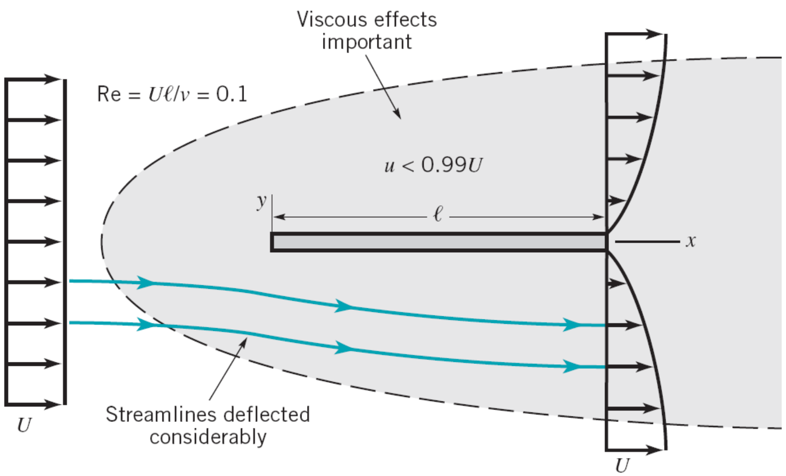
SPC 307
Introduction to Aerodynamics

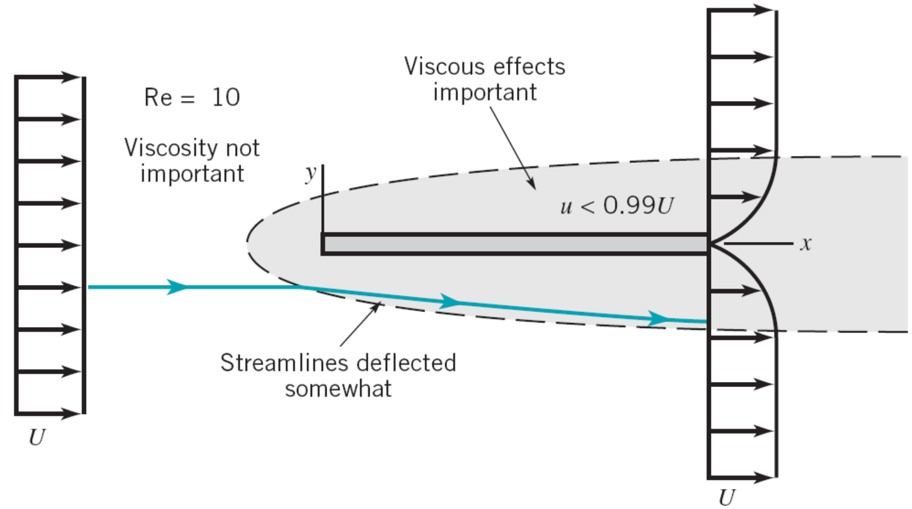
Lecture 7
Boundary Layer

April 9, 2017

Character of the steady, viscous flow past a flat plate parallel to the upstream velocity



(a)



(b)

Inertia force = ma $= \rho L^3 * V\, dV/dL = \rho V^2 L^2$

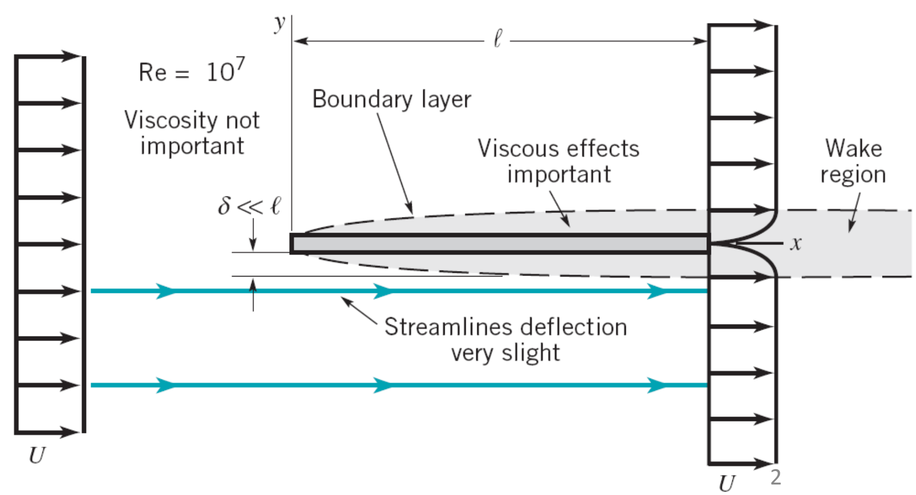
Viscous Force $= \mu\, L^2\, dV/dL = \mu\, V\, L$

$$\boxed{\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{Inertia\ Force}{Viscous\ Force}}$$

(a)low Reynolds number flow,

(b)moderate Reynolds number flow,

(c)large Reynolds number flow.



(c)

