Finite Element Analysis
Lecture 2

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Basic Steps in the Finite Element Methods

Preprocessing Phase

1. Create and discretize the solution domain into finite elements, that is, subdivide the problem into nodes and elements.
2. Assume a shape function to represent the physical behavior of an element; that is, a continuous function to represent the approximate behavior (solution) of an element. (Element Selection)
3. Develop equations for an element.
4. Assemble the elements to present to [resent the entire problem. Construct the global stiffness Matrix.
5. Apply Boundary Conditions, initial conditions and loading

Basic Steps in the Finite Element Methods

Solution Phase

6. Solve a set of linear or non linear algebraic equations simultaneously to otain nodal results, such as displacement values at different odes or temperature values at different nodes in heat transfer problem.

Postprocessing Phase

7. Obtain other important information such as principal stresses and heat fluxes.

Example 1-1 , Moaveni, P. 8

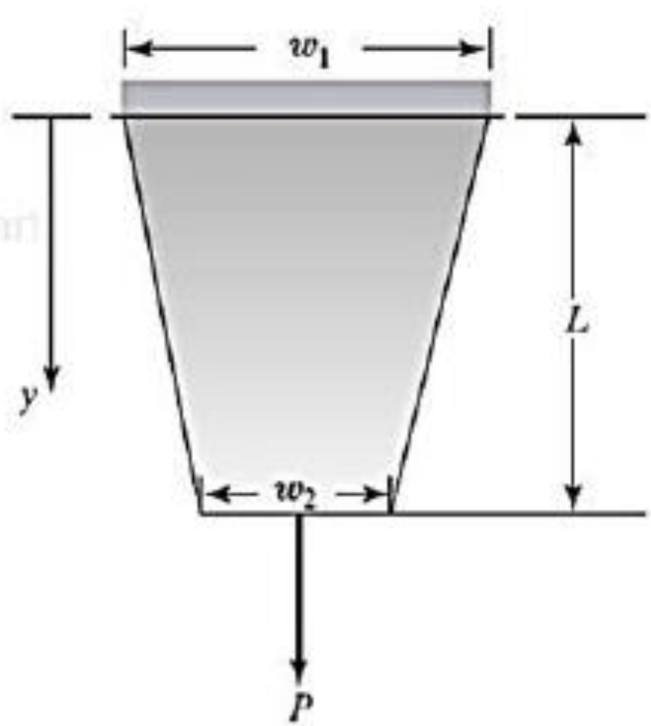


FIGURE 1.1 A bar under axial loading.

Example 1-1 , Moaveni, P. 8

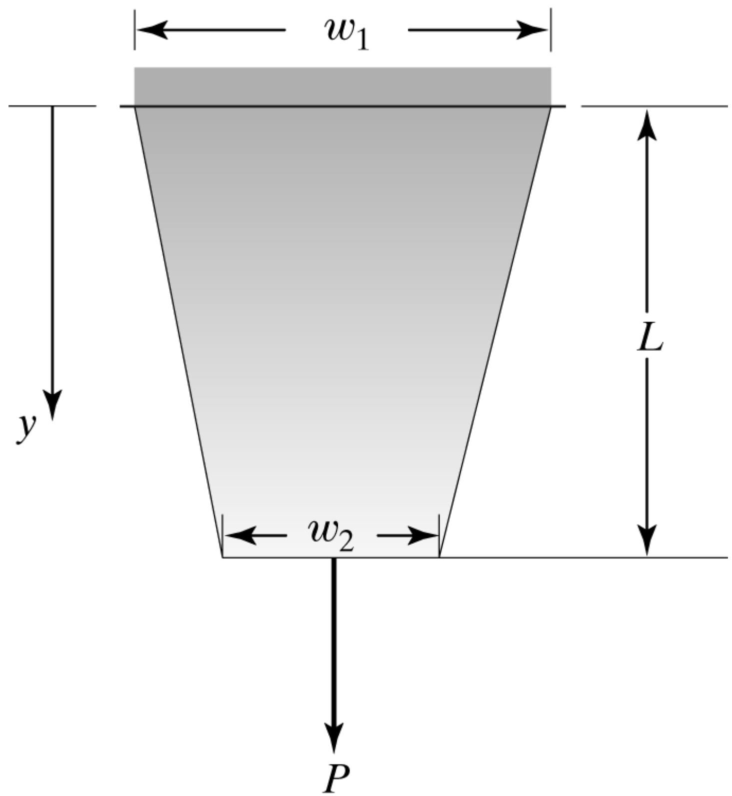


Figure 1-1
A bar under axial loading.

