EME 304– Mechanical Vibrations
Sheet 4
Two Degree of Freedom Systems

1- A rigid rod of negligible mass and length $2l$ is pivoted at the middle point and is constrained to move in the vertical plane by springs masses, as shown in Figure 1. Find the natural frequencies and mode shapes of the system.

2- Find the natural frequencies and the amplitude ratios of the system shown in Figure 2.

3- On type of seismograph, a device that records earthquakes can be modeled as shown in Figure 3. For this model, determine (a) the differential equations of motion; (b) the frequency equation and the natural frequencies; (c) the natural modes of vibration. (Assume small angles θ)

4- Determine the natural frequencies of the system shown in Figure 4 by assuming that the rope passing over the cylinder does not slip.

5- Determine the natural frequencies and the normal modes of the torsional system shown in Figure 5 for $k_{t2} = k_{t1}$ and $J_2 = J_1$.

6- A two-story building frame is modeled as shown in Figure 6. The girders are assumed to be rigid, and the columns have flexural rigidities EI_1 and EI_2 , with negligible masses. The stiffness of each column can be computed as $24EI_i / h_i$ (i =1,2). For $m_1 = 2m$, $m_1 = m$, $h_1 = h_2 = h$, and $EI_1 = EI_2 = EI$, determine the natural frequencies and mode shapes of the frame.

7- Figure 7 shows a system of two masses attached to a tightly stretched string, fixed at both ends. Determine the natural frequencies and mode shapes of the system for $m_1 = m_2 = m$, $l_1 = l_2 = l_3 = l$.

8- Write down the equations of motion and find the natural frequencies of the system shown in Figure 8.

9- A machine tool, having a mass of m = 1000 kg and a mass moment of inertia of J_o = 300 kg.m^2, is supported on elastic supports as shown in Figure 9. If the stiffnesses of the supports are given by k_1 = 3000 N/mm and k_2 = 2000 N/mm, and the supports are located at L_1 = 0.5 m and L_2 = 0.8 m, find the natural frequencies and mode shapes of the machine tool.

10- A machine mounted on elastic support has a rotating unbalance. The machine was at resonance when running at 1000 rpm. A vibration absorber of weight 22.5 N tuned to 1000 cpm is attached to the machine and the system was found to have

two natural frequencies of 860 and 1170 cpm. The amplitude of the absorber's mass was 2.5 mm. Find (a) the weight of the machine; (b) the force transmitted to the ground if the absorber's mass is removed and the machine runs at 1500 rpm.

11- A machine having a rotating unbalance is mounted on four springs having stiffness of 500 kN/m each. The machine was at resonance when running at 1000 rpm. To eliminate the resulting vibration, a vibration absorber, tuned at 1000 rpm, was attached to the machine. Find (i) the absorber's mass so that the natural frequencies should be at least ± 20 % from the operating speed; (ii) the unbalance in the machine if the amplitude of the absorber was 2 mm at the operating speed.

12- An air compressor weighing 250 kg mounted on springs of k_1 and carries a dynamic absorber with a spring stiffness of k_2 and a mass m_2. If the normal operating speed is 1750 rpm and the resonant frequencies should be at least ± 20 % from the operating speed. (a) Find the values of k_1, k_2, and m_2 (b) what is the amplitude of oscillation of the dynamic absorber. (c) What is the change in the force transmitted to the compressor foundation in the case with and without absorber?

13- An air compressor of mass 200 kg, with an unbalance of 0.01 kg-m, is found to have large amplitude of vibration while running at 1200 rpm. Determine the mass and spring constant of the absorber to be added if the natural frequencies of the system are to be at least ± 20 % from the impressed frequency.

14- An electric motor, having an unbalance of 2 kg-cm, is mounted at the end of a steel cantilever beam. The beam is observed to vibrate with large amplitudes at the operating speed of 1500 rpm of the motor. It is proposed to add a vibration absorber to reduce the vibration of the beam. Determine the ratio of the absorber mass to the mass of the motor needed in order to have the lower frequency of the resulting system equal to 75 % of the operating speed of the motor. If the mass of the motor is 300 kg, determine the stiffness and mass of the absorber. Also find the amplitude of vibration of the absorber mass.

15- A hollow steel shaft of outer diameter 5 cm., inner diameter 3.75 cm., and length 75 cm., carries a solid disc of diameter 40 cm and mass 50 kg. Another hollow steel shaft of length 50 cm., carrying a solid disc of diameter 16 cm., and mass 10 kg, is attached to the first disc as shown in Figure 10. Find the inner and outer diameters of the shaft such that the attached shaft and disc system acts as an absorber. (The two shafts have the same diameter ratio)

