EME 304– Mechanical Vibrations
Sheet 1
Free Undamped Single Degree of Freedom

1. The maximum velocity attained by the mass of a simple harmonic oscillator is 10 cm/s, and the period of oscillation is 2 s. If the mass is released with an initial displacement of 2 cm, find (a) the amplitude, (b) the initial velocity, (c) the maximum acceleration, and (d) the phase angle.

2. Three springs and a mass are attached to a rigid, weightless bar PQ as shown in Fig.1. Find the natural frequency of vibration of the system.

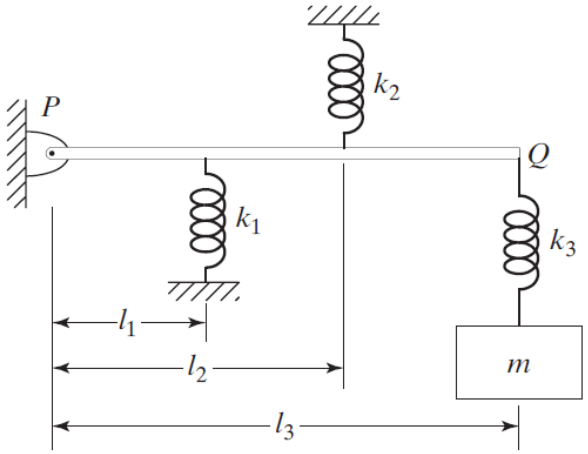


Fig.1.

3. An automobile having a mass of 2,000 kg deflects its suspension springs 0.02 m under static conditions. Determine the natural frequency of the automobile in the vertical direction by assuming damping to be negligible.

4. Find the natural frequency of vibration of a spring-mass system arranged on an inclined plane, as shown in Fig. 2.

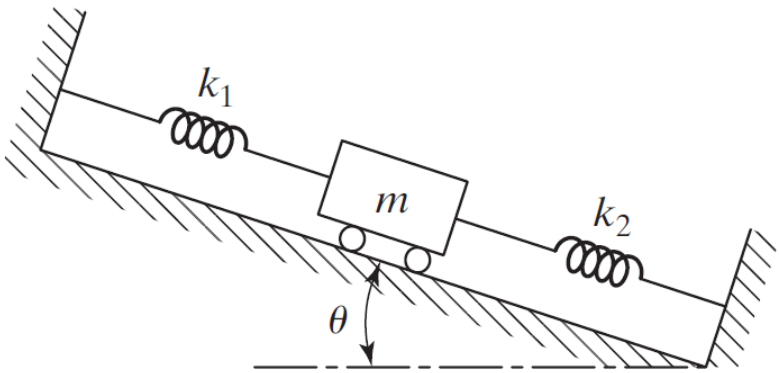


Fig.2

5. Figure 3 shows a steel traffic sign, of thickness 1/8 in, fixed to a steel post. The post is 72 in. high with a cross section 2 in. * 1/4 in and it can undergo torsional vibration (about the z-axis) or bending vibration (either in the zx-plane or the yz plane). Determine the mode of vibration of the post in a storm during which the wind velocity has a frequency component of 1.25 Hz.
   Hints:
   1. Neglect the weight of the post in finding the natural frequencies of vibration.
   2. Torsional stiffness of a shaft with a rectangular section (see Fig. 2.78(b)) is given by

$$k_t = 5.33\frac{ab^3G}{l}\left[1 - 0.63\frac{b}{a}\left(1 - \frac{b^4}{12a^4}\right)\right]$$

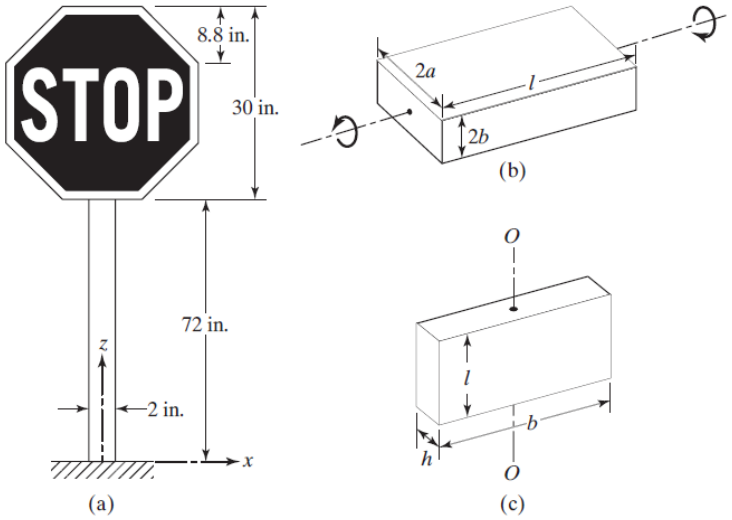


Fig.3.