Example 1-1 , Moaveni, P. 8

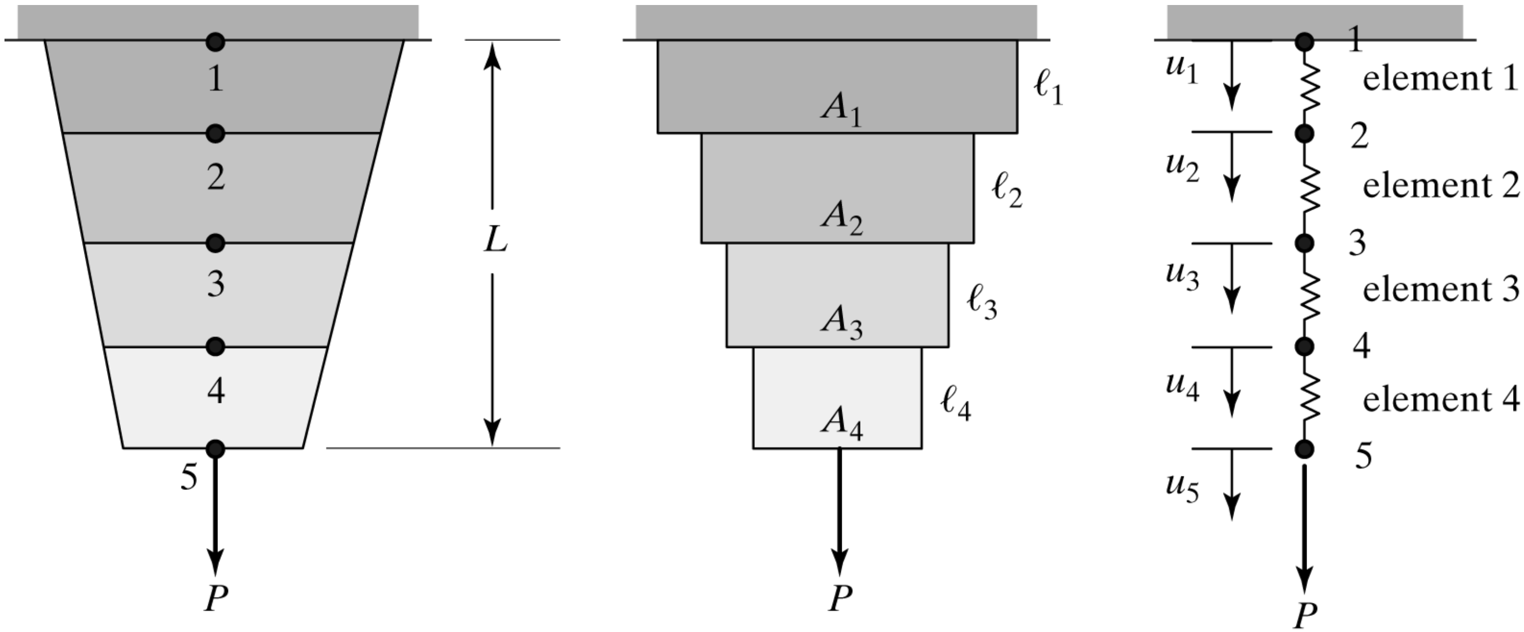


Figure 1-2
Subdividing the bar into elements and nodes.

Example 1-1 , Moaveni, P. 8

The average stress σ in the member is given by

$$\sigma = \frac{F}{A} \tag{1.1}$$

The average normal strain ε of the member is defined as the change in length per unit original length ℓ of the member:

$$\varepsilon = \frac{\Delta \ell}{\ell} \tag{1.2}$$

Over the elastic region, the stress and strain are related by Hooke's law, according to the equation

$$\sigma = E\varepsilon \tag{1.3}$$

where E is the modulus of elasticity of the material. Combining Eqs. (1.1), (1.2), and (1.3) and simplifying, we have

