Machine Design Course for Communication / Electrical Department
Sheet 1 Solution

Problem 1

Determine the maximum Moment in the steel I section shown in Figure 1. The flanges are 6 mm wide and the web is 3 mm thick.

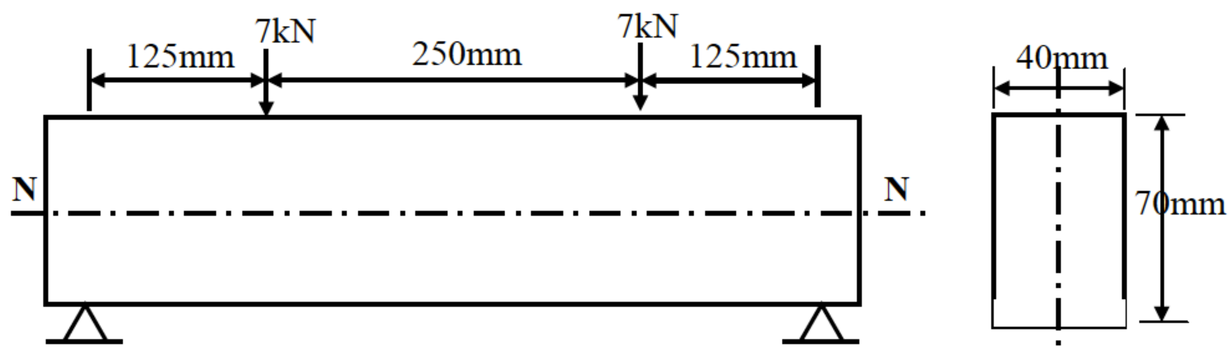
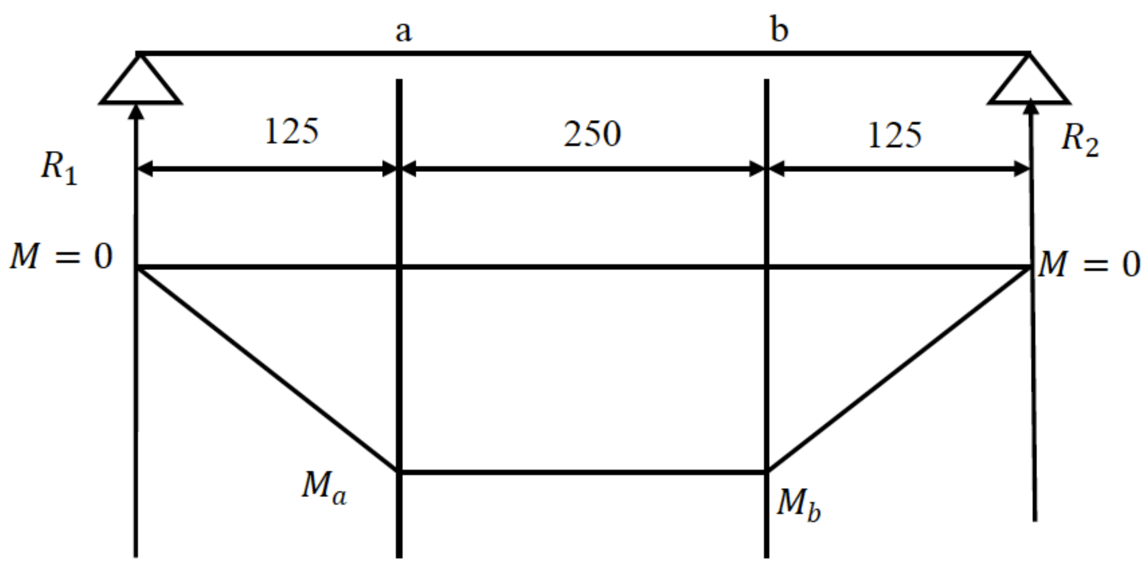


Figure 1.

Solution

1. The Force Analysis
   Free Body Diagram s shown in the following figure:



Beam reaction calculation:
$R_1 = R_2 = 7000\ N$

The Beam moment calculations:
$M_a = M_b = R_1 * 125 = 7000 * 125 = 875000\ N.mm$

2. The stress analysis:

The beam is subjected to bending stress.

$$\sigma_{max} = \frac{M y_{max}}{I_{min}}$$

To obtain y_{max} and the second moment of area I_{min} for symmetrical I-section shown in the figure must be applied the method in the Tables 2.1 and 2.2 about axis N-N.