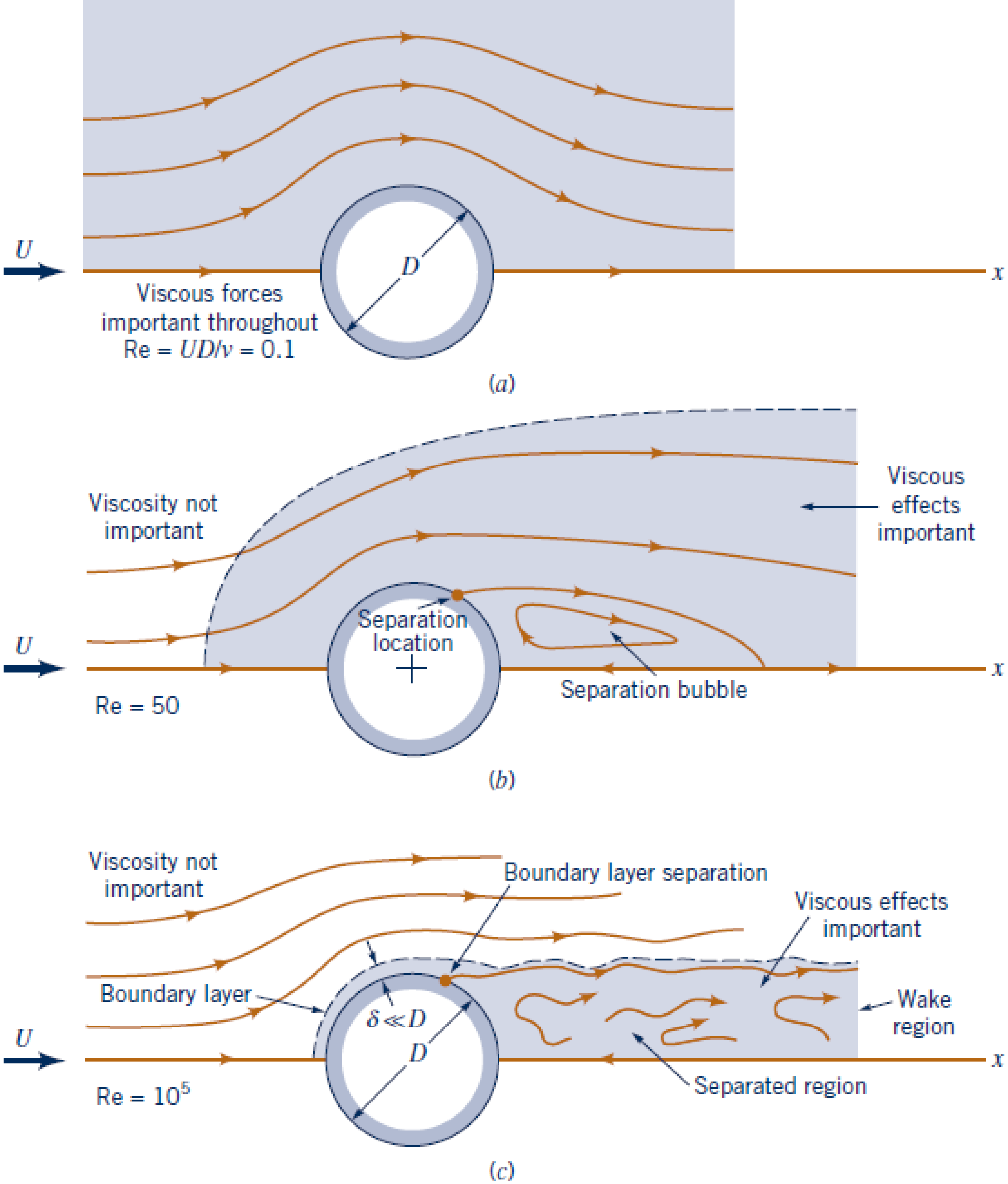


Figure 9.6 **Character of the steady, viscous flow past a circular cylinder: (*a*) low Reynolds number flow, (*b*) moderate Reynolds number flow, (*c*) large Reynolds number flow.**

Boundary Layer

The purpose of the boundary layer is to allow the fluid to change its velocity from the upstream value of U to zero on the surface. Thus, $\mathbf{V} = 0$ at $y = 0$ and $\mathbf{V} \approx U\,\hat{\mathbf{i}}$ at the edge of the boundary layer, with the velocity profile, $u = u(x, y)$ bridging the boundary layer thickness. This boundary layer characteristic occurs in a variety of flow situations, not just on flat plates. For example, boundary layers form on the surfaces of cars, in the water running down the gutter of the street, and in the atmosphere as the wind blows across the surface of the Earth (land or water).