$$F = \left(\frac{AE}{\ell}\right)\Delta \ell \tag{1.4}$$

Example 1-1 , Moaveni, P. 8

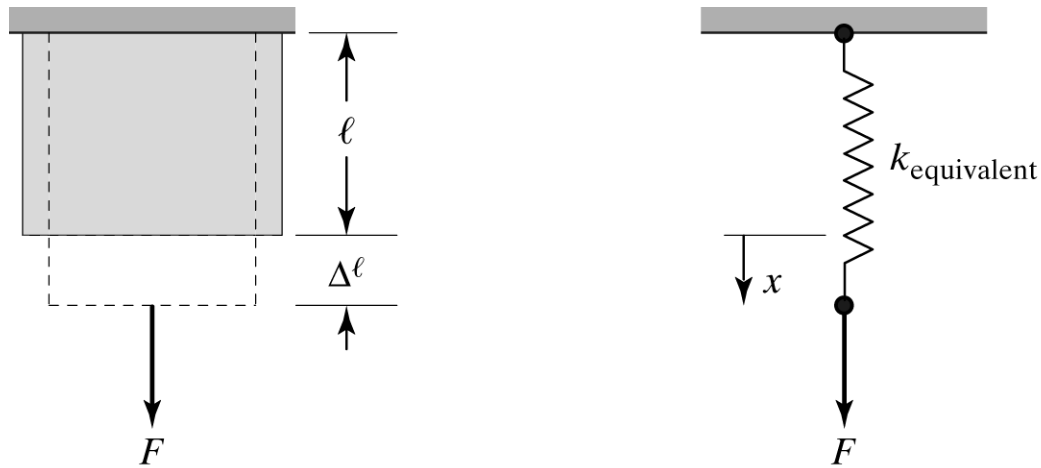


Figure 1-3

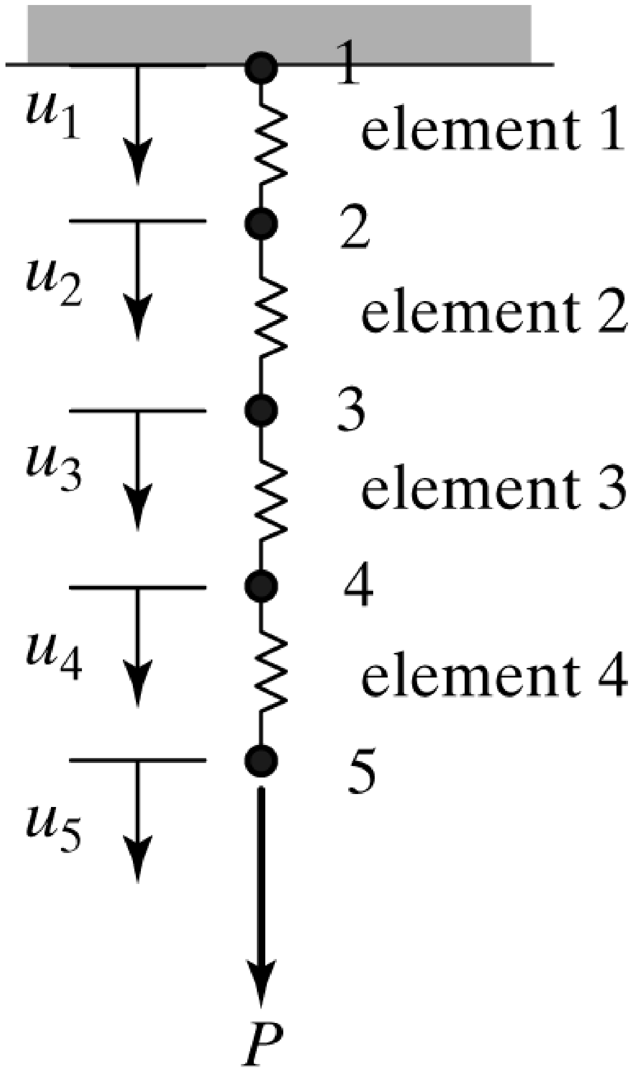
A solid member of uniform cross section subjected to a force F.

$$F = \left(\frac{AE}{\ell}\right)\Delta\ell \tag{1.4}$$

Note that Eq. (1.4) is similar to the equation for a linear spring, $F = kx$. Therefore, a centrally loaded member of uniform cross section may be modeled as a spring with an equivalent stiffness of

