# OPTIMUM DESIGN

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## Lecture 4

# Infinite/multiple solutions

When $f(\mathbf{x})$ is parallel to a binding constraint

$$f(\mathbf{x}) = -x_1 - 0.5x_2$$

$$g_1: \quad 2x_1 + 3x_2 \le 12$$

$$g_2: \quad 2x_1 + x_2 \le 8$$

$$g_3: \; -x_1 \le 0$$

$$g_4: \; -x_2 \le 0$$

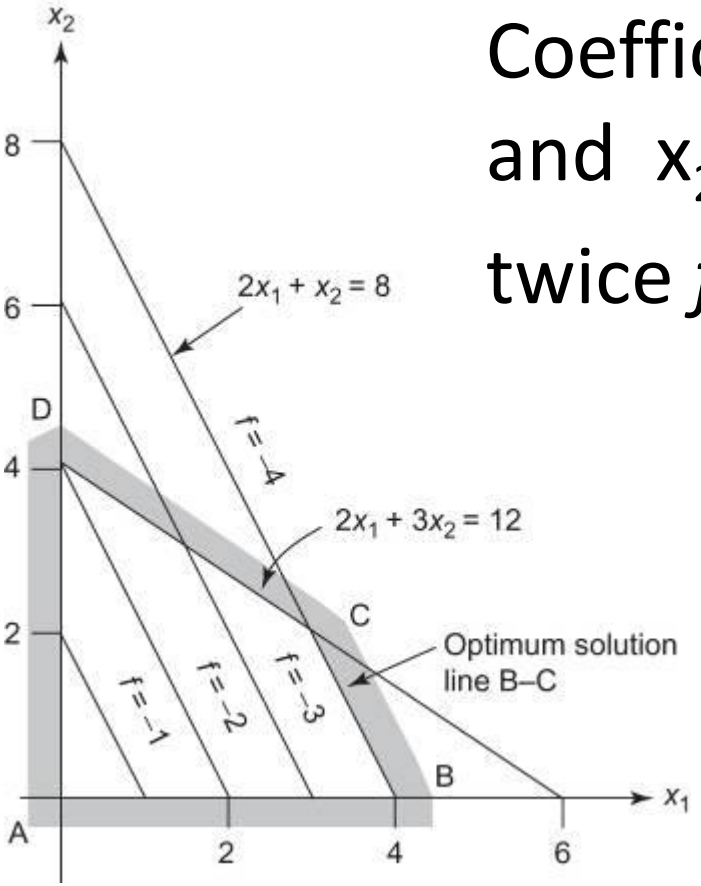
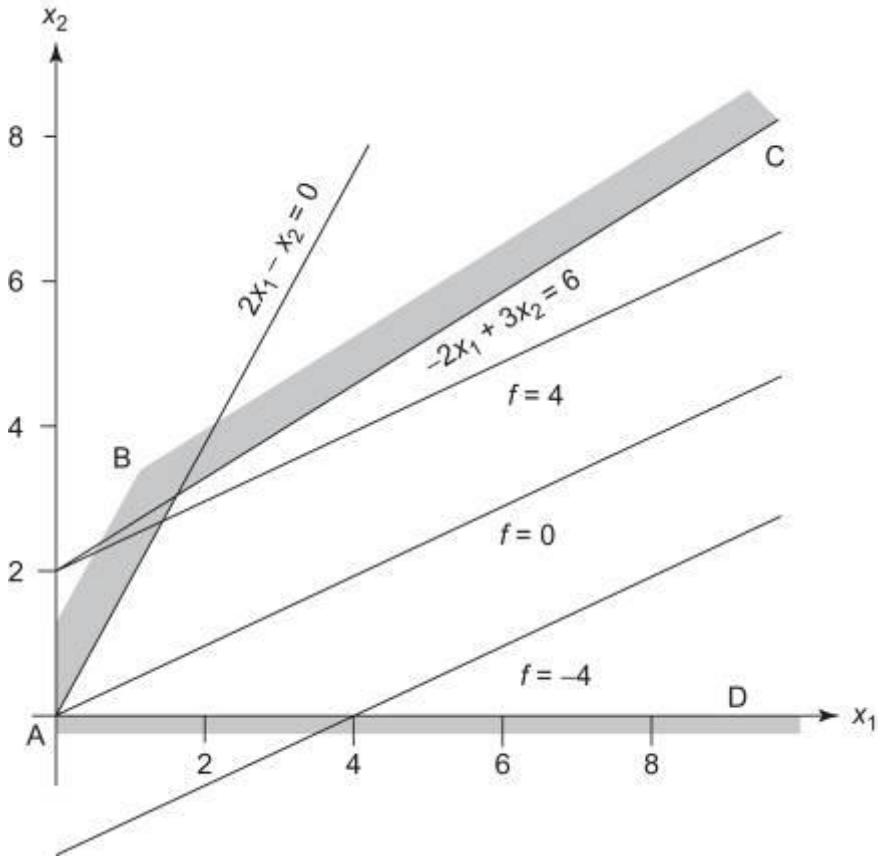