From Tables 2.1 and 2.2:

$$y_{max} = 35\ mm$$

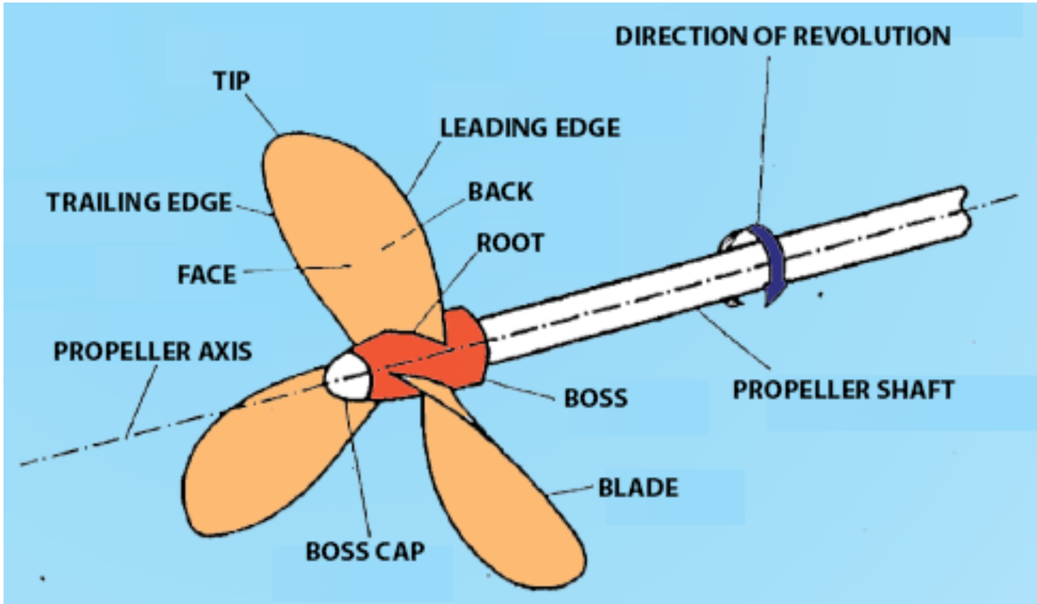
$$I = \frac{(40) * (70)^3}{12} = 143333.33\ mm^4$$

$$M = 875000\ N.mm$$

$$\therefore \sigma_{max} = \frac{875000 * 35}{143333.33} = 26.78\ MPa$$

Problem 2

A ship's propeller shaft transmits 8 MW at 5 rev/s. The shaft has the inner diameter of 150 mm. Calculate the maximum permissible external diameter if the shearing stress in the shaft is limited to 150 MPa.



Solution

1. The force analysis :

A ship's propeller shaft torque calculation:

$$T = \frac{Power}{w} = \frac{Power}{2\pi N}$$

$$Power = 8000000\, watt$$

$$N = 5\, rps$$

$$T = \frac{8000000}{2 * \pi * 5} = 254647.91\, N.m$$

2. The stress analysis:

A ship's propeller shaft is subjected to tortsional stress.

$$\tau_{max} = \frac{Tr}{J}$$

$$J = \frac{\pi}{32}(D_o^4 - D_i^4) \quad for\ hollow\ circle$$

$$\frac{254647.91\times\frac{D_o}{2}}{\frac{\pi}{32}(D_o^4-150^4)}=150\ MPa$$

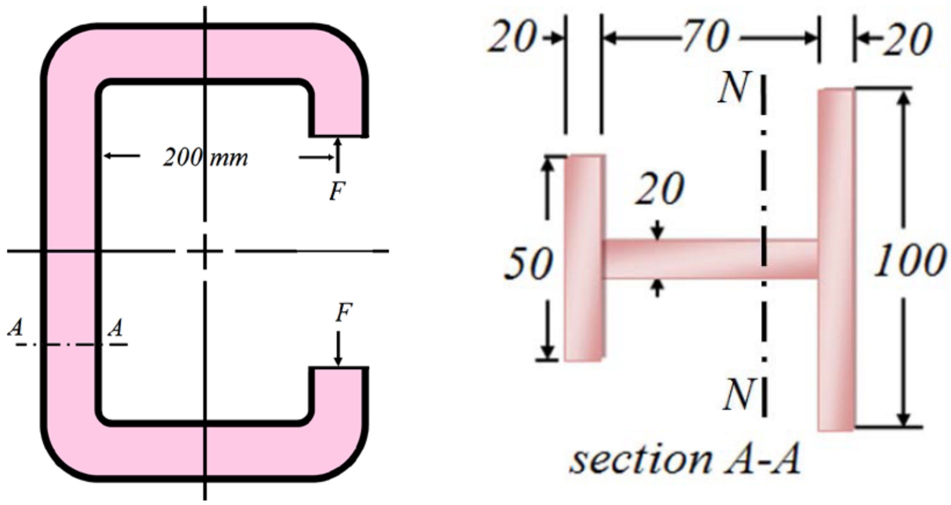
$$\frac{1296911\times D_o}{(D_o^4-150^4)}=150\ MPa$$