$$k_{eq} = \frac{AE}{\ell} \tag{1.5}$$

Example 1-1 , Moaveni, P. 8



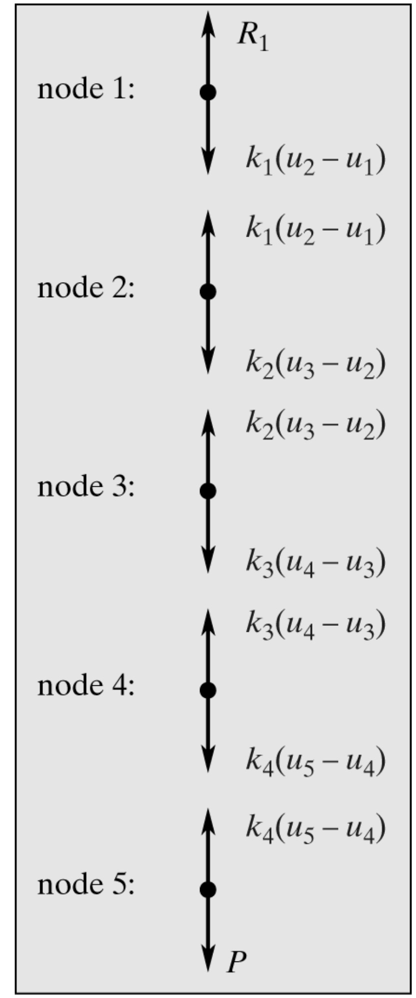


Figure 1-4
Free body diagram of the nodes in Example 1.1.

Example 1-1 , Moaveni, P. 8

Static equilibrium requires that the sum of the forces acting on each node be
ero. This requirement creates the following five equations:

$$\begin{aligned}
&\text{node 1:} \quad R_1 - k_1(u_2 - u_1) = 0 \qquad (1.8)\\
&\text{node 2:} \quad k_1(u_2 - u_1) - k_2(u_3 - u_2) = 0\\
&\text{node 3:} \quad k_2(u_3 - u_2) - k_3(u_4 - u_3) = 0\\
&\text{node 4:} \quad k_3(u_4 - u_3) - k_4(u_5 - u_4) = 0\\
&\text{node 5:} \quad k_4(u_5 - u_4) - P = 0
\end{aligned}$$

Example 1-1 , Moaveni, P. 8

Rearranging the equilibrium equations given by Eq. (1.8) by separating the reaction force R_1 and the applied external force P from the internal forces, we have

$$
\begin{aligned}
k_1u_1 - k_1u_2 &= -R_1 \\
-k_1u_1 + k_1u_2 + k_2u_2 - k_2u_3 &= 0 \\
-k_2u_2 + k_2u_3 + k_3u_3 - k_3u_4 &= 0 \\
-k_3u_3 + k_3u_4 + k_4u_4 - k_4u_5 &= 0 \\
-k_4u_4 + k_4u_5 &= P
\end{aligned}
\tag{1.9}
$$

Example 1-1 , Moaveni, P. 8

Presenting the equilibrium equations of Eq. (1.9) in a matrix form, we have

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} -R_1 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix} \qquad (1.10)$$

It is also important to distinguish between the reaction forces and the applied loads in the load matrix. Therefore, the matrix relation of Eq. (1.10) can be written as

$$\begin{Bmatrix} -R_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix} \qquad (1.11)$$

$$\{\mathbf{R}\} = [\mathbf{K}]\{\mathbf{u}\} - \{\mathbf{F}\} \qquad (1.12)$$

Example 1-1 , Moaveni, P. 8

We can readily show that under additional nodal loads and other fixed boundary conditions, the relationship given by Eq. (1.11) can be put into the general form

$$\{\mathbf{R}\} = [\mathbf{K}]\{\mathbf{u}\} - \{\mathbf{F}\} \tag{1.12}$$

which stands for

$$\{\textbf{reaction matrix}\} = [\textbf{stiffness matrix}]\{\textbf{displacement matrix}\} - \{\textbf{load matrix}\}$$

Note the difference between applied load matrix $\{\mathbf{F}\}$ and the reaction force matrix $\{\mathbf{R}\}$.

Example 1-1 , Moaveni, P. 8
Applying Boundary Conditions

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix} \quad (1.$$

The solution of the above matrix yields the nodal displacement values. It sho be clear from the above explanation and examining Eq. (1.13) that for sc mechanics problems, the application of boundary conditions to the finite elem formulations transforms the system of equations as given by Eq. (1.11) to a r general form that is made up of only the stiffness matrix, the displacement mat and the load matrix:

Example 1-1 , Moaveni, P. 8
Develop Equation for one Element

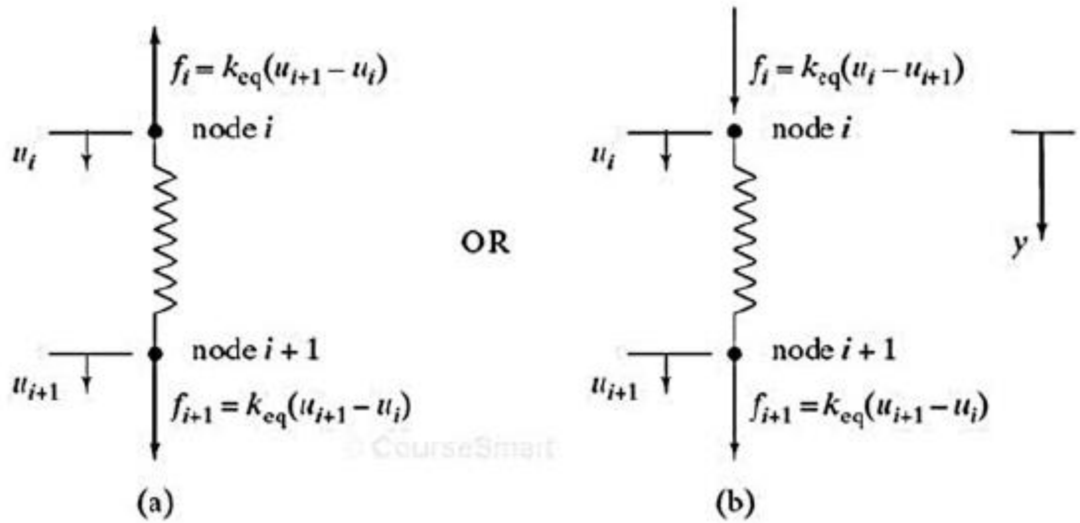


FIGURE 1.5 Internally transmitted forces through an arbitrary element.

nodes i and $i + 1$ according to the following equations:

$$f_i = k_{eq}(u_i - u_{i+1})$$
$$f_{i+1} = k_{eq}(u_{i+1} - u_i)$$

Equation (1.14) can be expressed in a matrix form by

$$\begin{Bmatrix} f_i \\ f_{i+1} \end{Bmatrix} = \begin{bmatrix} k_{eq} & -k_{eq} \\ -k_{eq} & k_{eq} \end{bmatrix} \begin{Bmatrix} u_i \\ u_{i+1} \end{Bmatrix}$$

Example 1-1 , Moaveni, P. 8
Elements Assembly

4. *Assemble the elements to present the entire problem.*
Applying the elemental description given by Eq. (1.15) to all elements and assembling them (putting them together) will lead to the formation of the global stiffness matrix. The stiffness matrix for element (1) is given by

$$[\mathbf{K}]^{(1)} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix}$$

and its position in the global stiffness matrix is given by

$$[\mathbf{K}]^{(1G)} = \begin{array}{c} \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}$$

Example 1-1 , Moaveni, P. 8
Elements Assembly

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$$[\mathbf{K}]^{(1G)} = \begin{array}{c} \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}$$

Example 1-1 , Moaveni, P. 8
Elements Assembly

The nodal displacement matrix is shown alongside the position of element 1 in the global stiffness matrix to aid us to observe the contribution of a node to its neighboring elements. Similarly, for elements (2), (3), and (4), we have

$$[\mathbf{K}]^{(2)} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

and its position in the global matrix

$$[\mathbf{K}]^{(2G)} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}$$

Example 1-1 , Moaveni, P. 8
Elements Assembly

$$[\mathbf{K}]^{(3)} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix}$$

and its position in the global matrix

$$[\mathbf{K}]^{(3G)} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}$$

Example 1-1 , Moaveni, P. 8
Elements Assembly

$$[\mathbf{K}]^{(4)} = \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix}$$

and its position in the global matrix

$$[\mathbf{K}]^{(4G)} = \begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \end{array} \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array}$$

Example 1-1 , Moaveni, P. 8
Elements Assembly

The final global stiffness matrix is obtained by assembling, or adding together, each element's position in the global stiffness matrix:

$$[\mathbf{K}]^{(G)} = [\mathbf{K}]^{(1G)} + [\mathbf{K}]^{(2G)} + [\mathbf{K}]^{(3G)} + [\mathbf{K}]^{(4G)}$$

$$[\mathbf{K}]^{(G)} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \quad (1.16)$$

Note that the global stiffness matrix obtained using elemental description, as given by Eq. (1.16), is identical to the global stiffness matrix we obtained earlier from the analysis of the free-body diagrams of the nodes, as given by the left-hand side of Eq. (1.10).

Example 1-1 , Moaveni, P. 8
Applying Boundary Conditions

The bar is fixed at the top, which leads to the boundary condition $u_1 = 0$. The external load P is applied at node 5. Applying these conditions results in the following set of linear equations.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P \end{Bmatrix} \qquad (1.17)$$

Example 1-1 , Moaveni, P. 8
Solution Phase

6. *Solve a system of algebraic equations simultaneously.*
In order to obtain numerical values of the nodal displacements, let us assume that $E = 10.4 \times 10^6$ lb/in^2 (aluminum), $w_1 = 2$ in, $w_2 = 1$ in, $t = 0.125$ in, $L = 10$ in, and $P = 1000$ lb. You may consult Table 1.5 while working toward the solution.

