EME 304– Mechanical Vibrations
Sheet 3 - Solution
Forced Single Degree of Freedom

1. A spring-mass system is subjected to a harmonic force whose frequency is close to the natural frequency of the system. If the forcing frequency is 39.8 Hz and the natural frequency is 40.0 Hz, determine the period of beating.

$$\tau_b = \frac{2\pi}{\omega_n - \omega} = \frac{2\pi}{2\pi(40.0 - 39.8)} = 5 \text{ sec}$$

2. A mass m is suspended from a spring of stiffness 4000 N/m and is subjected to a harmonic force having an amplitude of 100 N and a frequency of 5 Hz. The amplitude of the forced motion of the mass is observed to be 20 mm. Find the value of m.

$$\delta_{st} = \frac{F_o}{k} = \frac{100}{4000} = 0.025 \text{ m}$$

$$X = \delta_{st} \cdot \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}, \quad \left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right| = \frac{\delta_{st}}{X} = \frac{0.025}{20 \times 10^{-3}} = 1.25$$

$$\frac{\omega}{\omega_n} = \sqrt{1.25 + 1} = 1.5$$

$$\omega_n = \omega / 1.5 = 5(2\pi)/1.5 = 20.944 \text{ rad/sec}$$

$$m = k/\omega_n^2 = 4000/(20.944)^2 = 9.1189 \text{ kg}$$

3. The A four-cylinder automobile engine is to be supported on three shock mounts, as indicated in Fig. 1. The engine-block assembly weighs 500 lb. If the unbalanced force generated by the engine is given by 200 sin 100πt lb, design the three shock mounts (each of stiffness k and viscous-damping constant c) such that the amplitude of vibration is less than 0.1 in.

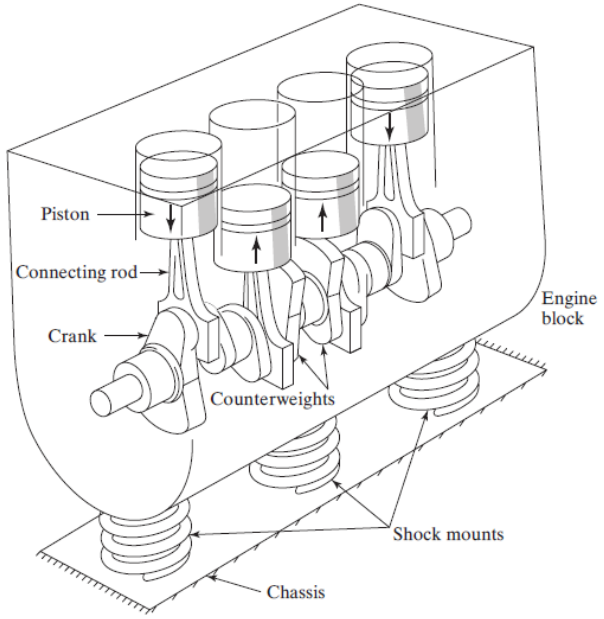


Fig. 1.

$m = \frac{500}{386.4}$ lb–sec^2/in, $F(t) = 200 \sin 100\pi t$ lb. Let $X_{max} = 0.05$ in < 0.1 in (maximum permissible value). From Eq. (3.33),

$$X_{max} = \delta_{st} \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 0.05 \tag{1}$$

Let $\zeta = 0.01$. Then $\delta_{st} = \frac{F_0}{k_{eq}} = \frac{200}{k_{eq}}$ and Eq. (1) gives

$$k_{eq} = \frac{200}{2\,(0.01)\sqrt{1-0.0001}\,(0.05)} = 20.0020\,(10^4) \text{ lb/in}$$

Since shock mounts are in parallel, stiffness of each mount $= k = \frac{k_{eq}}{3} = 6.6673\,(10^4)$ lb/in.

$$\zeta = \frac{c_{eq}}{c_c} = \frac{c_{eq}}{\sqrt{2\,k_{eq}\,m}}$$

$$\text{or} \quad c_{eq} = \zeta\sqrt{2\,k_{eq}\,m} = 0.01\sqrt{2\,(20.0020\,(10^4))\,(\frac{500}{386.4})} = 7.1948 \text{ lb–sec/in}$$

and hence $c = \frac{c_{eq}}{3} = 2.3983$ lb-sec/in

4. A spring-mass-damper system is subjected to a harmonic force. The amplitude is found to be 20 mm at resonance and 10 mm at a frequency 0.75 times the resonant frequency. Find the damping ratio of the system.