By using Matlab code or try and error

$D_o = 150.096\ mm$

Problem 3

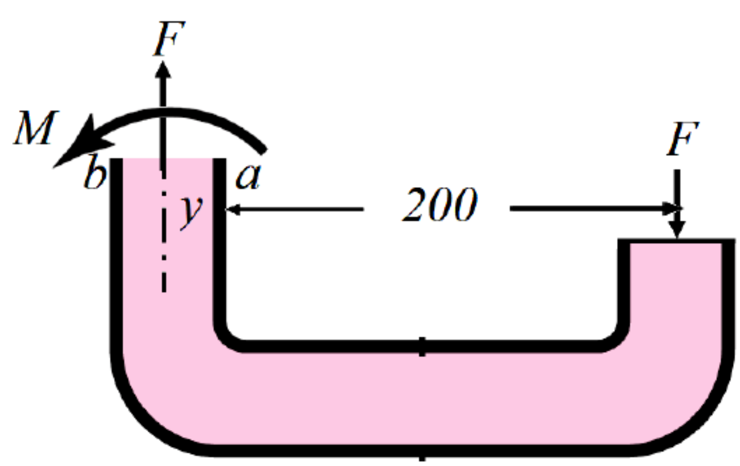
The press frame shown in the Figure is planned to be made of cast iron having ultimate strength of 240 MPa in tension and 900 MPa in compression. Based upon stress calculations made at section A-A, would the press frame be more likely to fail in tension or compression?



Solution

By study section A-A:

1. The force analysis:



2. The stress analysis:

The press at section A-A is subjected to:

1) Tension stress due to (F).

2) Bending stress due to (M).

1) **The tension stress:**

$$\sigma_T = \frac{F}{A}$$

From figure of section A-A

$$A = A_1 + A_2 + A_3 = (50 * 20) + (70 * 20) + (100 * 20) = 4400\ mm^2$$

$$\therefore \sigma_T = \frac{F}{4400}\ MPa$$

2) The bending stress:

$$\sigma_b = \frac{M y_{max}}{I_{min}}$$

To obtain y_{max} and the second moment of area I_{min} for un-symmetrical I-section shown in the figure must be applied the method in the Tables 2.1 and 2.2 about axis N-N.

From Table 2.1:

$$Y_1 = \frac{20}{2} = 10\ mm, \quad Y_2 = \frac{70}{2} + 20 = 55\ mm,$$

$$Y_3 = \frac{20}{2} + 70 + 20 = 100\ mm,$$

$$Y_c = \frac{\big((20 * 50) * 100\big) + \big((20 * 70) * 55\big) + \big((20 * 100) * 10\big)}{(20 * 50) + (20 * 70) + (20 * 100)}$$

$$= 44.77\ mm$$

$$\therefore y_{max} = 44.77\ mm$$

From Table 2.2:

$$I_{X-X} = \frac{(50) * (20)^3}{12} + \big((20 * 50) * (44.77 - 10)^2\big) + \frac{(20) * (70)^3}{12}$$
$$+ \big((20 * 70) * (55 - 44.77\,)^2\big) + \frac{(100) * (20)^3}{12}$$
$$+ \big((20 * 100) * (100 - 44.77)^2\big)$$

$$I_{min} = 30836886.23 + 718180.72 + 2484572.46 = 6286439.41\ mm^4$$

$$M = (200 + 44.7) * F = 244.77\ F\ N.mm$$

3. The design equation:

By using super position method:

1) The stress at point (a) is tension-tension:

$$\sigma_{total} = \sigma_b + \sigma_T$$

$$\therefore \sigma_{total} = \frac{F}{573.7} + \frac{F}{4400} = \frac{F}{507.5} \; MPa$$