6. Draw the free-body diagram and derive the equation of motion using Newton's second law of motion for each of the systems shown in Figs. 4 and 5.

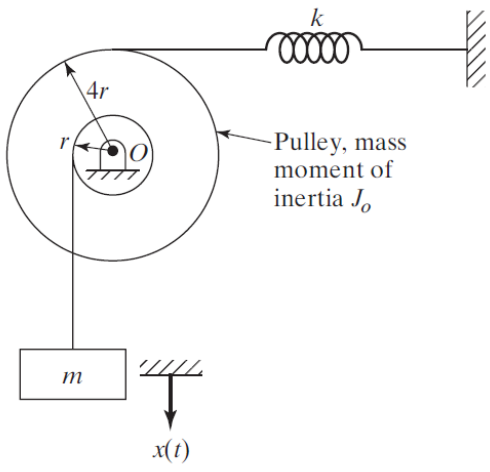


Fig. 4.

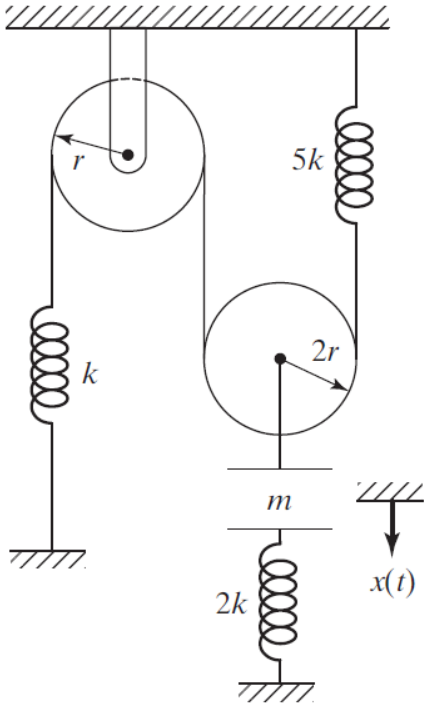
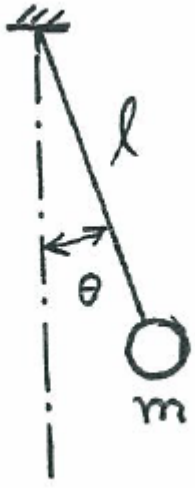


Fig. 5.

7. Determine the displacement x velocity $\dot{x}$ and acceleration $\ddot{x}$ of a spring-mass system with ω_n= 10 rad/s for the initial conditions x_o= 0.05 m and $\dot{x}_o$=1 m/s. Plot and from to 5 s.

8. A simple pendulum shown in Fig. 6 is set into oscillation from its rest position by giving it an angular velocity of 1 rad/s. It is found to oscillate with an amplitude of 0.5 rad. Find the natural frequency and length of the pendulum.



9. A mass m is attached at the end of a bar of negligible mass and is made to vibrate in three different configurations, as indicated in Fig. 7. Find the configuration corresponding to the highest natural frequency

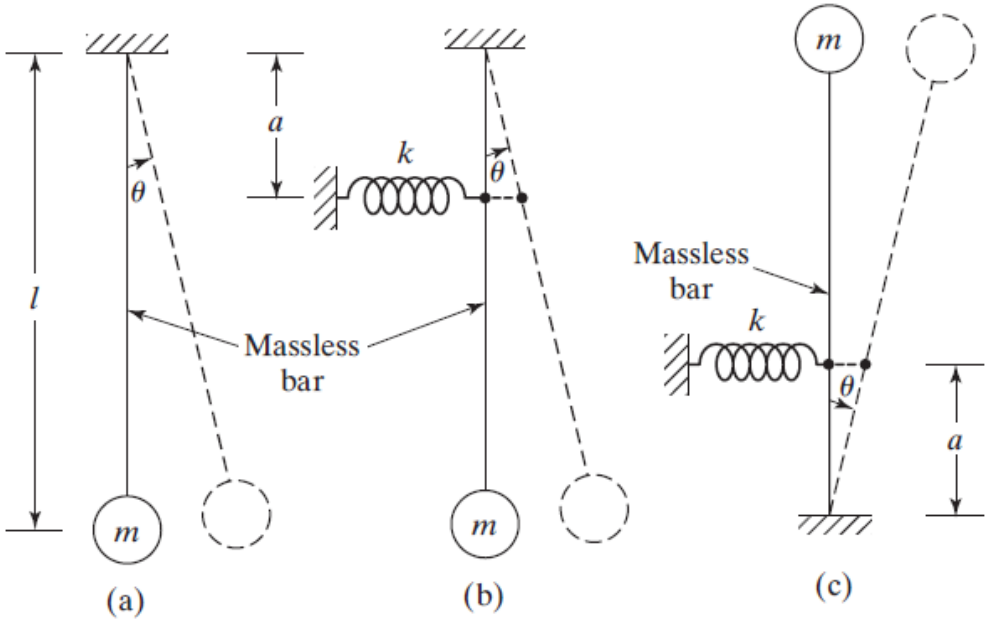


Fig. 7.