**Figure 3.7** Example problem with multiple solutions.

Coefficient of $x_1$ and $x_2$ in $g_2$ are twice $f(x)$

# Unbound Solution

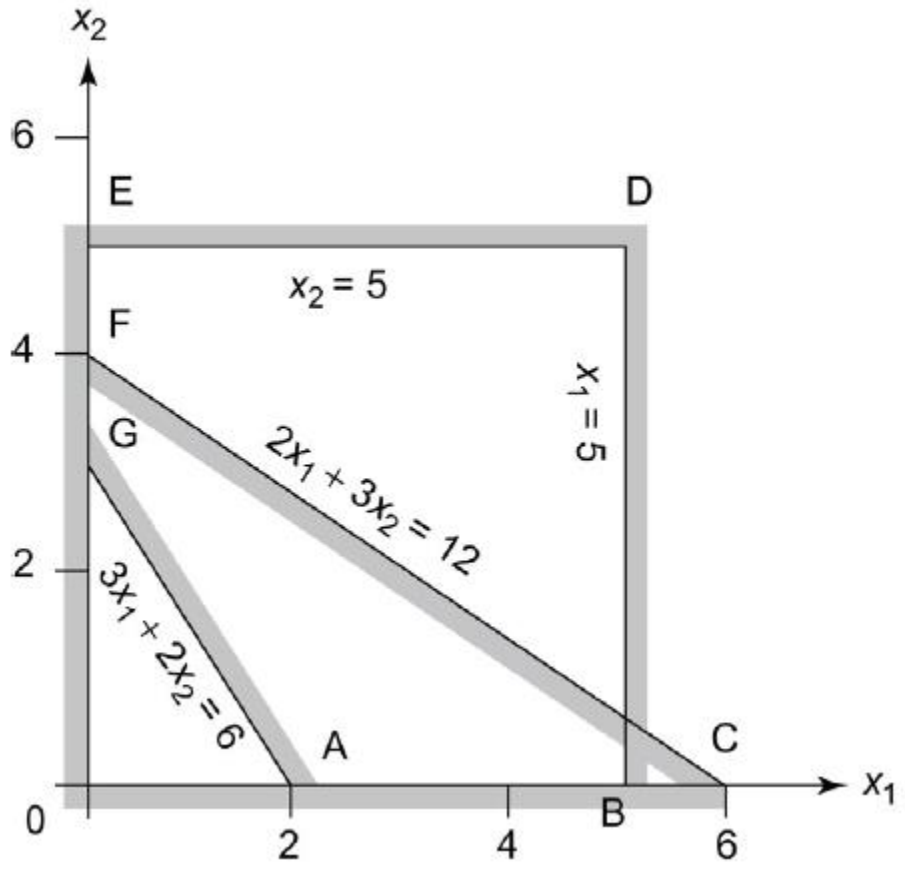
**Figure 3.8** Example problem with an unbounded solution.