The design equation:

$$\sigma_{total} \leq \frac{S_{U_T}}{f.o.s|_T}$$

$$\therefore f.o.s|_T = \frac{S_{U_T}}{\sigma_{total}} = \frac{240}{\sigma_{total}} = \frac{121806.14}{F} \qquad (1)$$

2) The stress at point (b) is tension-compression:

$$\sigma_{total} = \sigma_T - \sigma_b$$

$$\therefore \sigma_{total} = \frac{F}{4400} - \frac{F}{393.73} = -\frac{F}{361.4} \; MPa$$

The design equation:

$$\sigma_{total} \leq \frac{S_{U_C}}{f.o.s|_C}$$

$$\therefore f.o.s|_C = \frac{S_{U_C}}{\sigma_{total}} = \frac{900}{\sigma_{total}} = \frac{-325252.56}{F} \quad \textit{the sign means compression} \; (2)$$

From 1 and 2:

$$\therefore \frac{f.o.s|_C}{f.o.s|_T} = \frac{325252.56}{121806.14} = 2.7$$

The failure will occur due to tension stress

Problem 4

A steel column is 3m long and 0.4 m diameter. It carries a load of 50 MN. Given that the modulus of elasticity is 200 GPa, calculate the compressive stress and strain. Also determine how much the column is compressed.

Givens:

L = 3 m= 3000 mm, d = 0.4 m = 400 mm. F= 60 MN= 50 $\times 10^6$ N,
E= 200 GPa = 200,000 MPa.

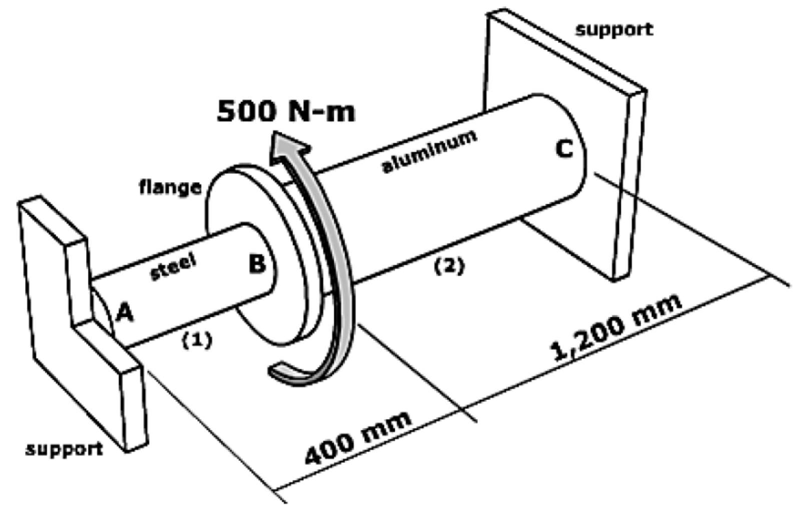
Solution:

$$\sigma_c = \frac{F}{A} = \frac{50\times10^6}{\frac{\pi}{4}\times d^2} = \frac{50\times10^6}{\frac{\pi}{4}\times400^2} = 397.88 \text{ MPa}$$

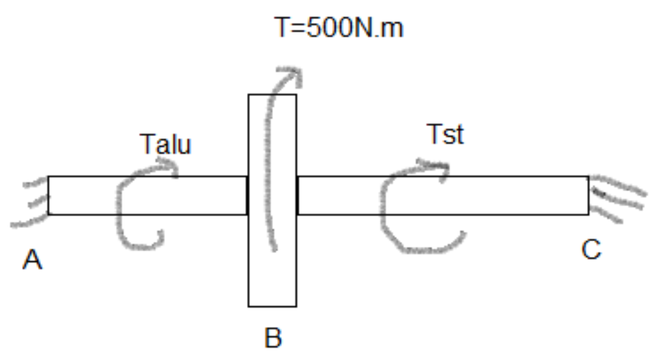
$$\epsilon_c = \frac{\sigma_c}{E} = \frac{397.88}{200\times10^3} = \frac{50\times10^6}{\frac{\pi}{4}\times400^2} = 0.00198$$

$$\delta = \frac{FL}{EA} = \frac{50\times10^6\times3000}{200\times10^3\times\frac{\pi}{4}\times400^2} = 5.69 \text{ mm}$$

Problem 5
A composite shaft AC is made by connecting a 20-mm diameter solid steel shaft AB to a 40-mm diameter solid aluminum shaft BC at flange B. The shear modulus of the steel is 72 GPa and the shear modulus of the aluminum is 24 GPa. A concentrated torque of 500 N.m is applied to the flange in the direction indicated.