Boundary Layer

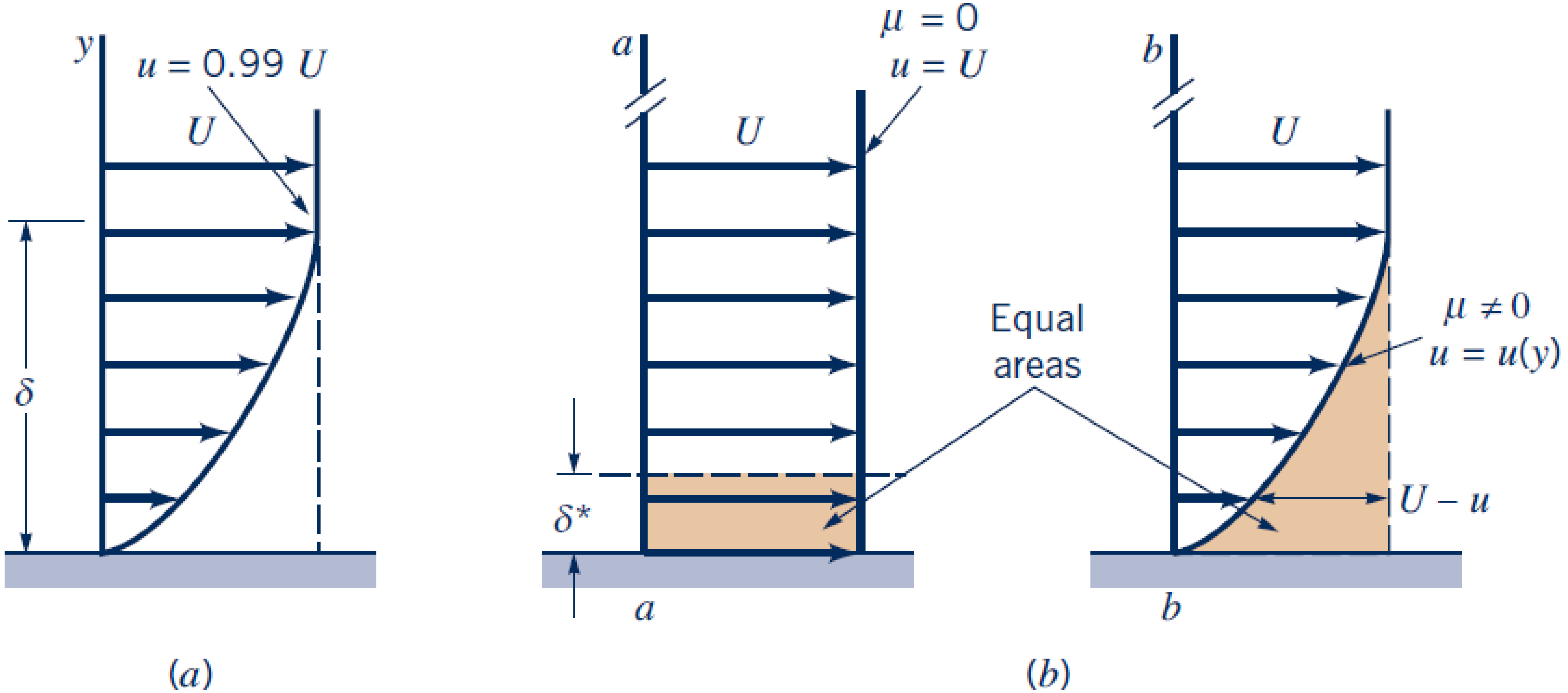


Figure 9.8 **Boundary layer thickness: (*a*) standard boundary layer thickness, (*b*) boundary layer displacement thickness.**

$$\delta = y \qquad \text{where} \qquad u = 0.99U$$

Boundary Layer

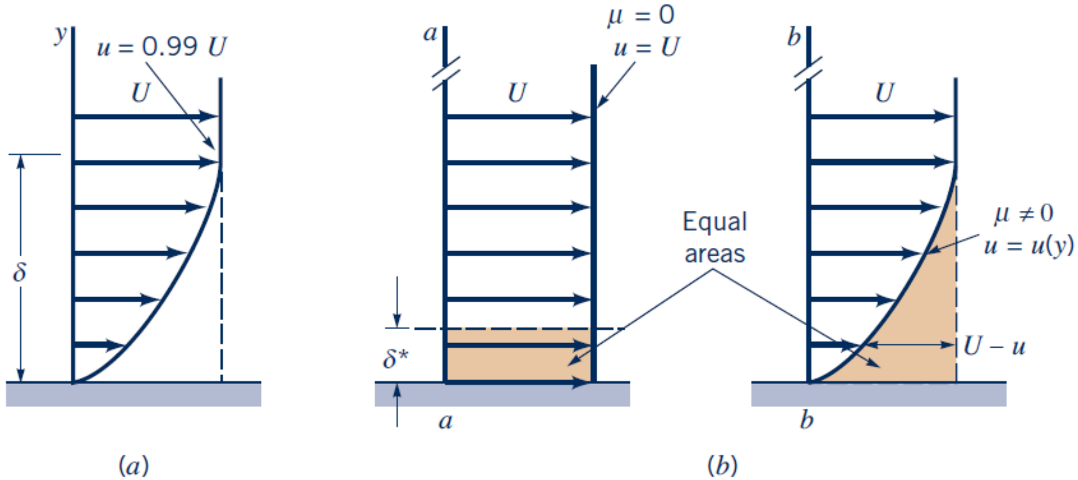


Figure 9.8 **Boundary layer thickness: (*a*) standard boundary layer thickness, (*b*) boundary layer displacement thickness.**

$$\delta^* b U = \int_0^\infty (U - u) b \, dy$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

The distance through which the external inviscid flow is displaced by the presence of the boundary layer.

Boundary Layer

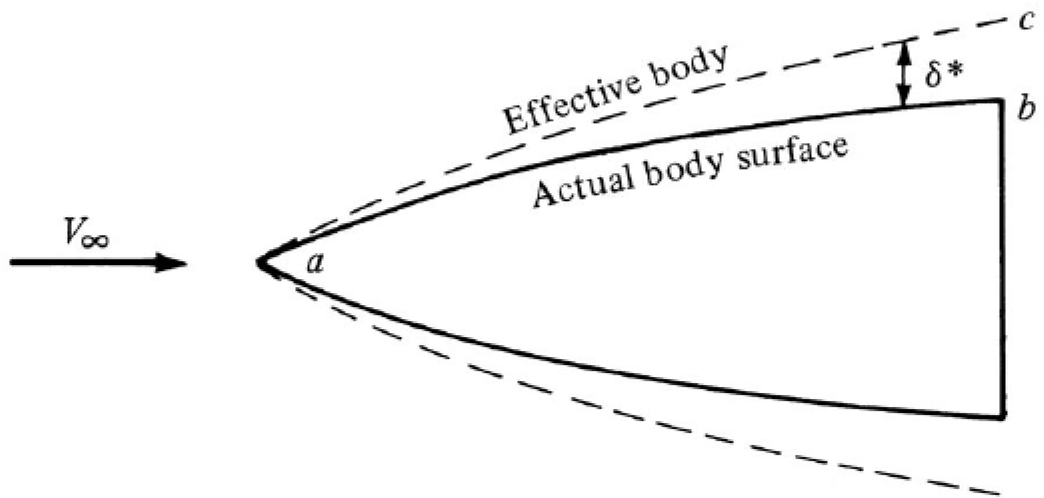


Figure 17.6 The "effective body," equal to the actual body shape plus the displacement thickness distribution.

Boundary Layer

Another boundary-layer property of importance is the *momentum thickness* θ, defined as

$$\boxed{\theta \equiv \int_0^{y_1} \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right) dy \qquad \delta \leq y_1 \to \infty} \tag{17.10}$$

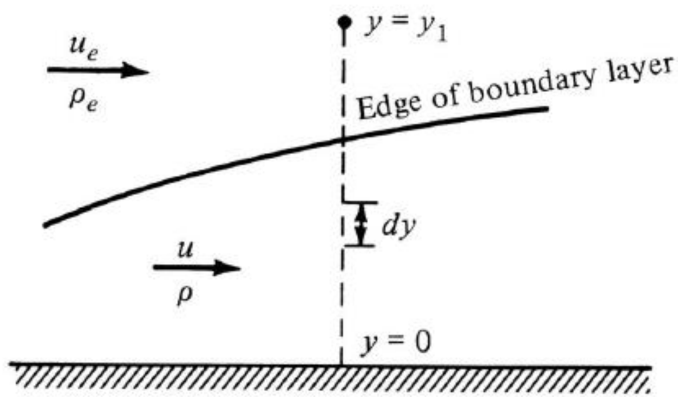


Figure 17.4 Construction for the discussion of displacement thickness.