Open region

On R.H.S.

# Infeasible design



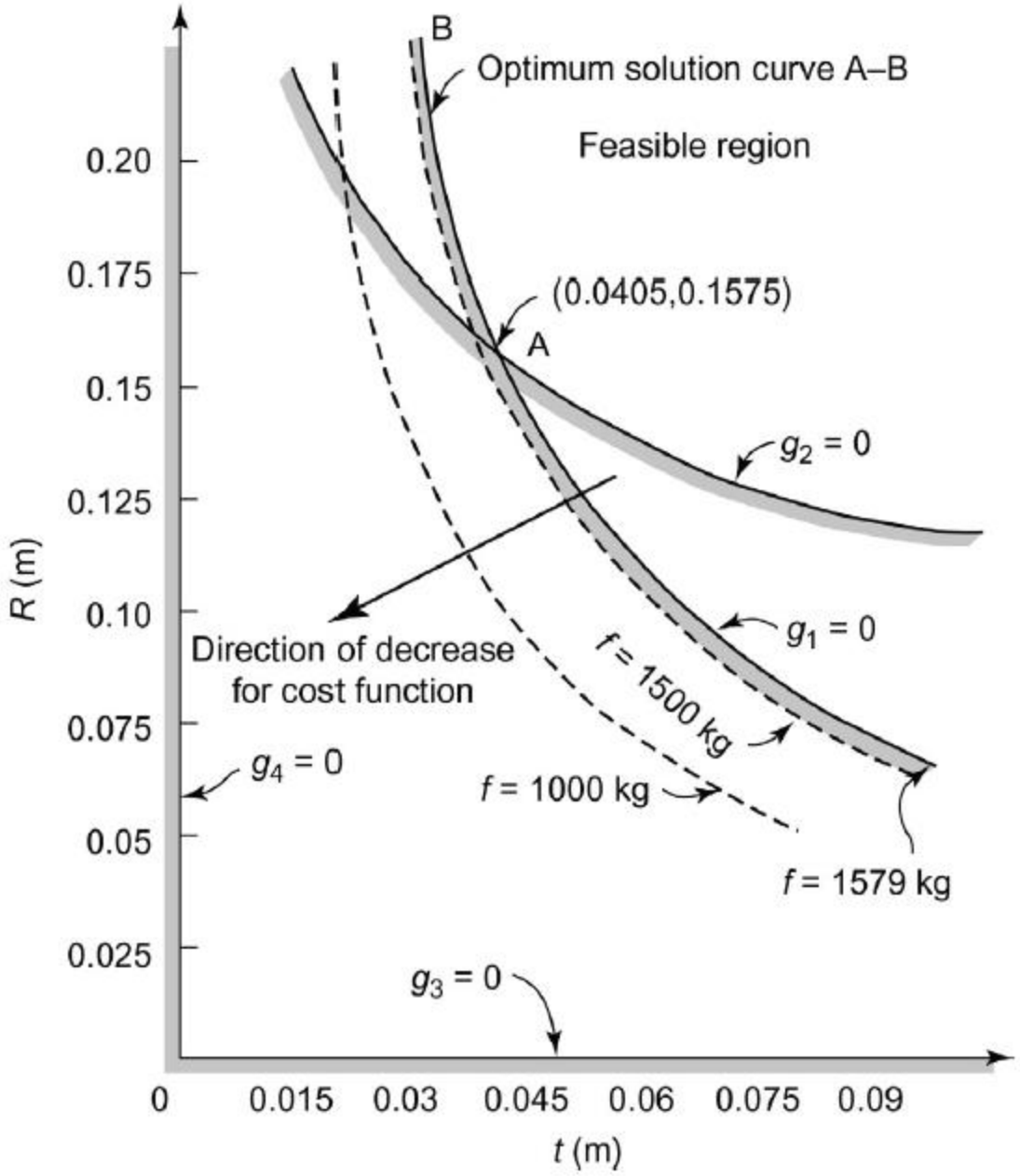
The design problem formulated in Section 2.7 will now be solved by the graphical method using the following data: $P = 10\,\text{MN}, E = 207\,\text{GPa}, \rho = 7833\,\text{kg/m}^3, l = 5.0\,\text{m}$, and $\sigma_a = 248\,\text{MPa}$. Using these data, formulation 1 for the problem is defined as, "find thickness $t$ (m) and mean radius $R$ (m) to minimize the mass function."

