



Answer the following questions:

QUESTION ONE (15 points):

A piping system transports a fluid and supports a vertical load of 9 kN and a horizontal load of 13 kN (acting in the +x direction) at flange A. The pipe has an outside diameter of $D = 200$ mm and an inside diameter of $d = 176$ mm. Determine the principal stresses, the maximum shear stress at points H and K.

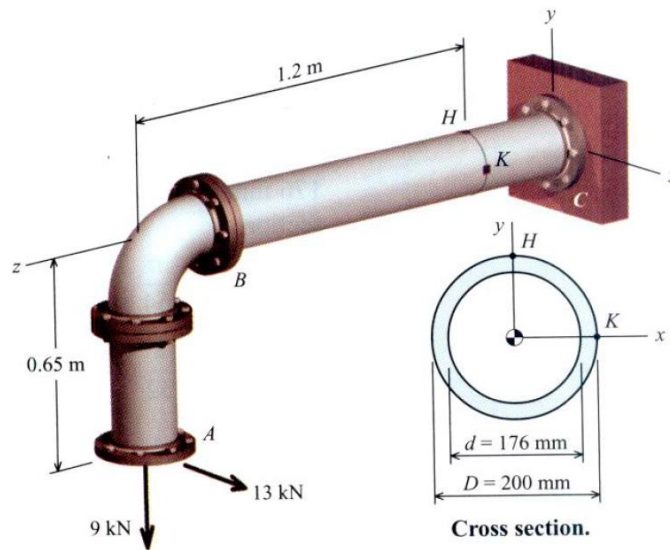
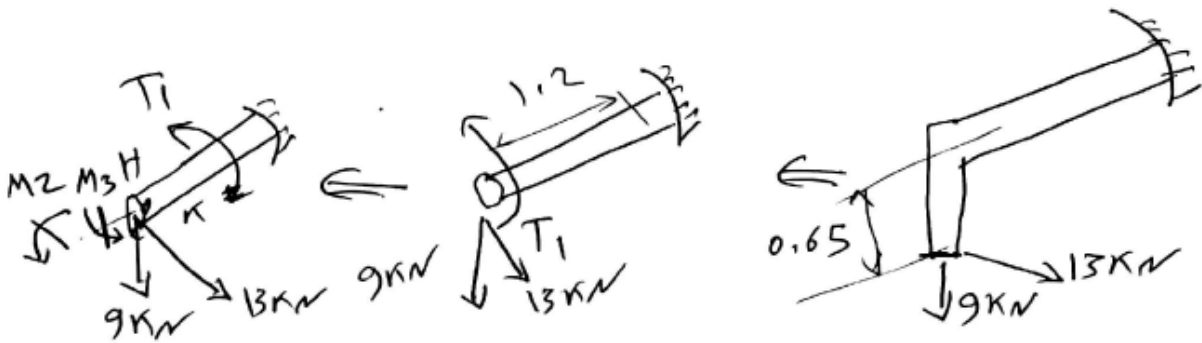


Figure 1.

Solution

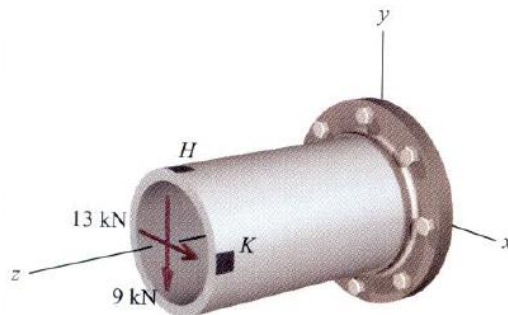
Given: two forces 9 kN, 13 kN, $d_o = 200$, $d_i = 176$ mm
 Req: $\sigma_{1,2}$, τ_{max} at H, K