TABLE 1.5 Properties of the elements in Example 1.1

Element	Nodes	Average Cross-Sectional Area (in^2)	Length (in)	Modulus of Elasticity (lb/in^2)	Element's Stiffness Coefficient (lb/in)
1	1 2	0.234375	2.5	10.4×10^6	975×10^3
2	2 3	0.203125	2.5	10.4×10^6	845×10^3
3	3 4	0.171875	2.5	10.4×10^6	715×10^3
4	4 5	0.140625	2.5	10.4×10^6	585×10^3

Example 1-1 , Moaveni, P. 8
Solution Phase

The variation of the cross-sectional area of the bar in the y-direction can be expressed by:

$$A(y) = \left(w_1 + \left(\frac{w_2 - w_1}{L}\right)y\right)t = \left(2 + \frac{(1-2)}{10}y\right)(0.125) = 0.25 - 0.0125y \quad (1.18)$$

Using Eq. (1.18), we can compute the cross-sectional areas at each node:

$A_1 = 0.25 \text{ in}^2$ $\quad A_2 = 0.25 - 0.0125(2.5) = 0.21875 \text{ in}^2$

$A_3 = 0.25 - 0.0125(5.0) = 0.1875 \text{ in}^2$ $\quad A_4 = 0.25 - 0.0125(7.5) = 0.15625 \text{ in}^2$

$A_5 = 0.125 \text{ in}^2$

Example 1-1 , Moaveni, P. 8
Solution Phase

Next, the equivalent stiffness coefficient for each element is computed from the equations

$$k_{eq} = \frac{(A_{i+1} + A_i)E}{2\ell}$$

$$k_1 = \frac{(0.21875 + 0.25)(10.4 \times 10^6)}{2(2.5)} = 975 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$k_2 = \frac{(0.1875 + 0.21875)(10.4 \times 10^6)}{2(2.5)} = 845 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$k_3 = \frac{(0.15625 + 0.1875)(10.4 \times 10^6)}{2(2.5)} = 715 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$k_4 = \frac{(0.125 + 0.15625)(10.4 \times 10^6)}{2(2.5)} = 585 \times 10^3 \frac{\text{lb}}{\text{in}}$$

Example 1-1 , Moaveni, P. 8
Solution Phase

and the elemental matrices are

$$[\mathbf{K}]^{(1)} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} = 10^3 \begin{bmatrix} 975 & -975 \\ -975 & 975 \end{bmatrix}$$

$$[\mathbf{K}]^{(2)} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} = 10^3 \begin{bmatrix} 845 & -845 \\ -845 & 845 \end{bmatrix}$$

$$[\mathbf{K}]^{(3)} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} = 10^3 \begin{bmatrix} 715 & -715 \\ -715 & 715 \end{bmatrix}$$

$$[\mathbf{K}]^{(4)} = \begin{bmatrix} k_4 & -k_4 \\ -k_4 & k_4 \end{bmatrix} = 10^3 \begin{bmatrix} 585 & -585 \\ -585 & 585 \end{bmatrix}$$

Example 1-1 , Moaveni, P. 8
Solution Phase

Assembling the elemental matrices leads to the generation of the global stiffness matrix:

$$[\mathbf{K}]^{(G)} = 10^3 \begin{bmatrix} 975 & -975 & 0 & 0 & 0 \\ -975 & 975 + 845 & -845 & 0 & 0 \\ 0 & -845 & 845 + 715 & -715 & 0 \\ 0 & 0 & -715 & 715 + 585 & -585 \\ 0 & 0 & 0 & -585 & 585 \end{bmatrix}$$

Example 1-1 , Moaveni, P. 8
Solution Phase

Applying the boundary condition $u_1 = 0$ and the load $P = 1000$ lb, we get

$$10^3 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -975 & 1820 & -845 & 0 & 0 \\ 0 & -845 & 1560 & -715 & 0 \\ 0 & 0 & -715 & 1300 & -585 \\ 0 & 0 & 0 & -585 & 585 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10^3 \end{Bmatrix}$$

Example 1-1 , Moaveni, P. 8
Solution Phase

Because in the second row, the −975 coefficient gets multiplied by $u_1 = 0$, we need only to solve the following 4×4 matrix:

$$10^3 \begin{bmatrix} 1820 & -845 & 0 & 0 \\ -845 & 1560 & -715 & 0 \\ 0 & -715 & 1300 & -585 \\ 0 & 0 & -585 & 585 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 10^3 \end{Bmatrix}$$

The displacement solution is $u_1 = 0$, $u_2 = 0.001026$ in, $u_3 = 0.002210$ in, $u_4 = 0.003608$ in, and $u_5 = 0.005317$ in.