$$f(R,t) = 2\rho l\pi Rt = 2(7833)(5)\pi Rt = 2.4608\times 10^5 Rt,\ \text{kg} \tag{a}$$

subject to the four inequality constraints

$$g_1(R,t) = \frac{P}{2\pi Rt} - \sigma_a = \frac{10\times 10^6}{2\pi Rt} - 248\times 10^6 \le 0\ \text{(stress constraint)} \tag{b}$$

$$g_2(R,t) = P - \frac{\pi^3 ER^3 t}{4l^2} = 10\times 10^6 - \frac{\pi^3 (207\times 10^9) R^3 t}{4(5)(5)} \le 0\ \text{(buckling load constraint)} \tag{c}$$


B
Optimum solution curve A–B
Feasible region
0.20
0.175
0.15
0.125
0.10
0.075
0.05
0.025
R (m)
(0.0405,0.1575)
A
$g_2 = 0$
$g_1 = 0$
Direction of decrease
for cost function
$f$ = 1500 kg
$g_4 = 0$
$f$ = 1000 kg
$f$ = 1579 kg
$g_3 = 0$
0
0.015
0.03
0.045
0.06
0.075
0.09
$t$ (m)

# Infeasible Problem

Constraints are:

inconsistent

conflicting

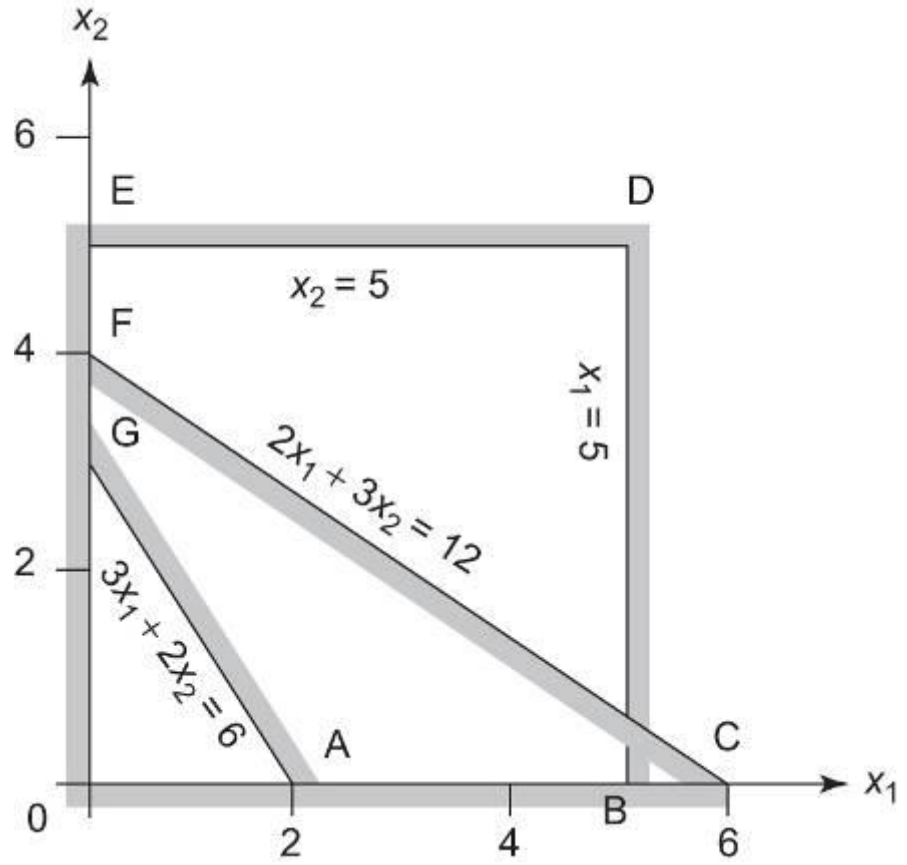