Boundary Layer

To understand the physical interpretation of θ, return again to Figure 17.4. Consider the mass flow across a segment dy, given by $dm = \rho u\, dy$. Then

$$A = \text{momentum flow across } dy = dm\, u = \rho u^2\, dy$$

If this same elemental mass flow were associated with the freestream, where the velocity is u_e, then

$$B = \begin{cases} \text{momentum flow at freestream} \\ \text{velocity associated with mass } dm = dm\, u_e = (\rho u\, dy)u_e \end{cases}$$

Hence,

$$B - A = \begin{cases} \text{decrement in momentum flow} \\ \text{(missing momentum flow) associated} = \rho u(u_e - u)\, dy \\ \text{with mass } dm \end{cases} \qquad (17.11)$$

The *total* decrement in momentum flow across the vertical line from $y = 0$ to $y = y_1$ in Figure 17.4 is the integral of Equation (17.11),

$$\left.\begin{array}{l} \text{Total decrement in momentum} \\ \text{flow, or missing momentum flow} \end{array}\right\} = \int_0^{y_1} \rho u(u_e - u)\, dy \qquad (17.12)$$

Boundary Layer

Assume that the missing momentum flow is the product of $\rho_e u_e^2$ and a height θ. Then,

$$\text{Missing momentum flow} = \rho_e u_e^2 \theta \tag{17.13}$$

Equating Equations (17.12) and (17.13), we obtain

$$\rho_e u_e^2 \theta = \int_0^{y_1} \rho u (u_e - u)\, dy$$

$$\theta = \int_0^{y_1} \frac{\rho u}{\rho_e u_e} \left(1 - \frac{u}{u_e}\right) dy \tag{17.14}$$

Therefore, θ is an index that is proportional to the decrement in momentum flow due to the presence of the boundary layer. It is the height of a hypothetical streamtube which is carrying the missing momentum flow at freestream conditions.

Reynolds Number

$$Re = \frac{\rho VL}{\mu} = \frac{Inertia\ Effect}{VIscosity\ Effect}$$

- Where ρ density, μ viscosity and V velocity.
- Area *L is the characteristic length*:
  for a flat plate: Plate Length
  For a circle or a sphere is Diameter

Reynolds Number for a flow over a Flate Plate

$$\boxed{Re = \frac{\rho V x}{\mu}}$$

- Where ρ density, μ viscosity and V velocity.
- Area x is the distance from the leading edge: for a flat plate: Plate Length

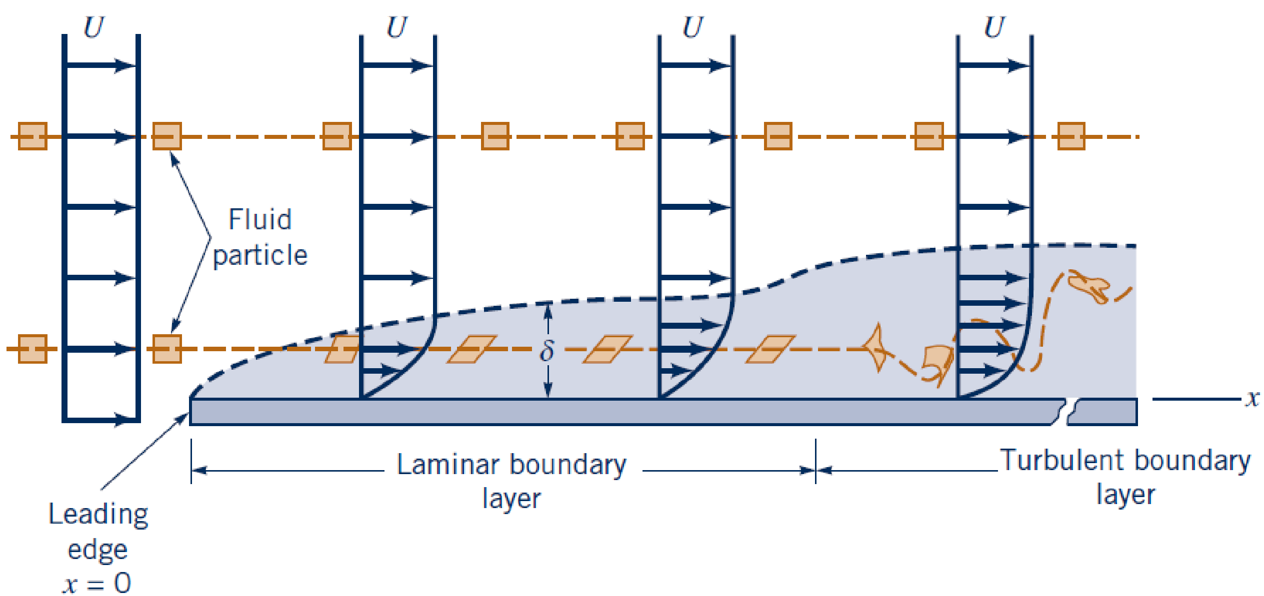
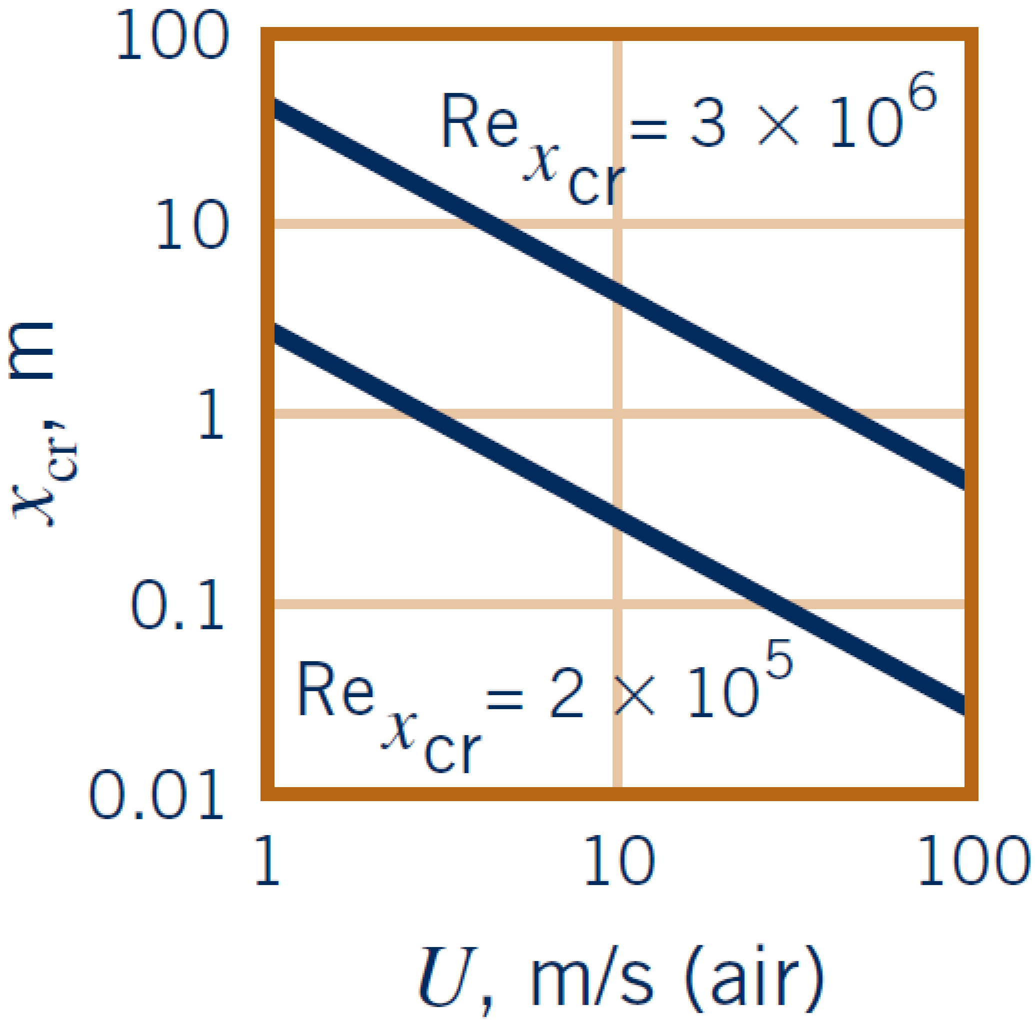


