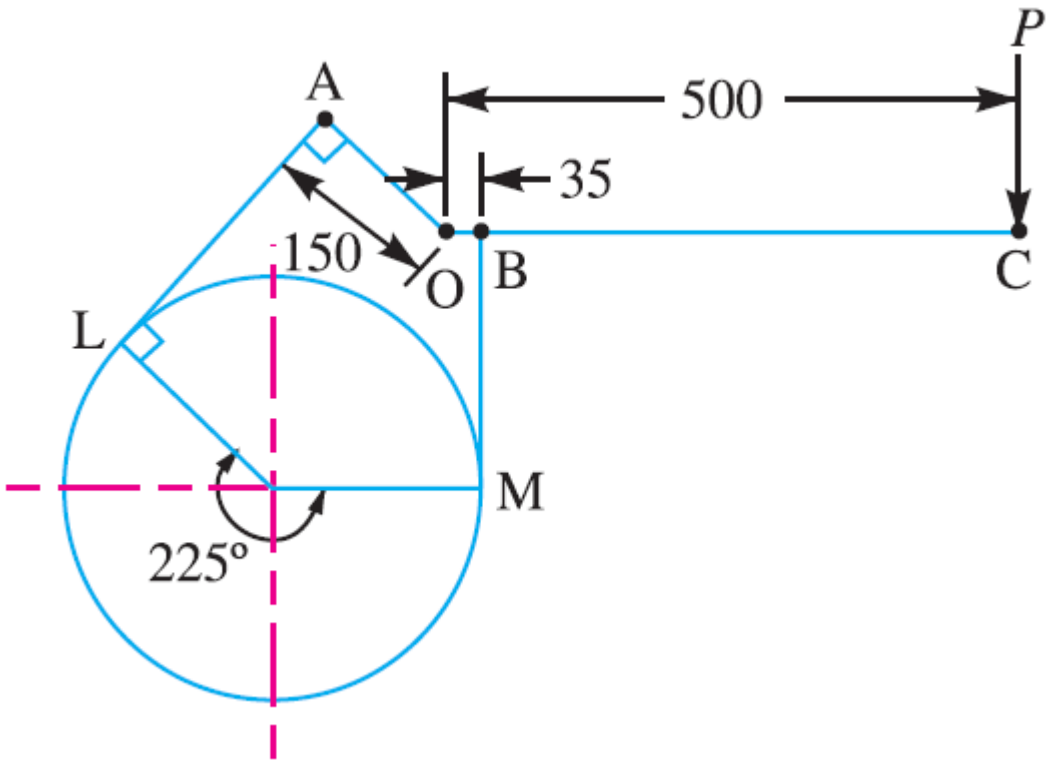


(*b*) Anticlockwise rotation of the drum.

Band Brake Example 2:

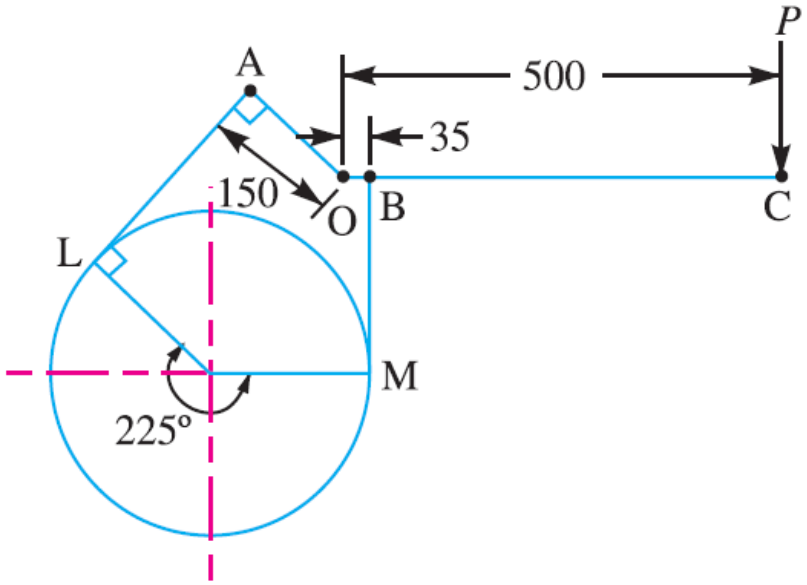
A differential band brake, as shown in Fig., has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find :