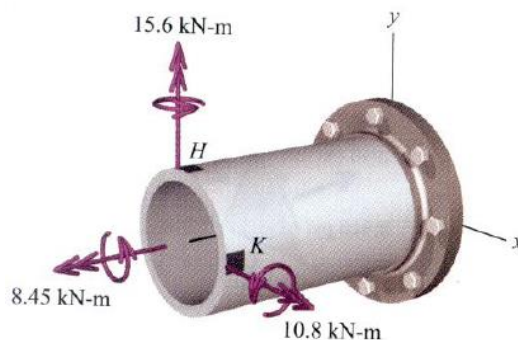
$$T_1 = 13,000 \times 0.65 \times 1000 = 8.45 \times 10^6 \text{ N}\cdot\text{mm}$$

$$M_2 = 9 \times 1000 \times 1.2 \times 1000 = 10.8 \times 10^6 \text{ N}\cdot\text{mm}$$

$$M_3 = 13 \times 1000 \times 1.2 \times 1000 = 15.6 \times 10^6 \text{ N}\cdot\text{mm}$$



Equivalent forces at the section that contains points H and K.



Equivalent moments at the section that contains points H and K.

$$I = \frac{\pi}{64} (d_o^4 - d_i^4) = \frac{\pi}{64} (200^4 - 176^4) = 31.439 \times 10^6 \text{ mm}^4 \quad (A)$$

$$J = \frac{\pi}{32} (d_o^4 - d_i^4) = \frac{\pi}{32} (200^4 - 176^4) = 62.879 \times 10^6 \text{ mm}^4 \quad (B)$$

Shear due to T_1

$$\tau_{T_1} = \frac{T_1 r}{J} = \frac{8.45 \times 10^6 \times 200/2}{62.879 \times 10^6} \quad (C)$$

$$\tau_{T_1} = 13.438 \text{ MPa} \quad (C)$$

Moment due to M_2

$$\sigma_{M_2} = \frac{M_2 y}{I} = \frac{10.8 \times 10^6 \times 200/2}{31.439 \times 10^6} = 34.35 \text{ MPa} \quad (D)$$

Shear due to Moment M_2 (force 9 kN)

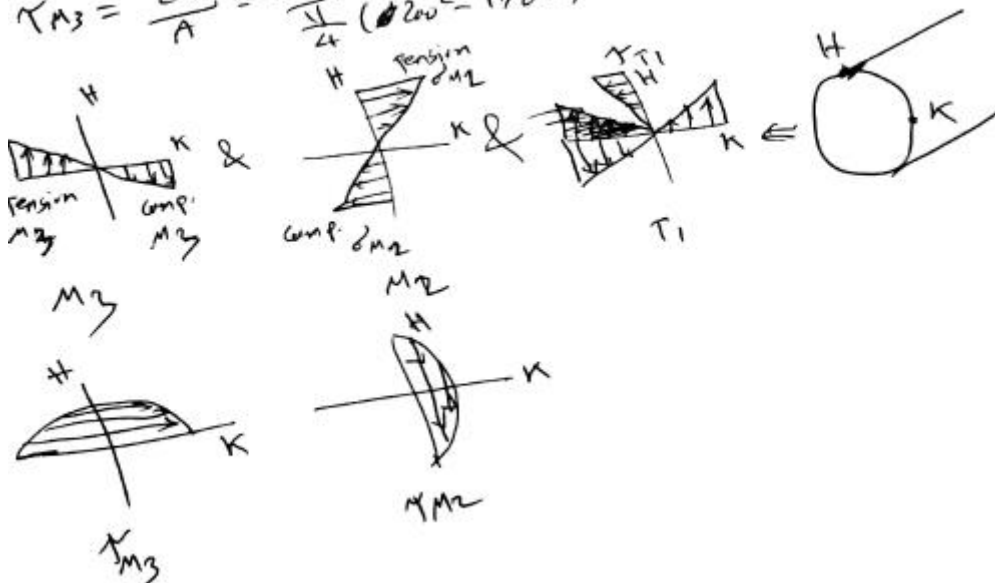
$$\tau_{M_2} = \frac{2V}{A} = \frac{2 \times 9000}{\frac{\pi}{4} (200^2 - 176^2)} = \frac{2 \times 9000}{\frac{\pi}{4} \times (200^2 - 176^2)} = 2.539 \text{ MPa} \quad (E)$$

Moment due to M_3

$$\sigma_{M_3} = \frac{M_3 y}{I} = \frac{15.6 \times 10^6 \times 200/2}{31.439 \times 10^6} = 49.61 \text{ MPa} \quad (F)$$