Eq. (3.34): $$\frac{X_{res}}{\delta_{st}} = \left.\frac{X}{\delta_{st}}\right|_{\omega=\omega_n} = \frac{1}{2\zeta}$$

i.e., $$\delta_{st} = 2\zeta\left(\frac{20}{1000}\right) = 0.04\,\zeta \qquad (E_1)$$

Eq. (3.30): $$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \; ; \quad r = 0.75 = \frac{\omega}{\omega_n}$$

i.e., $$\frac{0.01}{\delta_{st}} = \frac{1}{\sqrt{(1-0.75^2)^2 + (2\zeta\times 0.75)^2}} \qquad (E_2)$$

Eqs. (E_1) and (E_2) give

$$\frac{0.01}{0.04\,\zeta} = \frac{1}{\sqrt{0.1914 + 2.25\,\zeta^2}}$$

i.e., $$0.1914 + 2.25\,\zeta^2 = 16\,\zeta^2$$

i.e., $$\zeta = 0.1180$$

5. A torsional system consists of a disc of mass moment of inertia J_o = 10 kg.m^2, a torsional damper of damping constant c_t = 300 N.m.sec/rad, and a steel shaft of diameter 4 cm and length 1 m (fixed at one end and attached to the disc at the other end). A steady angular oscillation of amplitude 2° is observed when a harmonic torque of magnitude 1000 N-m is applied to the disc. (a) Find the frequency of the applied torque, and (b) find the maximum torque transmitted to the support.

$$k_t = \frac{\pi G}{32\,\ell} d^4 = \frac{\pi\,(79.3\times10^9)}{32\,(1)}\left(\frac{4}{100}\right)^4 = 19930.31 \text{ N-m/rad}$$

$$\omega_n = \sqrt{\frac{k_t}{J_o}} = \sqrt{\frac{19930.31}{10}} = 44.6434 \text{ rad/sec}$$

$$\theta_{st} = M_{to}/k_t = 1000/19930.31 = 0.0502 \text{ rad}$$

$$\zeta_t = \frac{c_t}{2 J_o \omega_n} = \frac{300}{2(10)(44.6434)} = 0.336$$

(a) Eq. (3.30), when written for a torsional system, gives

$$\frac{\Theta}{\theta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

i.e., $$\frac{(2/57.2956)}{0.0502} = \frac{1}{\sqrt{(1-r^2)^2 + (2\times 0.336\, r)^2}}$$

i.e., $$r^4 - 1.5484\, r^2 - 1.0679 = 0$$

i.e., $$r^2 = 2.0655,\ -0.5171$$

$$\therefore\ \omega = r\,\omega_n = \sqrt{2.0655}\,(44.6434) = 64.16 \text{ rad/sec}$$

(b) Maximum torque transmitted to the support:

$$M_t(t) = k_t\,\theta(t) + c_t\,\dot{\theta}(t)$$

$$= k_t\,\Theta\cos(\omega t - \phi) - c_t\,\Theta\,\omega\sin(\omega t - \phi)$$

$$(M_t)_{max} = \sqrt{(k_t\,\Theta)^2 + (c_t\,\Theta\,\omega)^2}$$

$$= \sqrt{\left\{19930.31\left(\frac{2}{57.2956}\right)\right\}^2 + \left\{300\left(\frac{2}{57.2956}\right)(64.16)\right\}^2}$$

$$= 967.2 \text{ N-m}$$

6. The landing gear of an airplane can be idealized as the spring-mass-damper system shown in Fig. 2. If the runway surface is described $y(t) = y_0 \cos\omega t$ determine the values of k and c that limit the amplitude of vibration of the airplane (x) to 0.1 m. Assume m = 2000 kg, y_0 = 0.2m, and ω = 157.08 rad/sec.

