Alexandria University
Faculty of Engineering
Mechanical Engineering Department
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جامعة الاسكندرية
كلية الهندسة
قسم الهندسة الميكانيكية
نوفمبر 2016

Optimum Design — التصميم الأمثل
Fourth Year — السنة الرابعة
Time Allowed: 1 Hour — الزمن: 1 ساعة

Answer the following two questions:

QUESTION ONE (10 points):

A refinery has three types of crude oil C_1, C_2 and C_3. Crude oil C_1costs \$0.40/L and there are at most 10,000 L/day of it available. Crude oil C_2 costs \$0.20/L and there are at most 12,000 L/day of it available. Crude oil C_3 costs \$0.10/L and there are at most 15,000 L/day of it available. The refinery can convert each type of crude oil to gasoline and can produce three types of gasoline: regular, plus, and premium. The maximum market demand for the regular, plus, and premium gasoline is 9,000 L/day, 8,000 L/day, and 7,000 L/day, respectively. The refinery can sell its gasoline to a distributor for \$0.70/L for regular, \$0.80/L for plus, and \$0.9/L for premium gasoline. It is found that (1) 1 L of C_1 crude oil yields 0.2 L of regular, 0.3 L of plus, and 0.5 L of premium gasoline, (2) 1 L of C_2 crude oil yields 0.5 L of regular, 0.3 L of plus gasoline, and 0.2 L of premium gasoline, and (3) 1 L of C_3 crude oil yields 0.7 L of regular, 0.3 L of plus, and no premium gasoline. We shall determine the number of liters of each of the crude oils C_1, C_2 and C_3 that the refinery should purchase to maximize its daily profit. Set the objective function as a minimization problem.

Let x_1, x_2, and x_3 represent the number of liters for crude oils C_1, C_2, and C_3, respectively, that is to be purchased. Then, the objective function f and constraints on gasoline demands and crude oil availability are

$$\begin{aligned}
\text{minimize } f(x_1, x_2, x_3) &= -[0.7(0.2\,x_1 + 0.5\,x_2 + 0.7\,x_3) + 0.8(0.3\,x_1 + 0.3\,x_2 + 0.3\,x_3) \\
&\quad +0.9(0.5x_1 + 0.2x_2) - 0.4x_1 - 0.2x_2 - 0.1x_3] \\
&= -0.43x_1 - 0.57x_2 - 0.63x_3
\end{aligned}$$

$$\begin{aligned}
\text{subject to: }\ & 0.2x_1 + 0.5x_2 + 0.7x_3 \le 9{,}000 \\
& 0.3x_1 + 0.3x_2 + 0.3x_3 \le 8{,}000 \\
& 0.5x_1 + 0.2x_2 \le 7{,}000 \\
& x_1 \le 10{,}000 \\
& x_2 \le 12{,}000 \\
& x_3 \le 15{,}000 \\
& (x_1, x_2, x_3) \ge 0
\end{aligned}$$

QUESTION TWO (10 points):

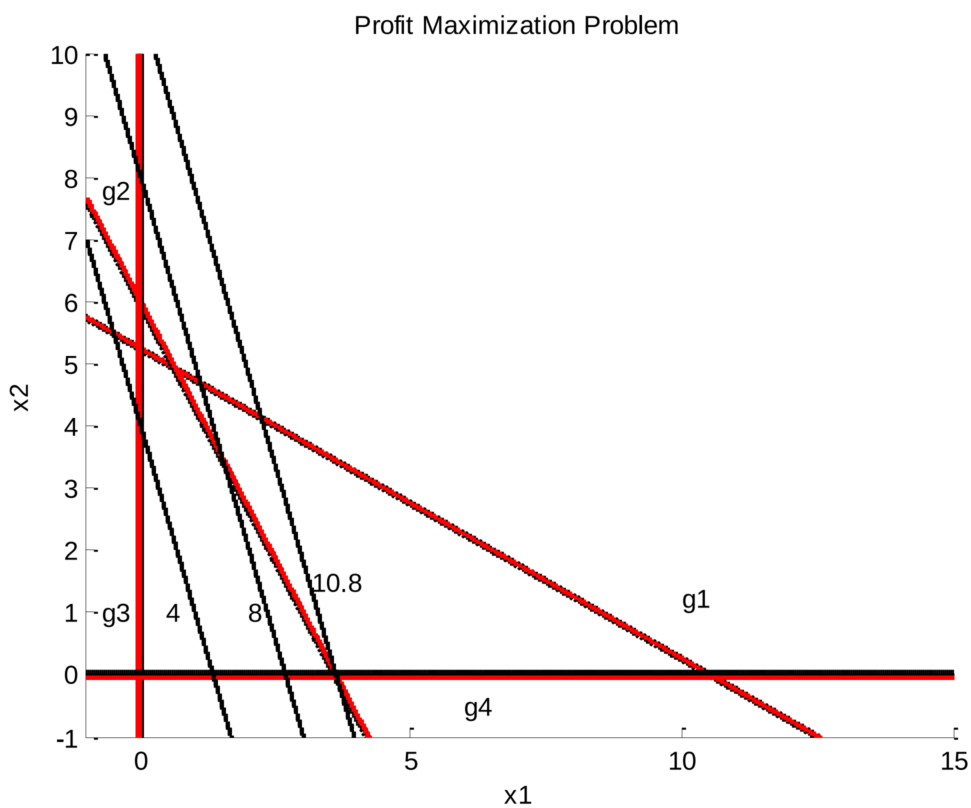
Graphically solve the optimal design problem given below.

$$\text{Maximize } f(x) = 3\,x_1 + x_2$$

Subject to

$$2\,x_1 + 4\,x_2 \le 21$$
$$5\,x_1 + 3\,x_2 \le 18$$
$$x_1, x_2 \ge 0$$

Show all constrains (with hatch marks to indicate the infeasible side)
Show at least two contours of $f(x)$
Find the Optimal Solution and Label it on the graph
Fill in the blanks at the bottom



Approximate optimal solution x_1^* = ___3.6___, x_2^* = ___0___.

Approximate Optimal value $f(x^*)$ = ___10.8___.

End of Exam