Figure 9.7 Distortion of a fluid particle as it flows within the boundary layer.

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{Inertia\ Force}{Viscous\ Force}$$

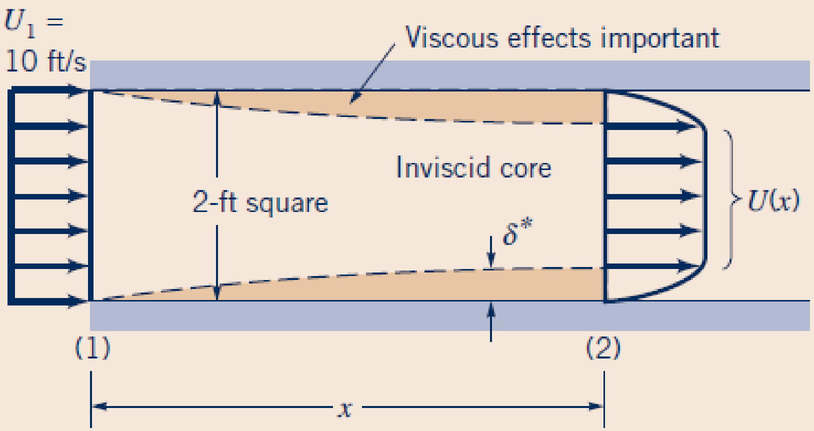


Example 9.3

GIVEN Air flowing into a 2-ft-square duct with a uniform velocity of 10 ft/s forms a boundary layer on the walls as shown in Fig. E9.3*a*. The fluid within the core region (outside the boundary layers) flows as if it were inviscid. From advanced calculations it is determined that for this flow the boundary layer displacement thickness is given by

$$\delta^* = 0.0070(x)^{1/2} \tag{1}$$

where δ^* and x are in feet.



FIND Determine the velocity $U = U(x)$ of the air within the duct but outside of the boundary layer.

Example 9.3 - Solution

If we assume incompressible flow (a reasonable assumption because of the low velocities involved), it follows that the volume flowrate across any section of the duct is equal to that at the entrance (i.e., $Q_1 = Q_2$). That is,

$$U_1A_1 = 10 \text{ ft/s } (2 \text{ ft})^2 = 40 \text{ ft}^3\text{/s} = \int_{(2)} u\, dA$$

According to the definition of the displacement thickness, δ^*, the flowrate across section (2) is the same as that for a uniform flow with velocity U through a duct whose walls have been moved inward by δ^*. That is,