Givens:
G_{ST}=72 GPa G_{alu}=24GPa
Solution:
(a):
T=T1+T2
500*1000=T_{st}+T_{alu} →1
$\Theta_B)_{st}$=$\Theta_B)_{Alu}$



$$\frac{T_{st} * L_{st}}{G_{st} * J_{st}} = \frac{T_{al} * L_{al}}{G_{al} * J_{al}}$$

$$\frac{T_{st} * 400 * 32}{72 * 10^3 * \pi * 20^4} = \frac{T_{al} * 1200}{24 * 10^3 * \pi * 40^4}$$

$T_{st} = 0.T_{al}$

→2

From 1&2:
T_{st} =180 N.m
T_{al}=320 N.m

(B):

τ_{max}=(T_{st}*r_{st})/J_{st} = $\frac{180*1000*20/2}{\frac{\pi}{32}*20^4}$=114.6 Mpa

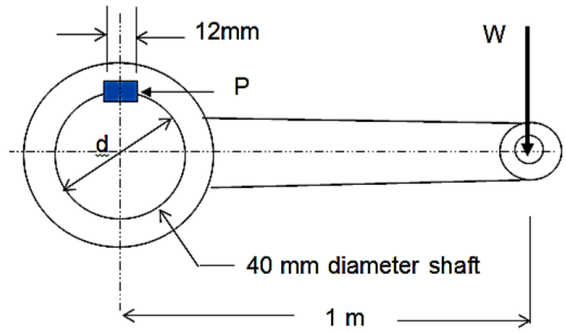
(C):

$\Theta = \frac{Tst*Lst}{Gst*Jst} = 0.063$ rad

$\Theta = 0.063*180/\pi = 3.65^o$

Problem 6

For the key length of 50 mm and allowable shear stress of 65 MPa; find the applied force W.



Given:

L_{eq}=50 mm , τ_{all}=65 Mpa

Solution:

Design eqn:

$\tau_{max} \leq \tau_{all}$

$P/A \leq 65$

$A=t*L=12*50=600\ mm^2$

$\therefore P \leq 3900N$

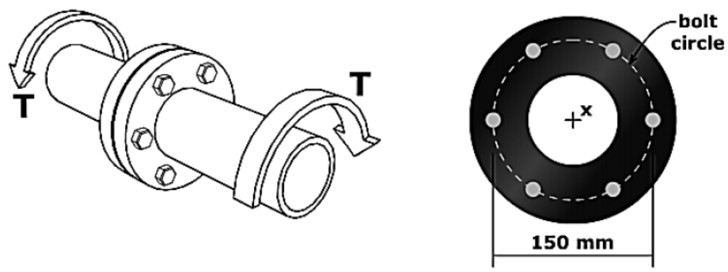
From equilibrium:

$W*1000=P*d/2$

$\therefore W \leq 780N$

Problem 7

Two lengths of pipe are joined with a bolted-flange connection consisting of six bolts. The diameter of the bolt circle (see sketch) is 150 mm. A torque of T = 12 kN.m is applied to the pipes as shown. Assume the allowable shear stress for the bolts is 200 MPa; determine the minimum diameter required for the bolts in the connection. (Disregard friction between the two flanges.)



Solution:

$T=12*10^6$ N.mm

$\tau_{\text{shear of bolts}}$=200 Mpa

$F=T/(r*n_{\text{of bolts}})=12*10^6/(150/2*6)=26.667*10^3$ N

$\tau_{\text{shear=Fb/Ab}} \rightarrow 200=26.667*10^3/(\Pi/4*d^2)$

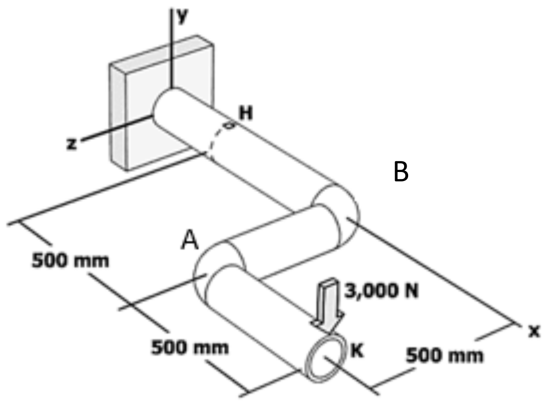
$\therefore$ d=12.029 mm

we will not find a bolt with a diameter of 12.029 in the market because bolts are manufactured in standard diameters. Therefore, we will look in the market for the next bigger diameter