Example 1-1 , Moaveni, P. 8
Post processing Phase
Obtaining Stress in each element

$$\sigma = \frac{f}{A_{avg}} = \frac{k_{eq}(u_{i+1} - u_i)}{A_{avg}} = \frac{\frac{A_{avg}E}{\ell}(u_{i+1} - u_i)}{A_{avg}} = E\left(\frac{u_{i+1} - u_i}{\ell}\right) \qquad (1.19)$$

Since the displacements of different nodes are known, Eq. (1.19) could have been obtained directly from the relationship between the stresses and strains,

$$\sigma = E\varepsilon = E\left(\frac{u_{i+1} - u_i}{\ell}\right) \qquad (1.20)$$

Example 1-1 , Moaveni, P. 8
Post processing Phase
Obtaining Stress in each element

Employing Eq. (1.20) in Example 1.1, we compute the average normal stress for each element as

$$\sigma^{(1)} = E\left(\frac{u_2 - u_1}{\ell}\right) = \frac{(10.4 \times 10^6)(0.001026 - 0)}{2.5} = 4268 \ \frac{\text{lb}}{\text{in}^2}$$

$$\sigma^{(2)} = E\left(\frac{u_3 - u_2}{\ell}\right) = \frac{(10.4 \times 10^6)(0.002210 - 0.001026)}{2.5} = 4925 \ \frac{\text{lb}}{\text{in}^2}$$

$$\sigma^{(3)} = E\left(\frac{u_4 - u_3}{\ell}\right) = \frac{(10.4 \times 10^6)(0.003608 - 0.002210)}{2.5} = 5816 \ \frac{\text{lb}}{\text{in}^2}$$

$$\sigma^{(4)} = E\left(\frac{u_5 - u_4}{\ell}\right) = \frac{(10.4 \times 10^6)(0.005317 - 0.003608)}{2.5} = 7109 \ \frac{\text{lb}}{\text{in}^2}$$