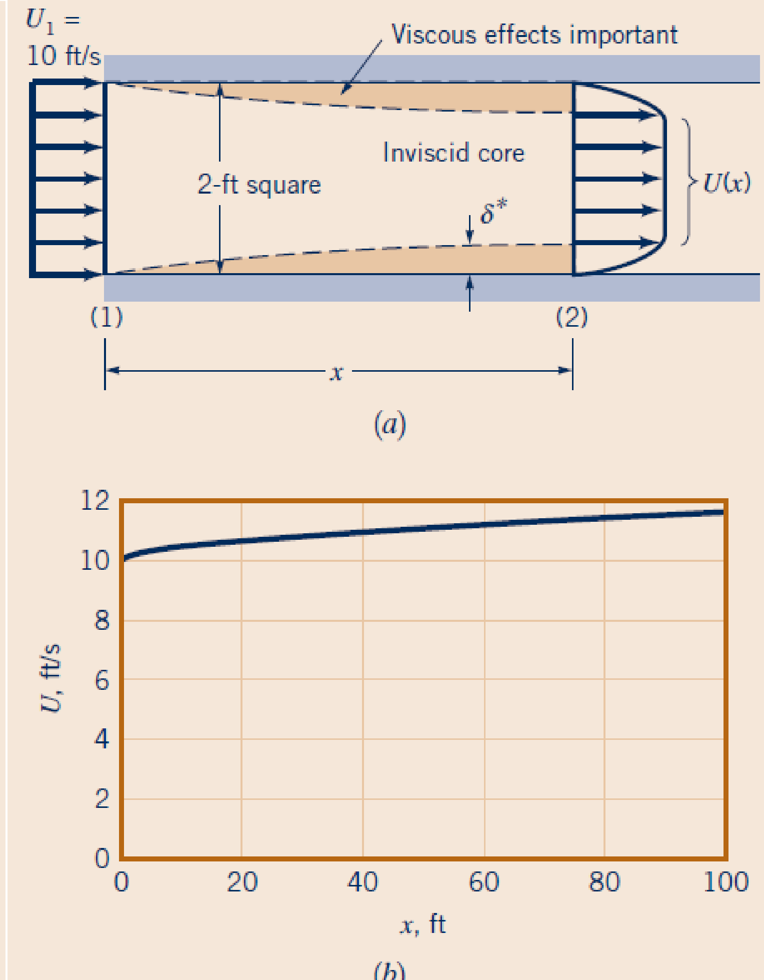
$$40 \text{ ft}^3\text{/s} = \int_{(2)} u\, dA = U(2 \text{ ft} - 2\delta^*)^2 \qquad \textbf{(2)}$$

By combining Eqs. 1 and 2 we obtain

$$40 \text{ ft}^3\text{/s} = 4U(1 - 0.0070x^{1/2})^2$$

or

$$U = \frac{10}{(1 - 0.0070x^{1/2})^2} \text{ ft/s} \qquad \textbf{(Ans)}$$



■ **Figure E9.3**

Example 9.3 - Solution

COMMENTS Note that U increases in the downstream direction. For example, as shown in Fig. E9.3*b*, $U = 11.6$ ft/s at $x = 100$ ft. The viscous effects that cause the fluid to stick to the walls of the duct reduce the effective size of the duct, thereby (from conservation of mass principles) causing the fluid to accelerate. The pressure drop necessary to do this can be obtained by using the Bernoulli equation (Eq. 3.7) along the inviscid streamlines from section (1) to (2). (Recall that this equation is not valid for viscous flows within the boundary layer. It is, however, valid for the inviscid flow outside the boundary layer.) Thus,

$$p_1 + \tfrac{1}{2}\rho U_1^2 = p + \tfrac{1}{2}\rho U^2$$

Hence, with $\rho = 2.38 \times 10^{-3}$ slugs/ft^3 and $p_1 = 0$ we obtain

$$\begin{aligned} p &= \frac{1}{2}\rho\,(U_1^2 - U^2) \\ &= \frac{1}{2}(2.38 \times 10^{-3}\ \text{slugs/ft}^3) \\ &\quad \times \left[(10\ \text{ft/s})^2 - \frac{10^2}{(1 - 0.0079x^{1/2})^4}\ \text{ft}^2/\text{s}^2\right] \end{aligned}$$