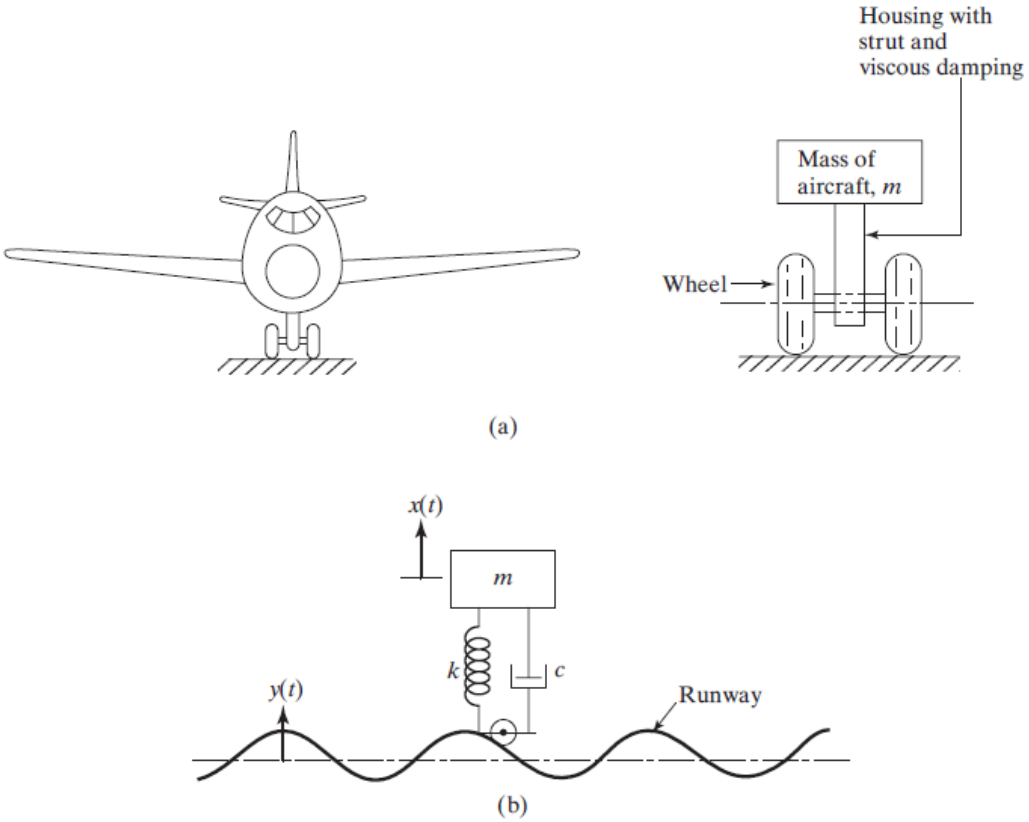


Fig. 2. Modeling of landing gear.

Amplitude of vibration under base excitation:

$$X = Y\left\{\frac{\sqrt{k^2 + (c\,\omega)^2}}{\left[\left(k - m\,\omega^2\right)^2 + (c\,\omega)^2\right]^{\frac{1}{2}}}\right\}$$

$$= \frac{(0.2)\sqrt{k^2 + c^2\,(157.08)^2}}{\left[\left\{k - 2000\,(157.08)^2\right\}^2 + c^2\,(157.08)^2\right]^{\frac{1}{2}}} = 0.1 \text{ m} \qquad (1)$$

Let $k = 5\,(10^6)$ N/m. Then Eq. (1) gives:

$$\frac{\sqrt{25\,(10^{12}) + 2.4674\,(10^4)\,c^2}}{\sqrt{1966.7717\,(10^{12}) + 2.4674\,(10^4)\,c^2}} = 0.5$$

i.e., $1.85055\,(10^4)\,c^2 = 466.6929\,(10^{12})$ i.e., $c = 158805.0$ N–s/m

7. A precision grinding machine (Fig. 3) is supported on an isolator that has a stiffness of 1 MN/m and a viscous damping constant of 1 kN-s/m. The floor on which the machine is mounted is subjected to a harmonic disturbance due to the operation of an unbalanced engine in the vicinity of the grinding machine. Find the maximum acceptable displacement amplitude of the floor if the resulting amplitude of vibration of the grinding wheel is to be restricted to 10^{-6} m. Assume that the grinding machine and the wheel are a rigid body of weight 5000 N.

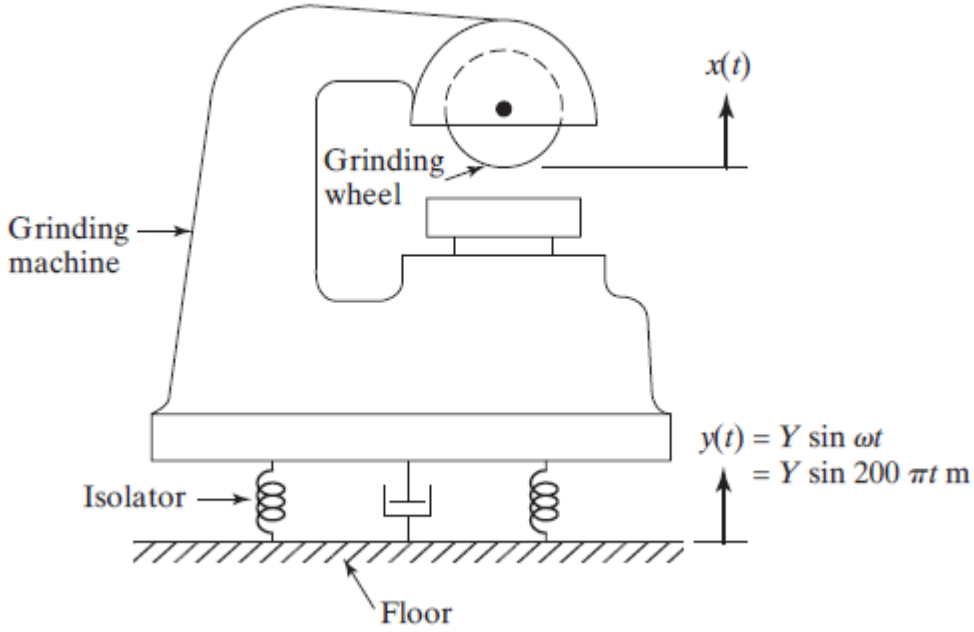


Fig. 3.