Example 1-1 , Moaveni, P. 8

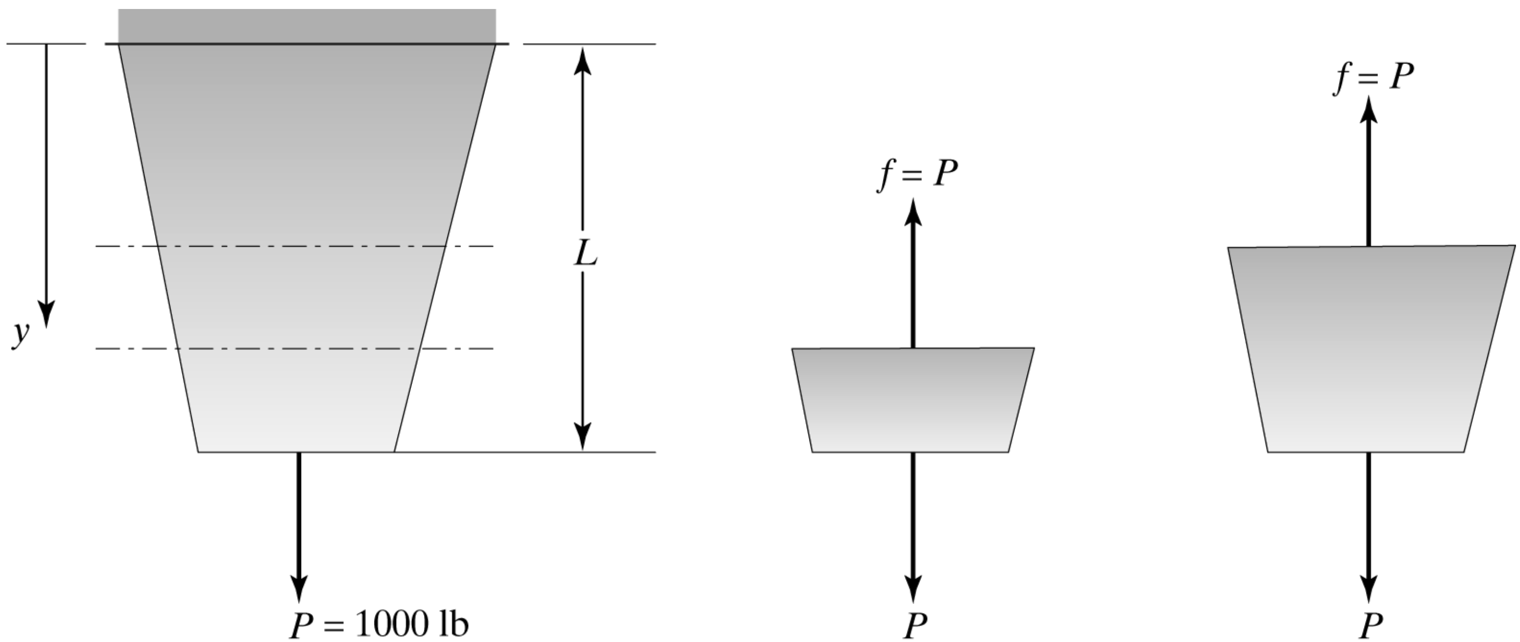

Figure 1-6
The internal forces in Example 1.1.

Example 1-1 , Moaveni, P. 8
Post processing Phase
Obtaining Stress in each element

In Figure 1.6, we note that for the given problem, regardless of where we cut a section through the bar, the internal force at the section is equal to 1000 lb. So,

$$\sigma^{(1)} = \frac{f}{A_{avg}} = \frac{1000}{0.234375} = 4267\,\frac{\text{lb}}{\text{in}^2}$$

$$\sigma^{(2)} = \frac{f}{A_{avg}} = \frac{1000}{0.203125} = 4923\,\frac{\text{lb}}{\text{in}^2}$$

$$\sigma^{(3)} = \frac{f}{A_{avg}} = \frac{1000}{0.171875} = 5818\,\frac{\text{lb}}{\text{in}^2}$$

$$\sigma^{(4)} = \frac{f}{A_{avg}} = \frac{1000}{0.140625} = 7111\,\frac{\text{lb}}{\text{in}^2}$$

Example 1-1 , Moaveni, P. 8
Post processing Phase
Obtaining Stress in each element

Reaction Forces For Example 1.1, the reaction force may be computed in a number of ways. First, referring to Figure 1.4, we note that the statics equilibrium at node 1 requires

$$R_1 = k_1(u_2 - u_1) = 975 \times 10^3(0.001026 - 0) = 1000 \text{ lb}$$

The statics equilibrium for the entire bar also requires that

$$R_1 = P = 1000 \text{ lb}$$

As you may recall, we can also compute the reaction forces from the general reaction equation

$$\{\mathbf{R}\} = [\mathbf{K}]\{\mathbf{u}\} - \{\mathbf{F}\}$$

or

$$\{\textbf{reaction matrix}\} = [\textbf{stiffness matrix}]\{\textbf{displacement matrix}\} - \{\textbf{load matrix}\}$$

Example 1-1 , Moaveni, P. 8
Post processing Phase
Obtaining Stress in each element

Because Example 1.1 is a simple problem, we do not actually need to go through the matrix operations in the aforementioned general equation to compute the reaction forces. However, as a demonstration, the procedure is shown here. From the general equation, we get

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix} = 10^3 \begin{bmatrix} 975 & -975 & 0 & 0 & 0 \\ -975 & 1820 & -845 & 0 & 0 \\ 0 & -845 & 1560 & -715 & 0 \\ 0 & 0 & -715 & 1300 & -585 \\ 0 & 0 & 0 & -585 & 585 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.001026 \\ 0.002210 \\ 0.003608 \\ 0.005317 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10^3 \end{Bmatrix}$$

where R_1, R_2, R_3, R_4, and R_5 represent the reactions forces at nodes 1 through 5 respectively. Performing the matrix operation, we have

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{Bmatrix} = \begin{Bmatrix} -1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Example 1-1 , Moaveni, P. 8
Matlab Code Results

```
K =

      975000     -975000           0           0           0
     -975000     1820000     -845000           0           0
           0     -845000     1560000     -715000           0
           0           0     -715000     1300000     -585000
           0           0           0     -585000      585000


F =

           0
           0
           0
           0
        1000
```

Example 1-1 , Moaveni, P. 8

```
d =

            0
    0.0010256
    0.0022091
    0.0036077
    0.0053171


reactions =

        -1000


results =

   0.00041026        4266.7        1066.7
   0.00047337        4923.1        1076.9
   0.00055944        5818.2        1090.9
   0.00068376        7111.1        1111.1
```