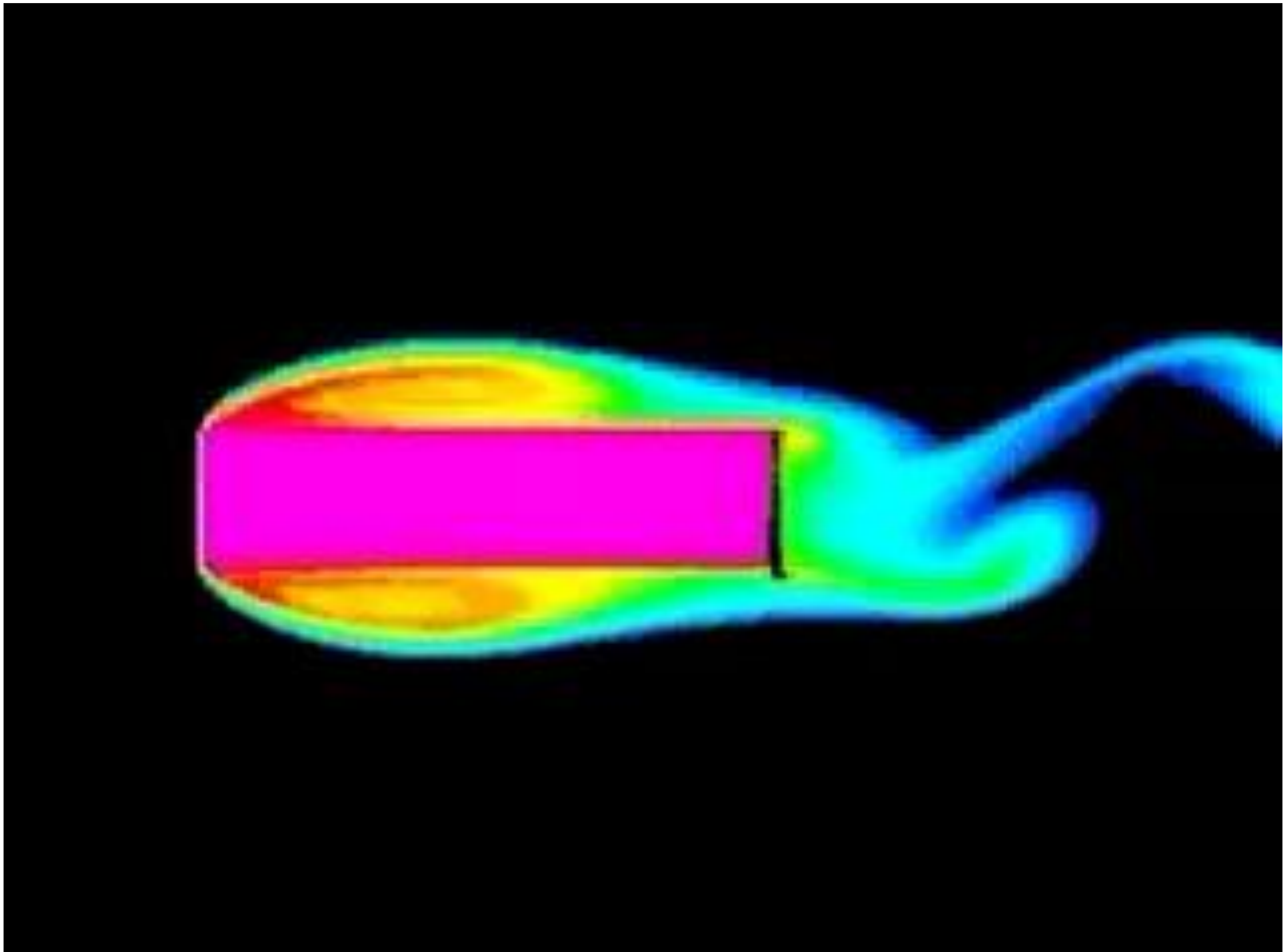
or

$$p = 0.119\left[1 - \frac{1}{(1 - 0.0070x^{1/2})^4}\right]\ \text{lb/ft}^2$$

For example, $p = -0.0401$ lb/ft^2 at $x = 100$ ft.

If it were desired to maintain a constant velocity along the centerline of this entrance region of the duct, the walls could be displaced outward by an amount equal to the boundary layer displacement thickness, δ^*.