$$\frac{X}{Y} = \left[\frac{k^2 + c^2\,\omega^2}{\left(k - m\,\omega^2\right)^2 + c^2\,\omega^2}\right]^{\frac{1}{2}} \text{.sp}$$

$$\text{or} \quad \frac{10^{-6}}{Y} = \left[\frac{(10^6) + (10^3\,(200\,\pi))^2}{\left\{10^6 - \left(\frac{5000}{9.81}\right)(200\,\pi)^2\right\}^2 + \left\{(10^3)\,(200\,\pi)\right\}^2}\right]^{\frac{1}{2}}$$

$$\text{or} \quad Y = 169.5294\,(10^{-6})\ \text{m}$$

8. Derive the equation of motion and find the steady-state response of the system shown in Fig. 4 for rotational motion about the hinge O for the following data: k=5000N/m, l = 1 m, c = 1000 N-s/m, m = 10 kg, M_0 = 100 N-m, ω = 1000 rpm..

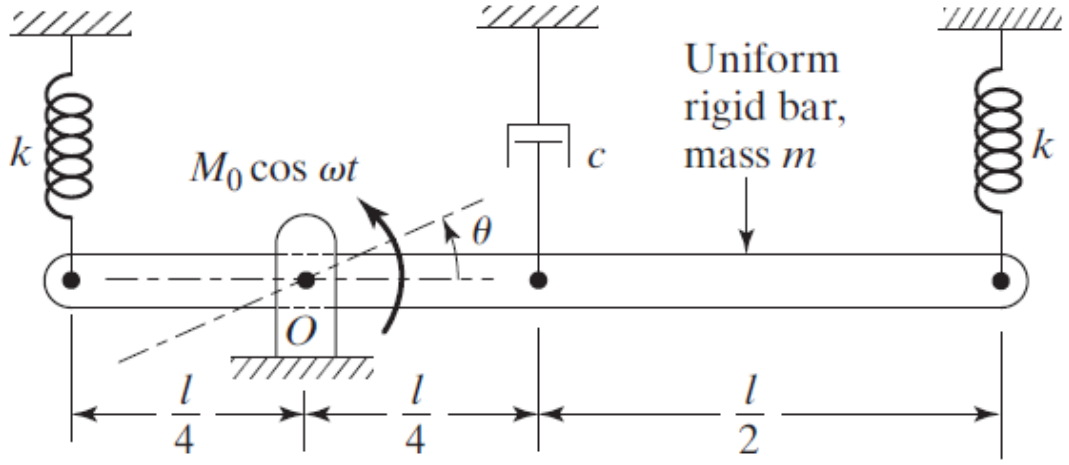


Fig. 4.

Equation of motion:

$$I_0 \ddot{\theta} + \left(k \frac{\ell}{4} \theta\right) \frac{\ell}{4} + \left(c \frac{\ell}{4} \dot{\theta}\right) \frac{\ell}{4} + \left(k \frac{3\ell}{4} \theta\right) \frac{3\ell}{4} = M_0 \cos \omega t$$

$$\text{or} \quad I_0 \ddot{\theta} + c \frac{\ell^2}{16} \dot{\theta} + \frac{5}{8} k \ell^2 \theta = M_0 \cos \omega t$$

$$\text{where } I_0 = \frac{m \ell^2}{12} + m \left(\frac{\ell}{4}\right)^2 = \frac{7}{48} m \ell^2 = \frac{7}{48} (10) (1^2) = 1.4583 \text{ kg–m}^2$$

$$\frac{c \ell^2}{16} = \frac{(1000) (1^2)}{16} = 62.5 \text{ N–m–s/rad}$$

$$\frac{5}{8} k \ell^2 = \frac{5}{8} (5000) (1^2) = 3125.0 \text{ N–m/rad}$$

$$\omega = \frac{1000 (2\pi)}{60} = 104.72 \text{ rad/sec}$$

Equation of motion becomes:

$$1.4583 \ddot{\theta} + 62.5 \dot{\theta} + 3125.0 \theta = 100 \cos 104.72 t$$

Steady state response is given by Eq. (3.28):

$$\theta_p(t) = \Theta \cos (\omega t - \phi) = \Theta \cos (104.72 t - \phi).\text{sp}$$

$$\text{where } \Theta = \frac{100}{\left[\left\{3125.0 - 1.4583 (104.72^2)\right\}^2 + \left\{62.5 (104.72)\right\}^2\right]^{\frac{1}{2}}} = 0.006927 \text{ rad}$$

$$\text{and } \phi = \tan^{-1} \left(\frac{62.5 (104.72)}{3125.0 - 1.4583 (104.72^2)}\right) = -0.4705 \text{ rad} = -26.9606^\circ$$