d ≈ 13 mm

Problem 8

The pipe shown has an outside diameter of 160 mm and an inside diameter of 144 mm. A concentrated load of 3,000 N acts at the free end K. Determine the normal and shear stresses produced at point H on the surface of the pipe. Show these stresses on a stress element.

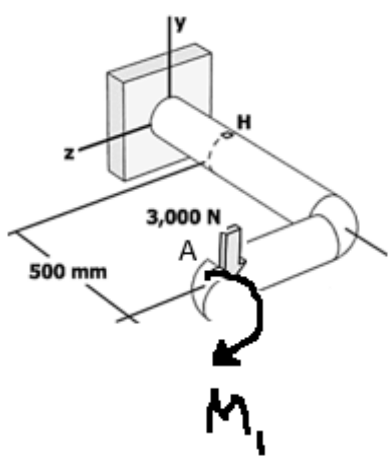


D_o=160mm ,D_i=144mm

Req:$\sigma_{max,}\tau_{max}$

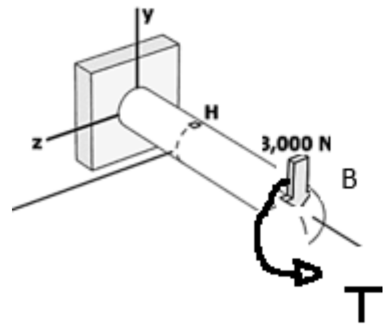
Solution

- move force from (k) → (A)
  F=3000N, M_1=F* AK =3000*500=1500000 N.mm

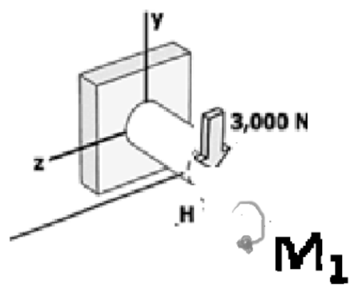


- move force from (A) → (B)

  F=3000N, T=F* AB =3000*500=1500000 N.mm



- move force from (B) → (H)
  F=3000N, M_2=F* BH =3000*500=1500000 N.mm



M= M_{2+} M_1=3*10^6 N.mm
T=15*10^5 N.mm
F=3000N

Bending stress

$$\sigma = \frac{M\,Y}{I} = \frac{3 * 10^6 * 160 * 0.5}{\frac{\pi}{64}(160^4 - 144^4)} = 21.6\ Mpa$$

Torsional stress

$$\tau = \frac{T * r}{J} = \frac{1.5 * 10^6 * 160 * 0.5}{\frac{\pi}{32}(160^4 - 144^4)} = 5.42337 Mpa$$