Shear due to Moment M_3 (force 13 kN)

$$\tau_{M_3} = \frac{2V}{A} = \frac{2 \times 13000}{\frac{\pi}{4} (200^2 - 176^2)} = 3.668 \text{ MPa} \quad (G)$$



for point K

$$\sigma_K = -\sigma_{M_3} = -49.61 \text{ MPa}$$

$$\tau_K = \tau_{T_1} - \tau_{M_2} = 13.438 - 2.539 = 10.89 \text{ MPa}$$

$$\therefore \sigma_{x,K} = -49.61 \text{ MPa} \quad \tau_{xy,K} = 10.89 \text{ MPa} \quad \sigma_y,K = 0$$

$$\sigma_{K \max \min} = \frac{\sigma_{x,K}}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2} = \frac{-49.61}{2} \pm \sqrt{\frac{(49.61)^2}{4} + (10.89)^2}$$

$$= -24.8 \pm 27.09$$

$$\therefore \sigma_{\max,K} = -51.89 \text{ MPa} \quad \sigma_{\min,K} = 2.285 \text{ MPa}$$

$$\tau_{\max,K} = \sqrt{\frac{\sigma_{x,K}^2}{4} + \tau_{xy}^2} = 27.09 \text{ MPa}$$

for point H

$$\sigma_H = \sigma_{M_2} = 34.35 \text{ MPa}$$

$$\tau_H = \tau_{T_1} - \tau_{M_3} = 13.438 - 3.668 = 9.769 \text{ MPa}$$

$$\therefore \sigma_{x,H} = 34.35 \text{ MPa} \quad \tau_{xy,H} = 9.769 \text{ MPa} \quad \sigma_y,H = 0$$

$$\therefore \sigma_{\max \min,H} = \frac{34.35}{2} \pm \sqrt{\frac{(34.35)^2}{4} + (9.769)^2}$$

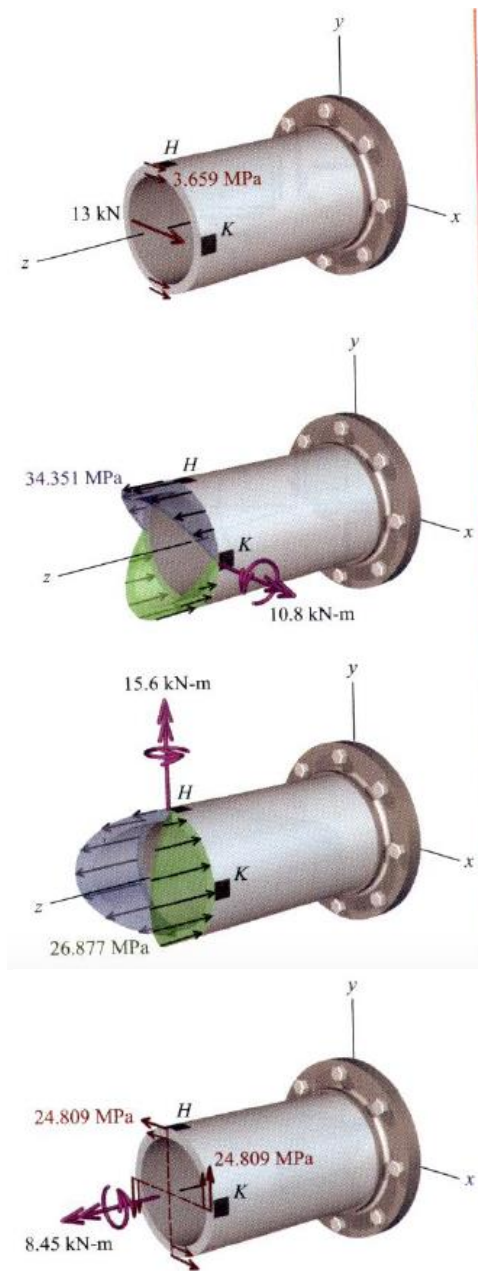
$$= 17.17 \pm 19.758$$

$$\sigma_{\max,H} = 36.93 \text{ MPa}$$

$$\sigma_{\min,H} = -2.583 \text{ MPa}$$

$$\tau_{\max,H} = \sqrt{\frac{(34.35)^2}{4} + (9.769)^2} = 19.758 \text{ MPa}$$

$$\boxed{\begin{array}{l} \text{for K} \Rightarrow \sigma_{\max} = -51.89 \text{ MPa}, \sigma_{\min} = 2.285 \text{ MPa}, \tau_{\max} = 27.09 \text{ MPa} \\ \text{for H} \Rightarrow \sigma_{\max} = 36.93 \text{ MPa}, \sigma_{\min} = -2.58 \text{ MPa}, \tau_{\max} = 19.758 \text{ MPa} \end{array}} \quad (3)$$