9. An air compressor of mass 100 kg is mounted on an elastic foundation. It has been observed that, when a harmonic force of amplitude 100 N is applied to the compressor, the maximum steady-state displacement of 5 mm occurred at a frequency of 300 rpm. Determine the equivalent stiffness and damping constant of the foundation.

$$\frac{X}{\delta_{st}} = \frac{1}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}} = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

The maximum value of X (when $r = \sqrt{1 - 2\zeta^2}$) is given by

$$\left(\frac{X}{\delta_{st}}\right)_{max} = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} \tag{3.33}$$

and the value of X at $\omega = \omega_n$ by

$$\left(\frac{X}{\delta_{st}}\right)_{\omega=\omega_n} = \frac{1}{2\zeta} \tag{3.34}$$

$m = 100$ kg, $F_0 = 100$ N, $X_{max} = 0.005$ m at $\omega = 300$ rpm $= 31.416$ rad/sec. Equations (3.33) and (3.34) yield:

$$\omega = \omega_n\sqrt{1 - 2\varsigma^2} = \sqrt{\frac{k}{m}}\sqrt{1 - 2\varsigma^2} = 31.416$$

$$\text{or} \quad k(1 - 2\varsigma^2) = (100)(31.416^2) = 98{,}696.5056 \tag{1}$$

$$\text{and} \quad X_{max} = \delta_{st}\frac{1}{2\varsigma\sqrt{1 - \varsigma^2}} = \frac{F_0}{k}\frac{1}{2\varsigma\sqrt{1 - \varsigma^2}} = 0.005$$

$$\text{or} \quad k\varsigma\sqrt{1 - \varsigma^2} = \frac{F_0}{2(0.005)} = 10{,}000.0 \tag{2}$$

Divide Eq. (1) by (2):

$$\frac{1 - 2\varsigma^2}{\varsigma\sqrt{1 - \varsigma^2}} = 9.8696 \tag{3}$$

Squaring Eq. (3) and rearranging leads to:

$$101.4090\,\varsigma^4 - 101.4090\,\varsigma^2 + 1 = 0 \quad \text{or} \quad \varsigma = 0.0998,\ 0.9950$$

Using $\varsigma = 0.0998$ in Eq. (1), we obtain

$$k = \frac{98696.5056}{1 - 2(0.0998^2)} = 100{,}702.4994 \text{ N/m}$$

Since $\varsigma = \frac{c}{2m\omega_n}$, we find

$$c = 2m\omega_n\varsigma = 2(100)\sqrt{\frac{100702.4944}{1000}}(0.0998) = 633.4038 \text{ N–s/m}$$

10. The Find the steady-state response of the system shown in Fig. 5 for the following data:
$k_1 = 1000$ N/m, $k_2 = 500$ N/m, $c = 500$ N-s/m, $m = 10$ kg, $r = 5$ cm, $J_0 = 1$ kg-m^2, $F_0 = 50$ N, $\omega = 20$ rad/s.

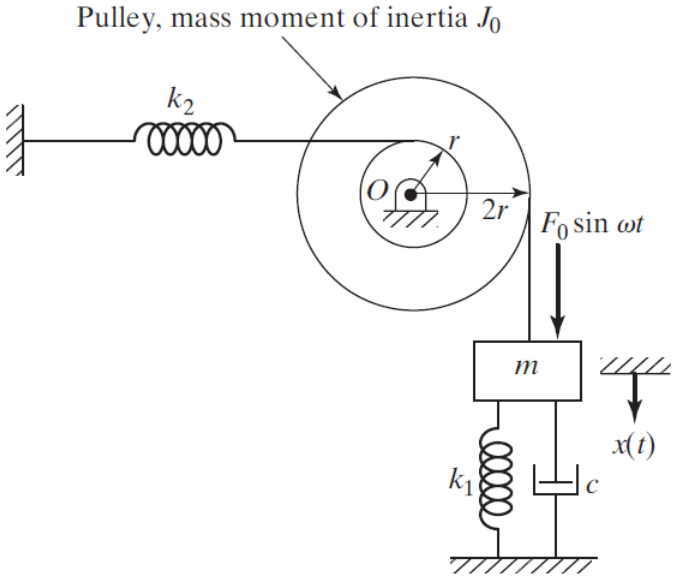
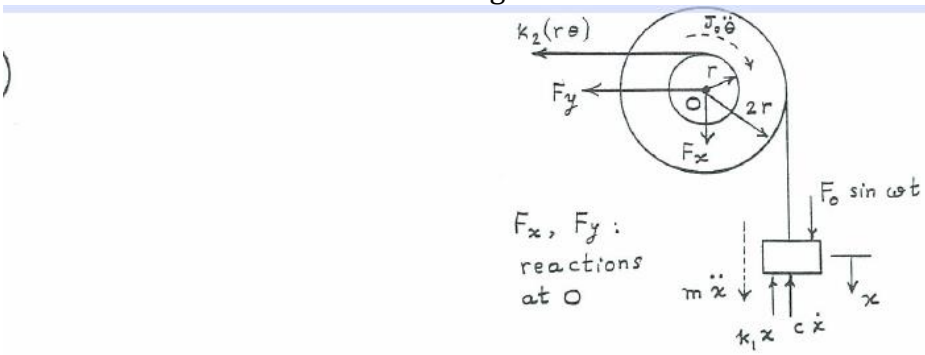