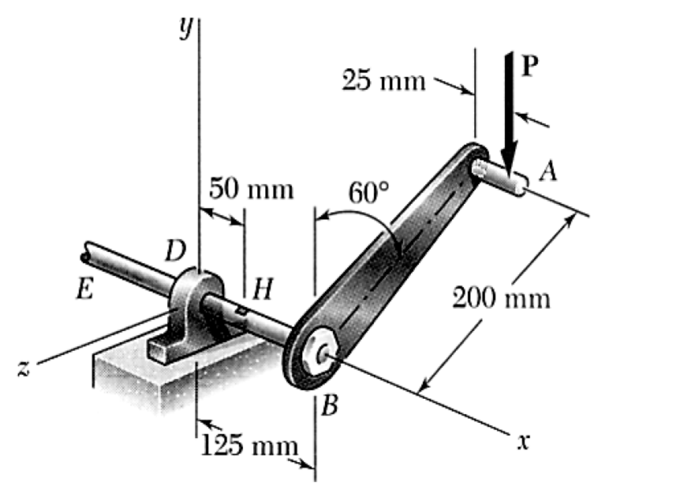
Shear due to bending

$$\tau = 0$$

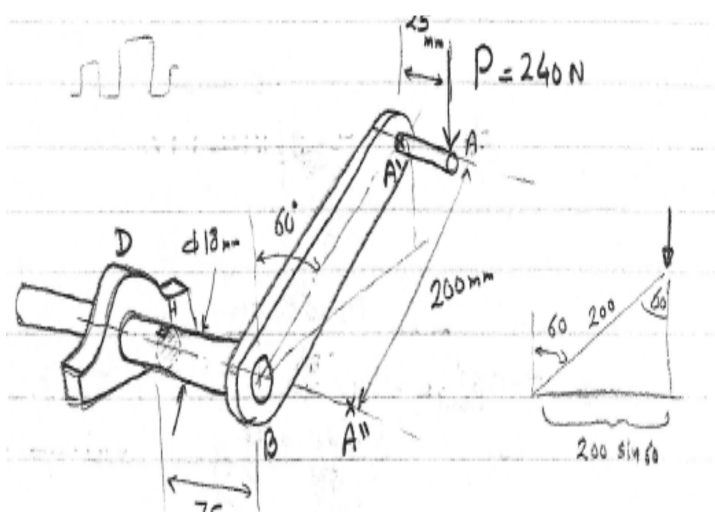
Problem 9

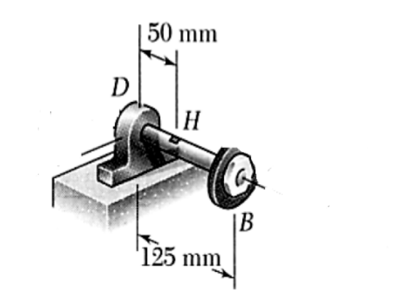
A vertical force P of magnitude 240 N is applied to the crank at point A. Knowing that the shaft BDE has a diameter of 18 mm. Determine the normal and shear stresses acting at point H located at the top of the shaft, 50 mm to the right of support D.

Request: σ_{max} , τ_{max} at point H



Solution





→ Move F from A to A''

It will be (F ,T=F*200*sin(60)=41559.2)

→ Move from A" to A

It will be (F +M=F*100=24000 N.mm)

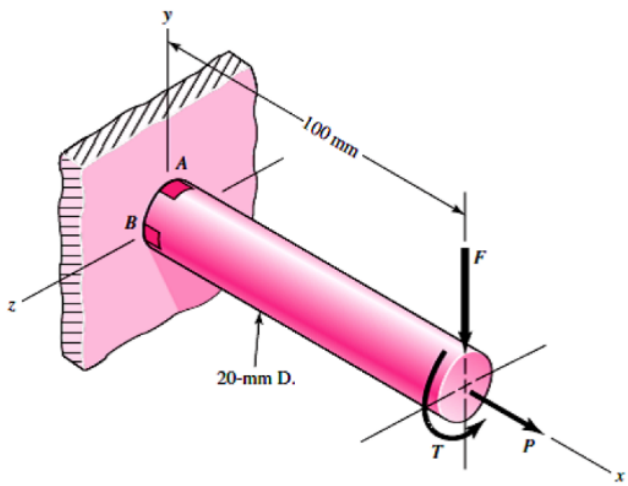
$\sigma=\sigma_{ben}=M*y/I=32*M/(\pi*18^3)=42Mpa$

$\sigma_x=\sigma_{ben}=42$ Mpa

$\tau=(16*T)/(\ /(\pi*d^3\ )=36.3$ Mpa

Problem 10

Determine the normal and shear stresses at points A and B of the solid shaft shown below. The vertical force F = 2 KN , the horizontal force P = 5 kN , the moment T = 4 N.m.



Solution :

F=2 KN

M=2*1000*100=2*10^5 N.mm

σ_t=f/A=5*1000/(π /4*20^2)=15.915 Mpa

T → τ

τ=T*r/J=(4*1000*20/2)/(π /32*20^4)=2.5465 Mpa

Point A:

σ_a=M*y/I=(2*10^5*20/2)/(π /64*20^4)=254.64 Mpa

σ_t=f/A=5*1000/(π /4*20^2)=15.915 Mpa

τ=T*r/J=(4*1000*20/2)/(π /32*20^4)=2.5465 Mpa

$\tau_{\text{shear due to bending}}$=0

Point B:

σ_B=0

σ_t=f/A=5*1000/(π /4*20^2)=15.915 Mpa

τ=T*r/J=(4*1000*20/2)/(π /32*20^4)=2.5465 Mpa

$\tau_{\text{shear due to bending}}$=(4/3)*(f/A)=8.485 Mpa