QUESTION TWO (15 points):

Determine the required weld size for Figure 2 using a yield stress of 345 MPa and a safety factor of 2.5. All dimensions are in mm.

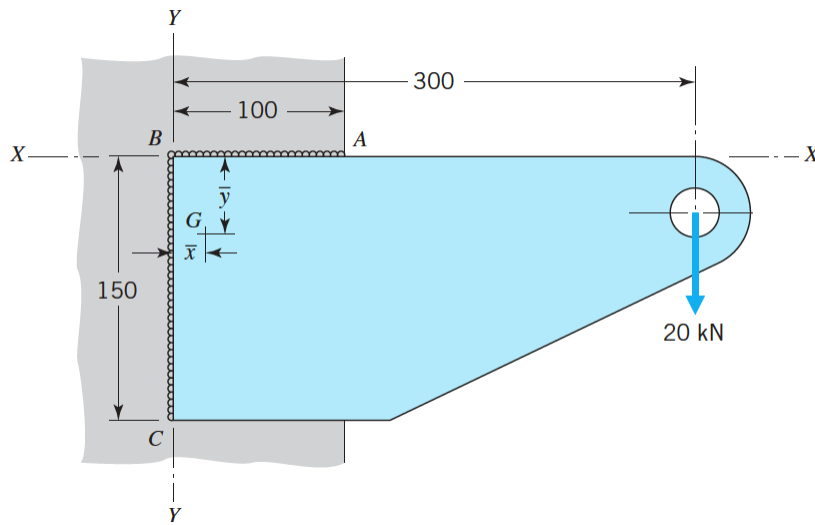


Figure 2.

Solution

Givens:

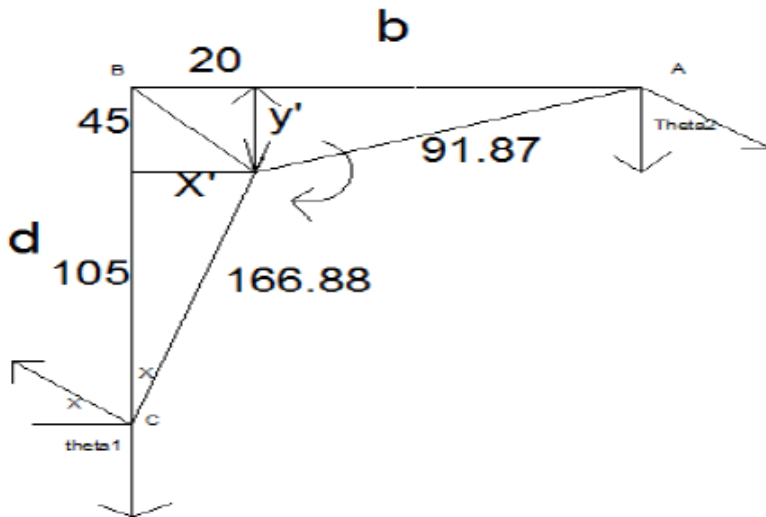
$S_y = 345 \text{ Mpa}$

f.o.s=2.5

Solution

$$T = 20 \times 10^3 \times (300 - X') = 20 \times 10^3 \times (300 - 20) = 5.6 \times 10^6 \text{ N.mm}$$