Fig. 5.



Equation of motion for rotation of pulley about O:

$$-k_2(\theta r)\,r - J_0\ddot{\theta} - k_1 x(2r) - c\dot{x}(2r) + F_0 \sin \omega t\,(2r) - m\ddot{x}(2r) = 0 \quad (1)$$

where $\theta = x/(2r)$. Equation (1) can be rearranged as:

$$\left(\frac{J_0}{2r} + 2mr\right)\ddot{x} + 2crx + \left(2k_1 r + \frac{1}{2}k_2 r\right)x = 2rF_0 \sin \omega t \quad (2)$$

For given data, Eq. (2) becomes

$$11\ddot{x} + 50\dot{x} + 112.5x = 5 \sin 20t \quad (3)$$

Steady state response is given by Eq. (3.25):

$$x_p(t) = X \cos(\omega t - \phi)$$

where $$X = \frac{5}{\left[\left\{112.5 - 11(20^2)\right\}^2 + \left\{50(20)\right\}^2\right]^{\frac{1}{2}}} = 0.001136 \text{ m}$$

and $$\phi = \tan^{-1}\left(\frac{50(20)}{112.5 - 11(20^2)}\right) = -0.2291 \text{ rad} = -13.1287^\circ$$

11. A uniform slender bar of mass m may be supported in one of two ways as shown in Fig. 6. Determine the arrangement that results in a reduced steady-state response of the bar under a harmonic force, F0 sin ωt, applied at the middle of the bar, as shown in the figure.

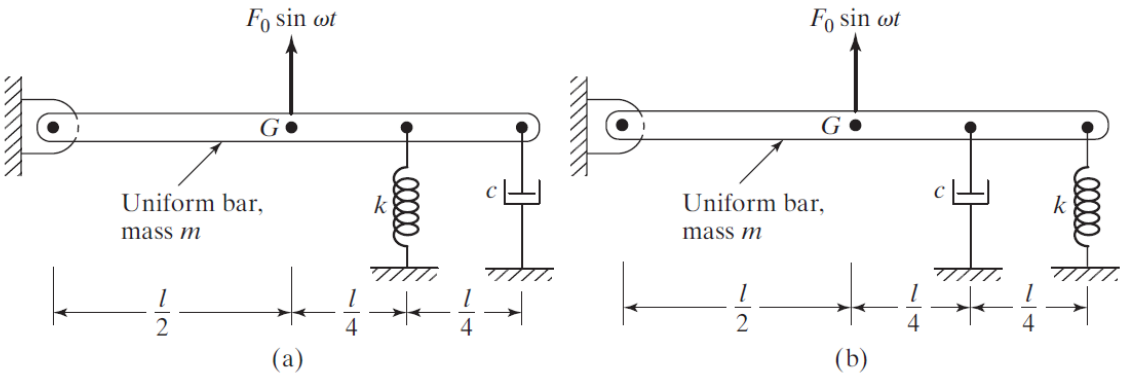


Fig. 6.

(a)

$$\sum M_0 = 0 \quad \text{(about hinge):}$$

$$I_0 \ddot{\theta} + \left(k\,\theta\,\frac{3\ell}{4}\right)\frac{3\ell}{4} + (c\,\ell\,\dot{\theta})\,\ell = \frac{\ell}{2} F_0 \sin \omega t$$

$$\text{or} \quad I_0 \ddot{\theta} + c\,\ell^2\,\dot{\theta} + \frac{9}{16} k\,\ell^2\,\theta = \frac{F_0\,\ell}{2} \sin \omega t$$

Magnitude of steady state response:

$$\Theta_a = \left(\frac{F_0\,\ell}{2}\right) \Big/ \left[\left\{\frac{9}{16} k\,\ell^2 - I_0\,\omega^2\right\}^2 + (c\,\ell^2\,\omega)^2\right]^{\frac{1}{2}} \tag{1}$$