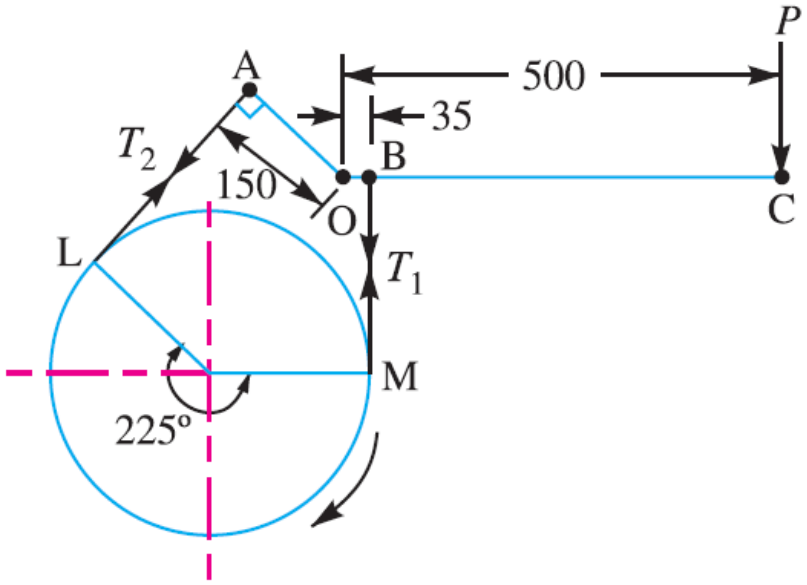
1. The necessary force (P) for the clockwise and anticlockwise rotation of the drum;
2. The value of 'OA' for the brake to be self locking, when the drum rotates clockwise



Solution:

Given : θ = 225° = 225 × π / 180 = 3.93 rad ; d = 350 mm or r = 175 mm ;
T = 350 N-m = 350 × 10^3 N-mm ; f = 0.3





Solution:

Necessary force (P) for the clockwise and anticlockwise rotation of the drum

When the drum rotates in the clockwise direction, the end of the band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. First of all, let us find the values of tensions T_1 and T_2. We know that

and $$\frac{T_1}{T_2} = e^{f\phi} = e^{0.3*3.93} = 0.5126$$

and braking torque (T),

$$350 \times 103 = (T_1 - T_2) * r = (T_1 - T_2) * 175$$

$$\therefore T_1 - T_2 = 350 \times 10^3 / 175 = 2000 \text{ N}$$

From the previous two equations:

$$T_1 = 2886.5 \text{ N ; and } T_2 = 886.5 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 35$$