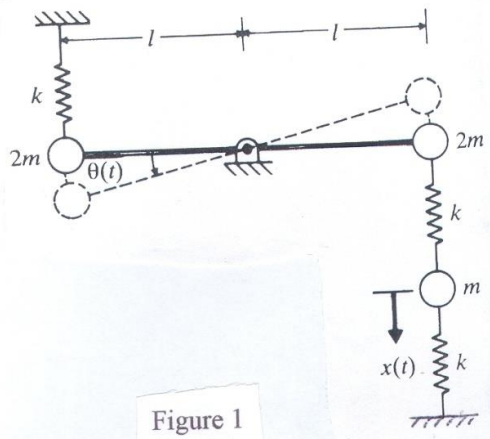

Figure 1

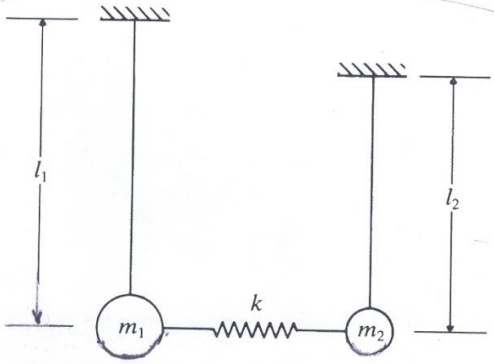

Figure 2

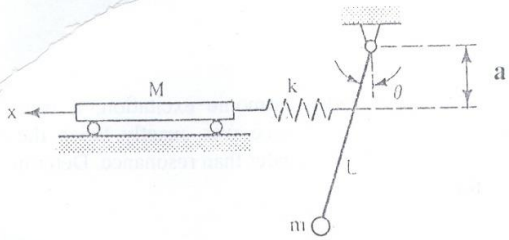

Figure 3

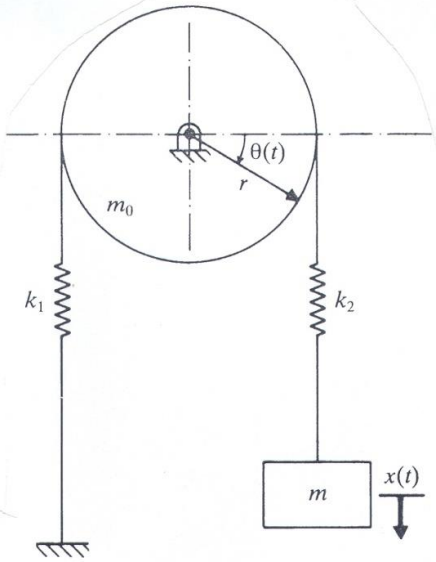

Figure 4

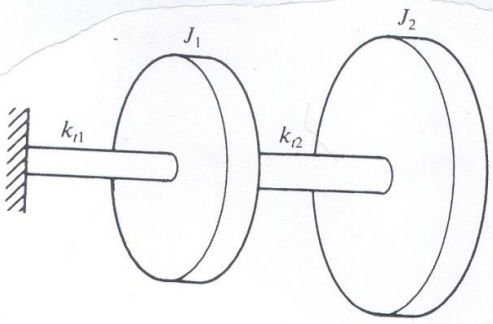

Figure 5

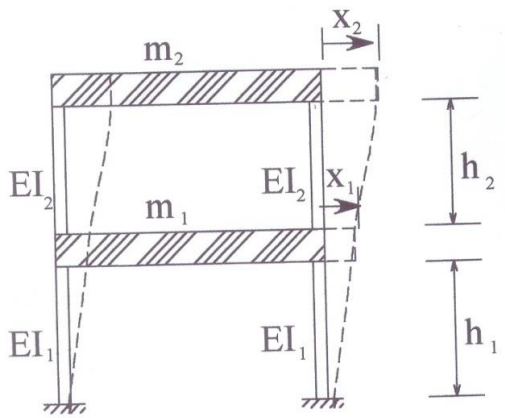

Figure 6

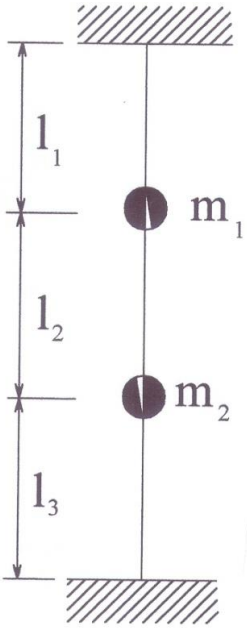

Figure 7

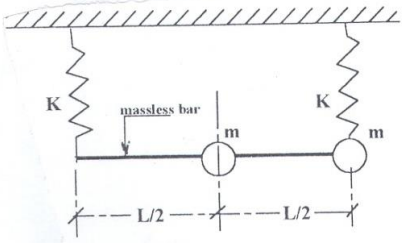

Figure 8

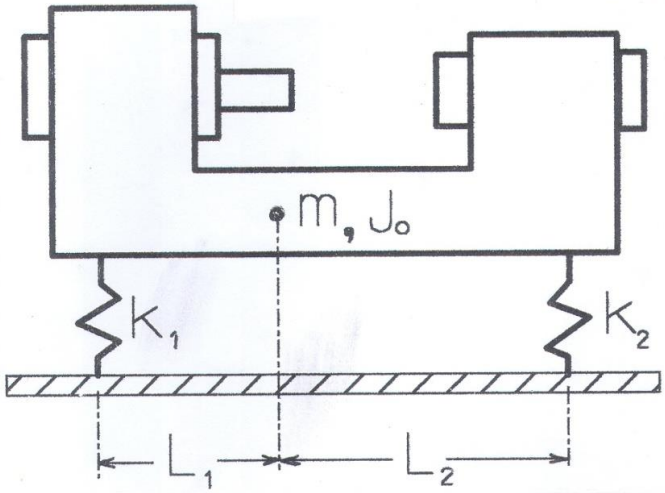

Figure 9

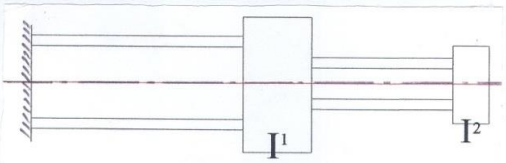

Figure 10