(b)

$$\sum M_0 = 0 \quad \text{(about hinge):}$$

$$I_0 \ddot{\theta} + (k\,\ell\,\theta)\,\ell + \left(c\,\frac{3\ell}{4}\,\dot{\theta}\right)\frac{3\ell}{4} = \frac{\ell}{2} F_0 \sin \omega t$$

$$\text{or} \quad I_0 \ddot{\theta} + \frac{9}{16} c\,\ell^2\,\dot{\theta} + k\,\ell^2\,\theta = \frac{F_0\,\ell}{2} \sin \omega t$$

Magnitude of steady state response:

$$\Theta_b = \left(\frac{F_0\,\ell}{2}\right) \Big/ \left[\left\{k\,\ell^2 - I_0\,\omega^2\right\}^2 + \left\{\frac{9}{16} c\,\ell^2\,\omega\right\}^2\right]^{\frac{1}{2}} \tag{2}$$

Usually, c is small compared to k. If the term containing c is negligible, Θ_a will be smaller than Θ_b. Hence arrangement (a) is desirable.

12. A single-cylinder air compressor of mass 100 kg is mounted on rubber mounts, as shown in Fig. 7. The stiffness and damping constants of the rubber mounts are given by and 2000 N-s/m, respectively. If the unbalance of the compressor is equivalent to a mass 0.1 kg located at the end of the crank (point A), determine the response of the compressor at a crank speed of 3000 rpm. Assume r = 10 cm and *l* = 40 cm.

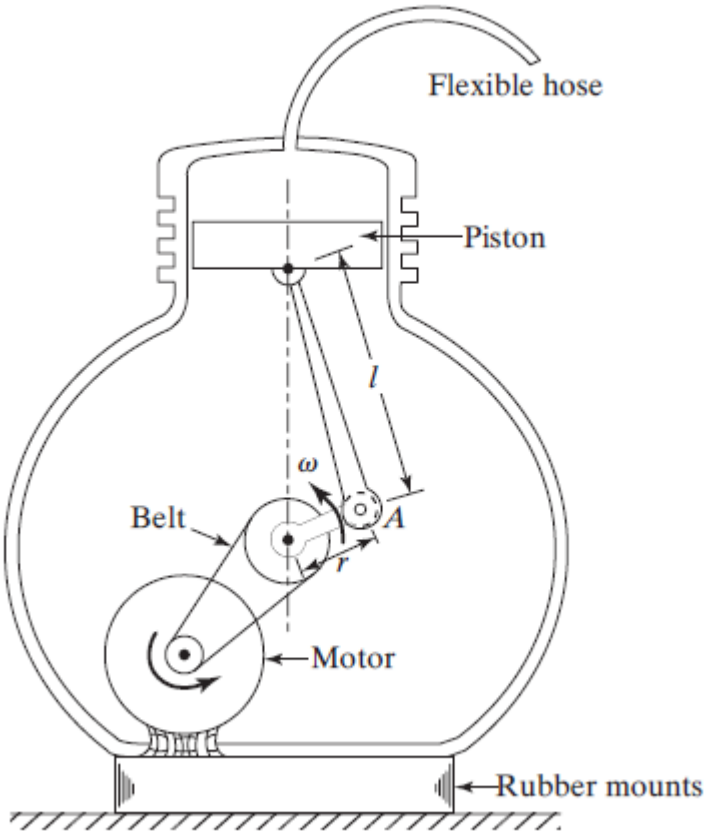


Fig. 7.

Equation of motion: $\mathrm{M}\ddot{\mathrm{x}} + \mathrm{c}\dot{\mathrm{x}} + \mathrm{k}\,\mathrm{x} = \mathrm{m}\,\mathrm{e}\,\omega^2 \sin \omega\,\mathrm{t}$

where $\omega = \dfrac{3000\ (2\ \pi)}{60} = 314.16$ rad/sec, $\mathrm{M} = 100$ kg, $\mathrm{c} = 2000$ N-s/m, $\mathrm{k} = 10^6$ N/m, $\mathrm{m} = 0.1$ kg and $\mathrm{e} = \mathrm{r} = 0.1$ m. Steady state response is:

$$\mathrm{x_p(t)} = \mathrm{X} \sin\ (\omega\,\mathrm{t} - \phi)$$

$$\text{where} \quad \mathrm{X} = \frac{\mathrm{m}\,\mathrm{e}\,\omega^2}{\left[(\mathrm{k} - \mathrm{M}\,\omega^2)^2 + (\mathrm{c}\,\omega)^2\right]^{\frac{1}{2}}}$$

$$= \frac{0.1\ (0.1)\ (314.16^2)}{\left[\left\{10^6 - 100\ (314.16^2)\right\}^2 + (2000\ (314.16))^2\right]^{\frac{1}{2}}} = 110.9960\ (10^{-6})\ \mathrm{m}$$

$$\text{and} \quad \phi = \tan^{-1}\left(\frac{\mathrm{c}\,\omega}{\mathrm{k} - \mathrm{M}\,\omega^2}\right) = \tan^{-1}\left(\frac{2000\ (314.16)}{10^6 - 100\ (314.16^2)}\right)$$

$$= -0.07072 \text{ rad} = -4.0520^\circ$$

13. When an exhaust fan of mass 380 kg is supported on springs with negligible damping, the resulting static deflection is found to be 45 mm. If the fan has a rotating unbalance of 0.15 kg-m, find (a) the amplitude of vibration at 1750 rpm, and (b) the force transmitted to the ground at this speed.