$$\text{or } P \times 500 = 886.5 \times 150 - 2886.5 \times 35 = 31\,947.5$$

$$\therefore P = 31\,947.5 / 500 = 64 \text{ N}$$

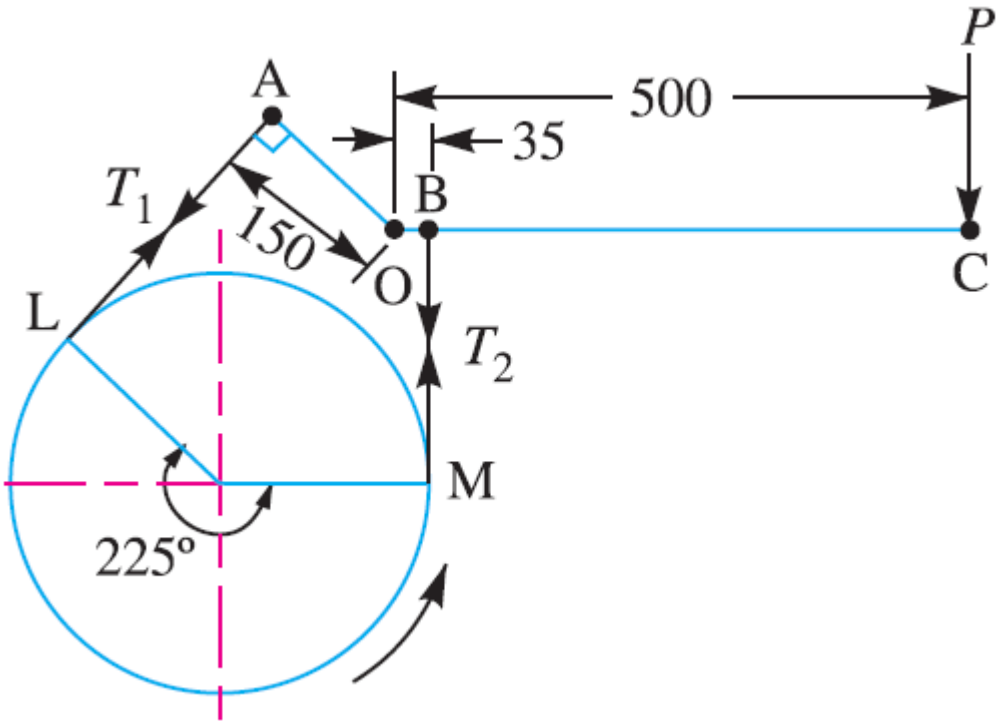
Solution:

When the drum rotates in the anticlockwise direction, the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2, as shown in Fig. Taking moments about the fulcrum O, we have

$$P \times 500 = T_1 \times 150 - T_2 \times 35$$

$$\text{or } P \times 500 = 2886.5 \times 150 - 886.5 \times 35$$

$$\therefore P = 401947.5/500 = 804 \text{ N}$$



Solution:

Value of ‘OA’ for the brake to be self locking, when the drum rotates clockwise

The clockwise rotation of the drum is shown in Fig.

$$P \times 500 + T1 \times OB = T2 \times OA$$

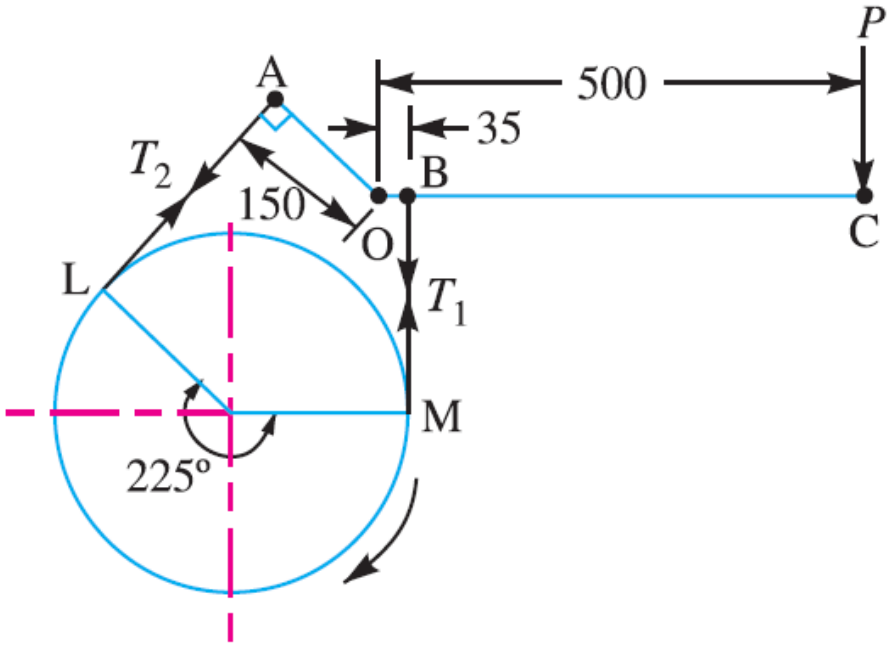
$$P \times 500 = T2 \times OA - T1 \times OB$$