10. A uniform slender rod of mass m and length l is hinged at point A and is attached to four linear springs and one torsional spring, as shown in Fig. 8. Find the natural frequency of the system if k = 2000 N/m, kt and l = 5 m.

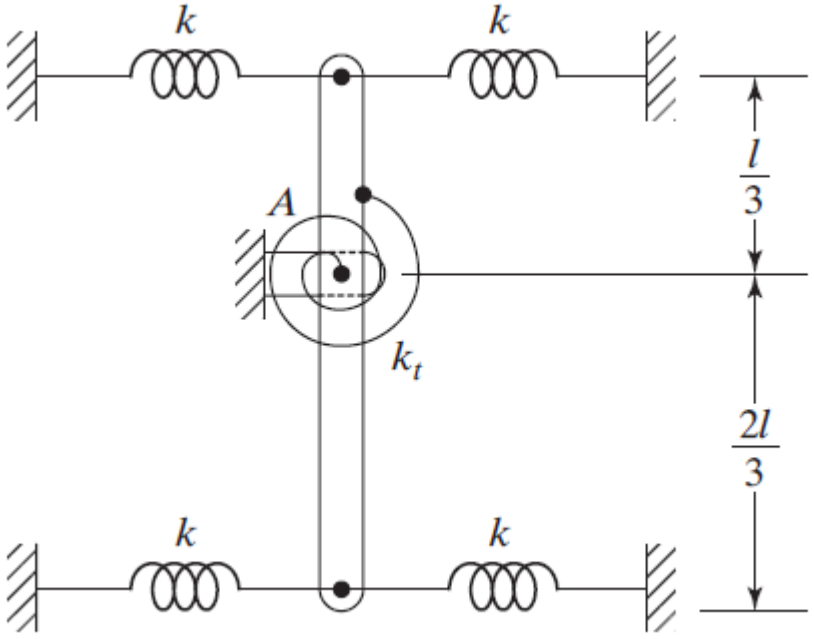


Fig. 8.

11. Derive the equation of motion of the system shown in Fig. 9.

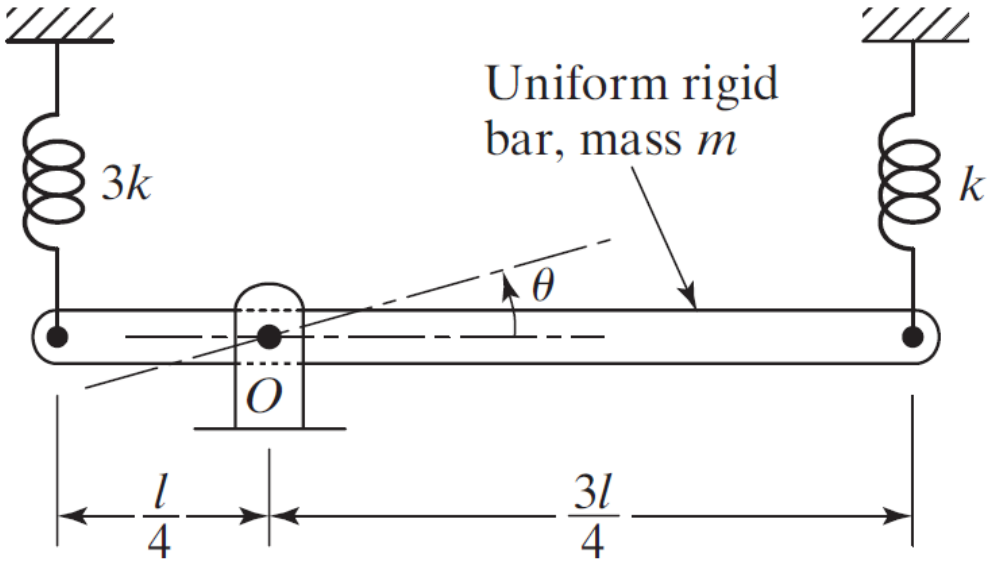


Fig. 9.