$\delta_{st} = \frac{45}{1000} m = \frac{Mg}{k} = \frac{380 \times 9.81}{k}$

i.e., $k = 82{,}840$ N/m

$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{82{,}840}{380}} = 14.7648$ rad/sec ; $\omega = \frac{2\pi(1750)}{60} = 183.26$ rad/sec

$r = \frac{\omega}{\omega_n} = \frac{183.26}{14.7648} = 12.412$; $r^2 = 154.0566$

(i) Amplitude of vibration

$X = \frac{me}{M} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{0.15}{380} \frac{154.0566}{\sqrt{(153.0566)^2 + 0}}$

$= 3.9732 \times 10^{-4}$ m

(ii) Force transmitted to ground

$= kX = (82840)(3.9732 \times 10^{-4}) = 32.9140$ N

14. A centrifugal pump, weighing 600 N and operating at 1000 rpm, is mounted on six springs of stiffness 6000 N/m each. Find the maximum permissible unbalance in order to limit the steady-state deflection to 5 mm peak-to-peak.

$m = (600/9.81)$ N , $\omega = 2\pi(1000)/60 = 104.72$ rad/sec

$k = 6(6000) = 36{,}000$ N/m

$\omega_n = \sqrt{k/m} = \sqrt{36000/\left(\frac{600}{9.81}\right)} = 24.2611$ rad/sec

$r = \omega/\omega_n = 104.72/24.2611 = 4.3164$, $r^2 = 18.6311$

$X = \frac{F_o}{k\,|r^2-1|} = \frac{m_o e \omega^2}{k\,|r^2-1|}$ where m_o = unbalanced mass and e = eccentricity

i.e., $2.5 \times 10^{-3} = \frac{m_o e\ (104.72)^2}{36000\,|17.6311|}$

i.e., $m_o e = 0.1447$ kg-m

∴ Unbalance = $W_o e = m_o g e = 0.1447(9.81) = 1.4195$ N-m

15. An air compressor, weighing 1000 lb and operating at 1500 rpm, is to be mounted on a suitable isolator. A helical spring with a stiffness of 45,000 lb/in., another helical spring with a stiffness of 15,000 lb/in., and a shock absorber with a damping ratio of 0.15 are available for use. Select the best possible isolation system for the compressor.

$m = \frac{1000}{386.4} = 2.588 \ \frac{lb-s^2}{in}$, $\omega = \frac{2\pi(1500)}{60} = 157.08 \ \frac{rad}{s}$

Possible isolators are: (i) $k = 45\,000$ lb/in , $\zeta = 0$

(ii) $k = 90\,000$ lb/in , $\zeta = 0$

(iii) $k = 45\,000$ lb/in , $\zeta = 0.15$

(iv) $k = 90\,000$ lb/in , $\zeta = 0.15$

We will compare the force transmissibilities of these isolators.

Force transmissibility $= T_r = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$

(i) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{45000}{2.588}} = 131.8634$ rad/sec

$r = \omega/\omega_n = 157.08/131.8634 = 1.1912$, $r^2 = 1.4190$

$T_r = \frac{1}{|1-r^2|} = \frac{1}{0.419} = 2.3866$

(ii) $\omega_n = \sqrt{\frac{90\,000}{2.588}} = 186.4829$ rad/sec

$r = \omega/\omega_n = 157.08/186.4829 = 0.8423$, $r^2 = 0.7095$

$T_r = \frac{1}{|1-r^2|} = \frac{1}{0.2905} = 3.4423$

(iii) $\omega_n = \sqrt{\frac{45\,000}{2.588}} = 131.8634$ rad/sec

$r = 1.1912$, $r^2 = 1.4190$, $\zeta = 0.15$

$T_r = \sqrt{\frac{1+(2\times 1.1912\times 0.15)^2}{(1-1.4190)^2+(2\times 1.1912\times 0.15)^2}} = 1.9282$

(iv) $\omega_n = 186.4829$ rad/sec , $r = 0.8423$, $r^2 = 0.7095$, $\zeta = 0.15$

$T_r = \sqrt{\frac{1+(2\times 0.8423\times 0.15)^2}{(1-0.7095)^2+(2\times 0.8423\times 0.15)^2}} = 2.6789$

$\therefore$ Isolation (iii) is best.