For the brake to be self-locking, P must be equal to zero or

$$T2 \times OA = T1 \times OB$$

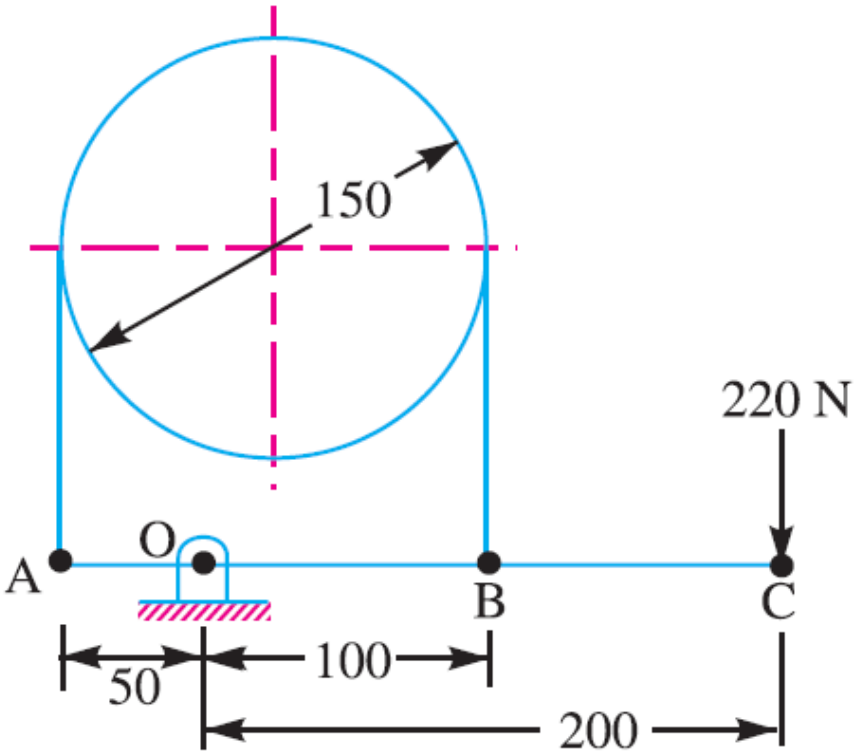
$$\text{or } OA = 2886.5 * 35/886.5$$

$$OA = 114 \text{ mm}$$

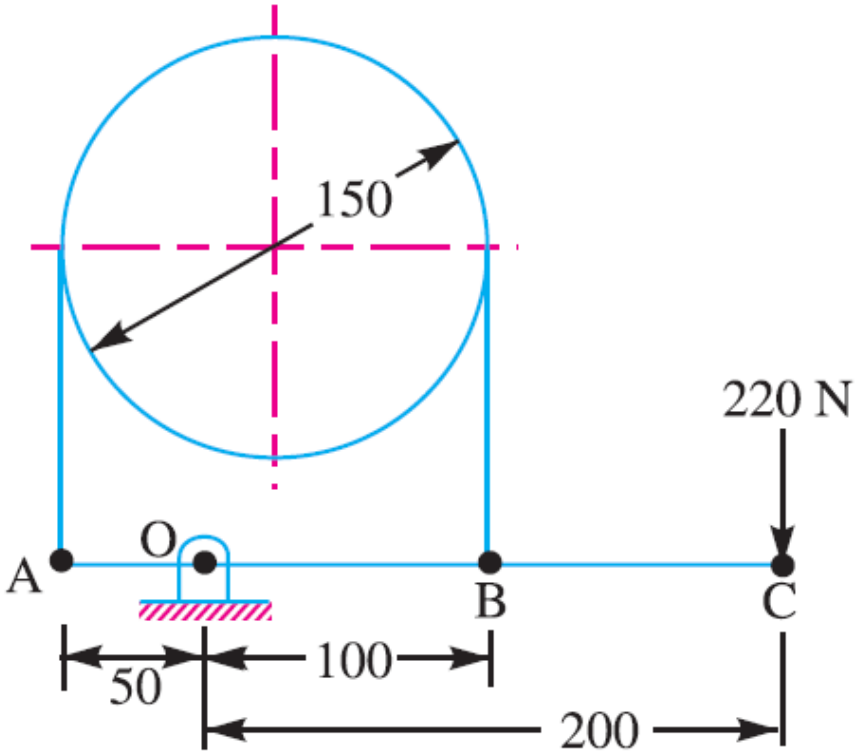


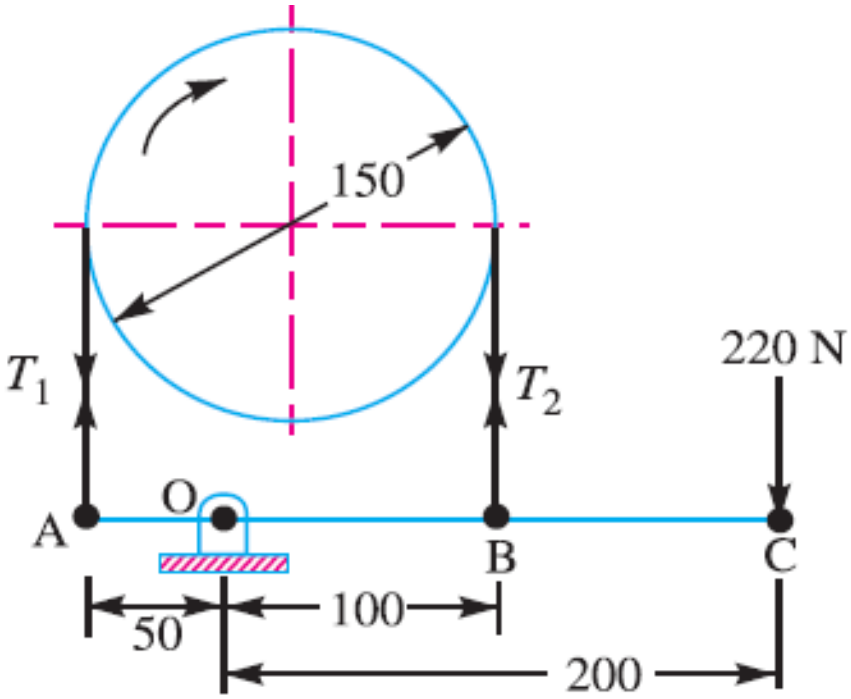
Band Brake Example 3:

A differential band brake has a force of 220 N applied at the end of the lever as shown in figure. The coefficient of friction is 0.4. The wrapping angle is 180˚. If a clockwise torque of 450 N.m is applied to the drum. Determine the maximum and minimum force in the band. What is the maximum toque that the brake may be sustain for anticlockwise direction? Check if the brake is self locking in both direction of rotation.



Example:





Solution:

For clockwise rotation:

Taking summation of moments about the fixed pivot equal to zero and using the torque relationship,

$220 * 200 = F_2 * 100 - F_1 * 50$

and $450 * 1000 = (F_1 - F_2) * 75$

from which $F_1 = 12.88\ \text{kN}$, $F_2 = 0.688\ \text{kN}$

check to see if the brake is self-locking, The brake is self-locking if $\frac{b}{a} \leq e^{f\emptyset}$, where

$e^{f\emptyset} = e^{0.4*180*\pi/180} = 3.5$. here $\frac{b}{a} = \frac{100}{50} = 2$;hence the brake is self-locking.

For anticlockwise rotation:

$$220 * 200 = \mathrm{F_1}' * 100 - \mathrm{F_2}' * 50$$

and $\frac{F_1'}{F_2'} = e^{f\emptyset} = 3.5$, from which

$$\mathrm{F_1}' = 513\ \mathrm{N} \quad , \qquad \mathrm{F_2}' = 146\ \mathrm{N}$$