**Figure 3.9** Infeasible design optimization problem.

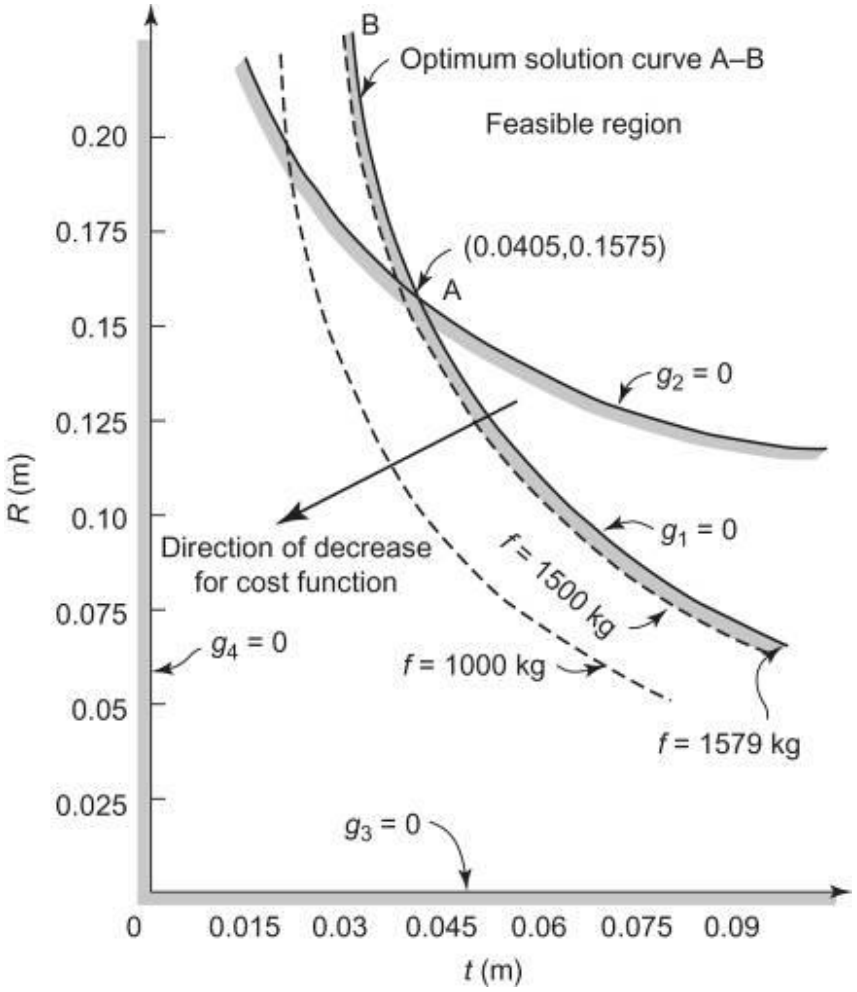
How many inequality constraints can we have?

How many active inequality constraints?

# Non-linear constraints & Inf. Solns

**Figure 3.10** A graphical solution to the problem of designing a minimum-weight tubular column.

Which constraint(s) are active?



# Graphical Solution

1. Sketch coordinate system
2. Plot constraints
3. Determine feasible region
4. Plot f(**x**) contours (2 or 3)
5. Find opt solution **x*** & opt value f(**x***)

# Summary

- Graphical solution – 5 step process
- Feasible region may not exist resulting in an infeasible problem
- When obj function is ll to active/binding $g_i$ an infinite number of solutions exist
- Feasible region may be unbounded
- An unbounded region may result in an unbounded solution