Point ©



$$x = \frac{b^2}{2(b+d)} = \frac{100^2}{2(100+150)} = 20$$

$$y = \frac{d^2}{2(b+d)} = \frac{150^2}{2(100+150)} = 45$$

$$J_u = \frac{(b+d)^4 - 6 \cdot b^2 \cdot d^2}{12(b+d)} = \frac{(100+150)^4 - 6 \cdot 100^2 \cdot 150^2}{12(100+150)} = 852.0833 \cdot 10^3$$

$$\cos(x) = \frac{105}{106.88}$$

$$X = 10.762^\circ$$

$$R_a = \sqrt{45^2 + 80^2} = 91.7877 \text{ mm}$$

$$\cos(\theta_2) = \frac{80}{91.78}$$

$$z' = \frac{F}{A} = \frac{\frac{91.78}{20 \times 10^3}}{(100 + 150) \times 0.707h} = \frac{133.154}{h}$$

$$z'' = \frac{T \cdot r_c}{J_u \cdot 0.707h} = \frac{5.6 \times 10^6 \cdot 106.88}{852.0833 \times 10^3 \cdot 0.707h} = \frac{993.53}{h}$$

$$z = \sqrt{z'^2 + z''^2 + 2z'z''\cos(\theta_1)}$$

$$= \sqrt{\left(\frac{133.154}{h}\right)^2 + \left(\frac{993.53}{h}\right)^2 + 2 \cdot \frac{133.154}{h} \cdot \frac{993.53}{h} \cdot \cos(100.76)} = \frac{977.4}{h}$$

Point (A)

$$z' = \frac{133.154}{h}$$

$$z'' = \frac{T \cdot r_a}{J_u \cdot 0.707h} = \frac{5.6 \times 10^6 \cdot 91.7877}{852.0833 \times 10^3 \cdot 0.707h} = \frac{853.239}{h}$$

$$z = \sqrt{z'^2 + z''^2 + 2z'z''\cos(\theta_1)}$$

$$= \sqrt{\left(\frac{133.154}{h}\right)^2 + \left(\frac{853.239}{h}\right)^2 + 2 \cdot \frac{133.154}{h} \cdot \frac{853.239}{h} \cdot \frac{80}{91.78}} = \frac{969}{h}$$

Point B

$$\frac{977.4}{h} = \frac{0.577 \cdot S_y}{SF} = \frac{0.577 \cdot 345}{2.5}$$

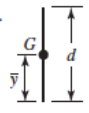
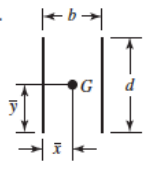
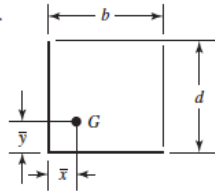
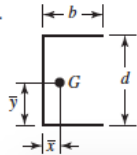
$$h = 12.27 \text{ mm}$$

Take $h = 13 \text{ mm}$

Comment: It is important to understand the approximation in the preceding procedure, which is used conventionally for simplicity. Throat dimension t is assumed to be in a 45° orientation when calculating transverse shear and axial stresses; but the *same* dimension is assumed to be in the plane of the weld pattern when computing torsional stresses. After these stresses are vectorially added and a value for t is determined, the final answer for weld dimension h again assumes dimension t to be in the 45° plane. Although of course not rigorously correct, this convenient procedure is considered to be justified when used by the engineer who understands what he or she is doing and who interprets the results accordingly. Note that the same simplifying approximation appears when handling bending loads, as illustrated by the next sample problem.

Design Equations:

$$d_o = \left\{ \frac{16}{\pi \tau_{all} (1-k^4)} \sqrt{\left(K_m M + \frac{\alpha F_a d_o (1+k^2)}{8} \right)^2 + (K_T T)^2} \right\}^{1/3}$$

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
1. 	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
2. 	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \frac{d(3b^2 + d^2)}{6}$
3. 	$A = 0.707h(b + d)$	$\bar{x} = \frac{b^2}{2(b + d)}$ $\bar{y} = \frac{d^2}{2(b + d)}$	$J_u = \frac{(b + d)^4 - 6b^2d^2}{12(b + d)}$
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \frac{b^2}{2b + d}$ $\bar{y} = d/2$	$J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$I = \frac{\pi}{64} (D_{out}^4 - D_{in}^4)$$

$$J = \frac{\pi}{32} (D_{out}^4 - D_{in}^4)$$

End of the Design Part Questions. Check the following Pages for the Fluid and Thermodynamics questions.