The amount of torque that may be sustained is

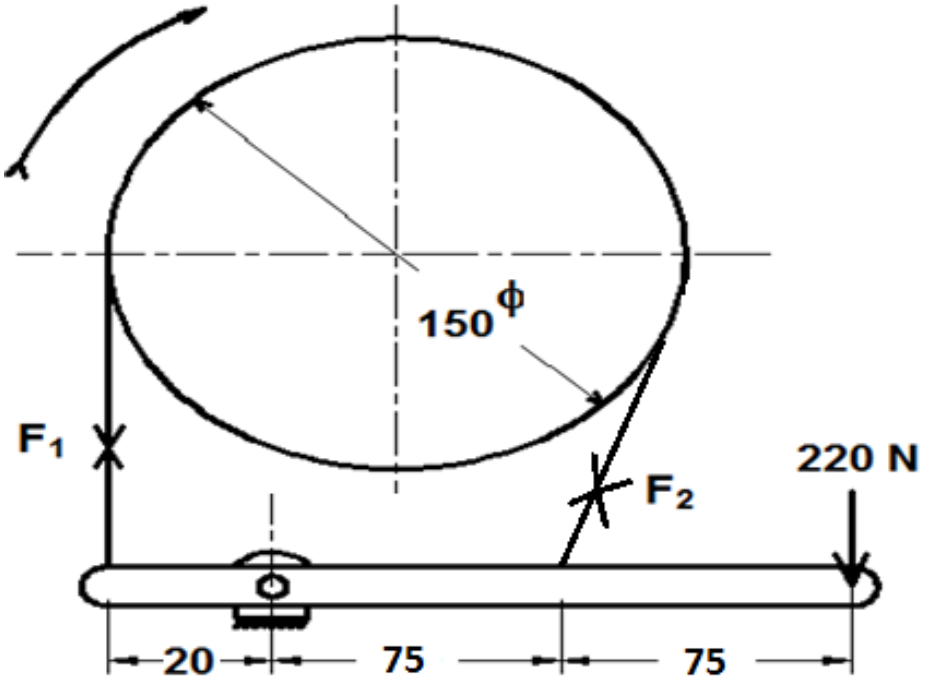
$$T = (513 - 146) * 0.075$$

$$T = 27.5\ \mathrm{N.\ m}$$

For anticlockwise rotation, The brake is self-locking if $\frac{a}{b} \leq e^{f\emptyset}$, where $e^{f\emptyset} = e^{0.4*180*\pi/180} = 3.5$. here $\frac{a}{b} = \frac{50}{100} = 0.5$, hence the brake is not self-locking.

Band Brake Example 4:

A differential band brake has a force of 220 N applied at the end of the lever as shown in figure. The coefficient of friction is 0.4. The wrapping angle is 240˚. If a clockwise torque of 450 N.m is applied to the drum. Determine the maximum and minimum force in the band. What is the maximum toque that the brake may be sustain for anticlockwise direction? Check if the brake is self locking in both direction of rotation.



Solution:

For clockwise rotation:

Taking summation of moments about the fixed pivot equal to zero and using the torque relationship,

$220 * 1500 = F_2 * 75 - F_1 * 20$

and $450 * 1000 = (F_1 - F_2) * 75$

from which $F_1 = 1327.27\ N$, $F_2 = 0793.94\ N$

Check to see if the brake is self-locking, The brake is self-locking if $\frac{b}{a} \leq e^{f\emptyset}$, where $e^{f\emptyset} = e^{0.4*240*\pi/180} = 4.55$.

here $\frac{b}{a} = \frac{75}{20} = 3.75$;hence the brake is self-locking.

For anticlockwise rotation:

$$220 * 150 = \mathrm{F_1}' * 75 - \mathrm{F_2}' * 20$$

and $\frac{F_1'}{F_2'} = e^{f\emptyset} = 4.55$, from which

$$\mathrm{F_1}' = 102.72\ \mathrm{N} \qquad \mathrm{F_2}' = 22.5\ \mathrm{N}$$

The amount of torque that may be sustained is

$$T = (102.7 - 22.5) * 0.075$$

$$T = 6.0112\ \mathrm{N.\ m}$$

For anticlockwise rotation, The brake is self-locking if $\frac{a}{b} \leq e^{f\emptyset}$, where $e^{f\emptyset} = e^{0.4*240*\pi/180} = 4.55$. here $\frac{a}{b} = \frac{20}{75} = 0.26$, hence the brake is not self-locking.