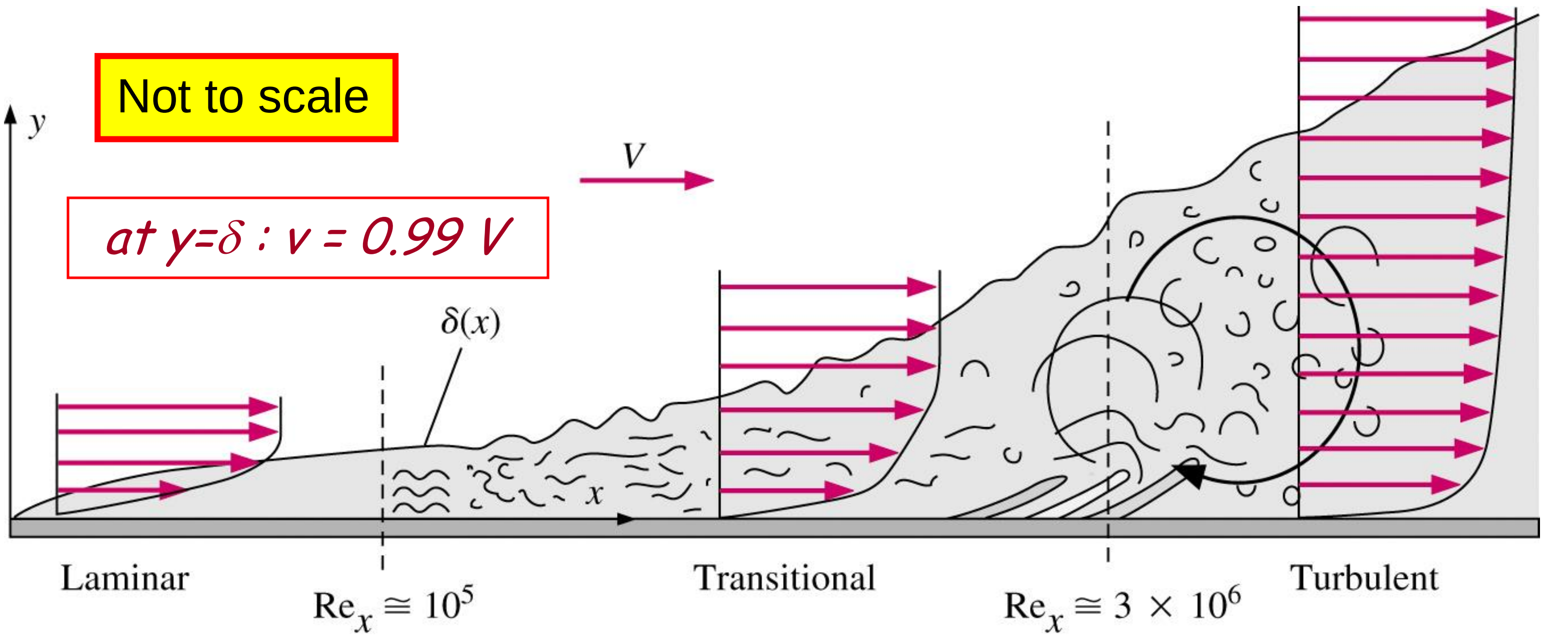
Boundary Layer over a flat plate

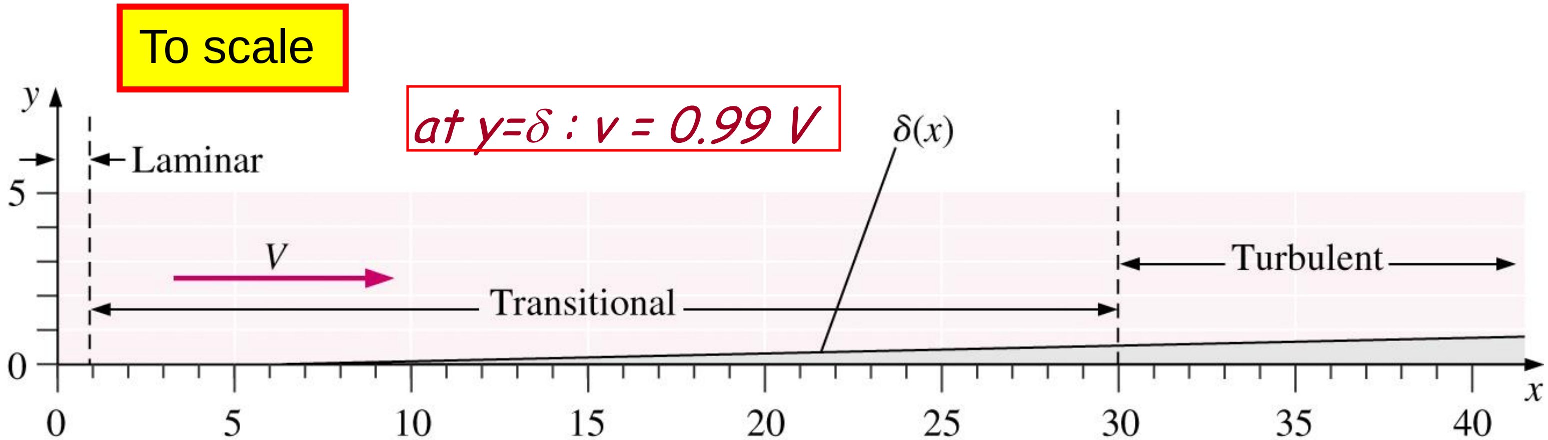


Boundary Layer transition from laminar to turbulent

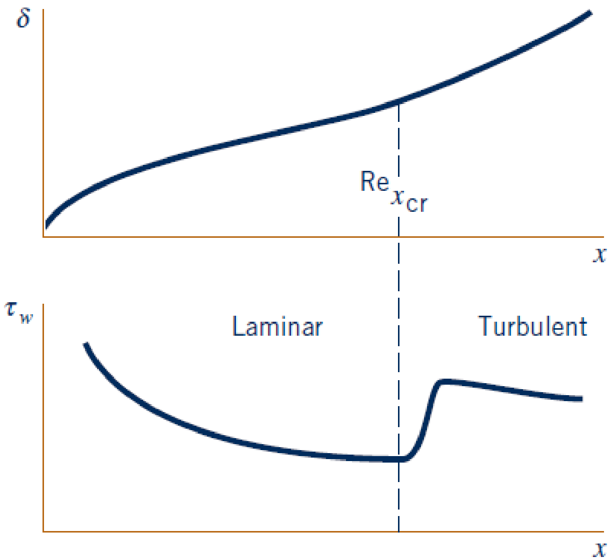
The transition from a ***laminar boundary layer*** to a ***turbulent boundary layer*** occurs at a critical value of the Reynolds number, Re_{xcr} on the order of 2×10^5 to 3×10^6 depending on the roughness of the surface and the amount of turbulence in the upstream flow

Boundary Layer on a Flat Plate



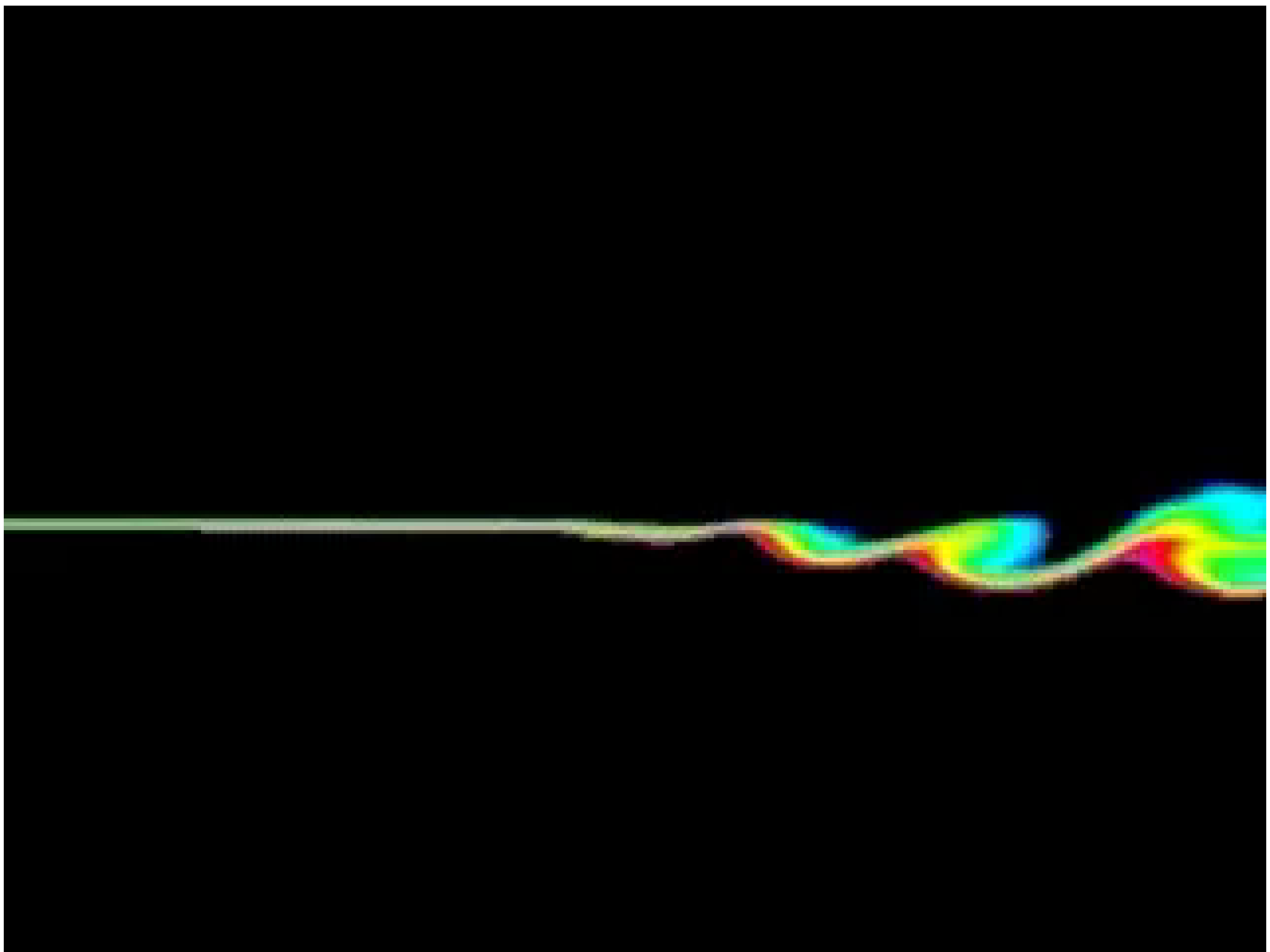


Boundary Layer on a Flat Plate

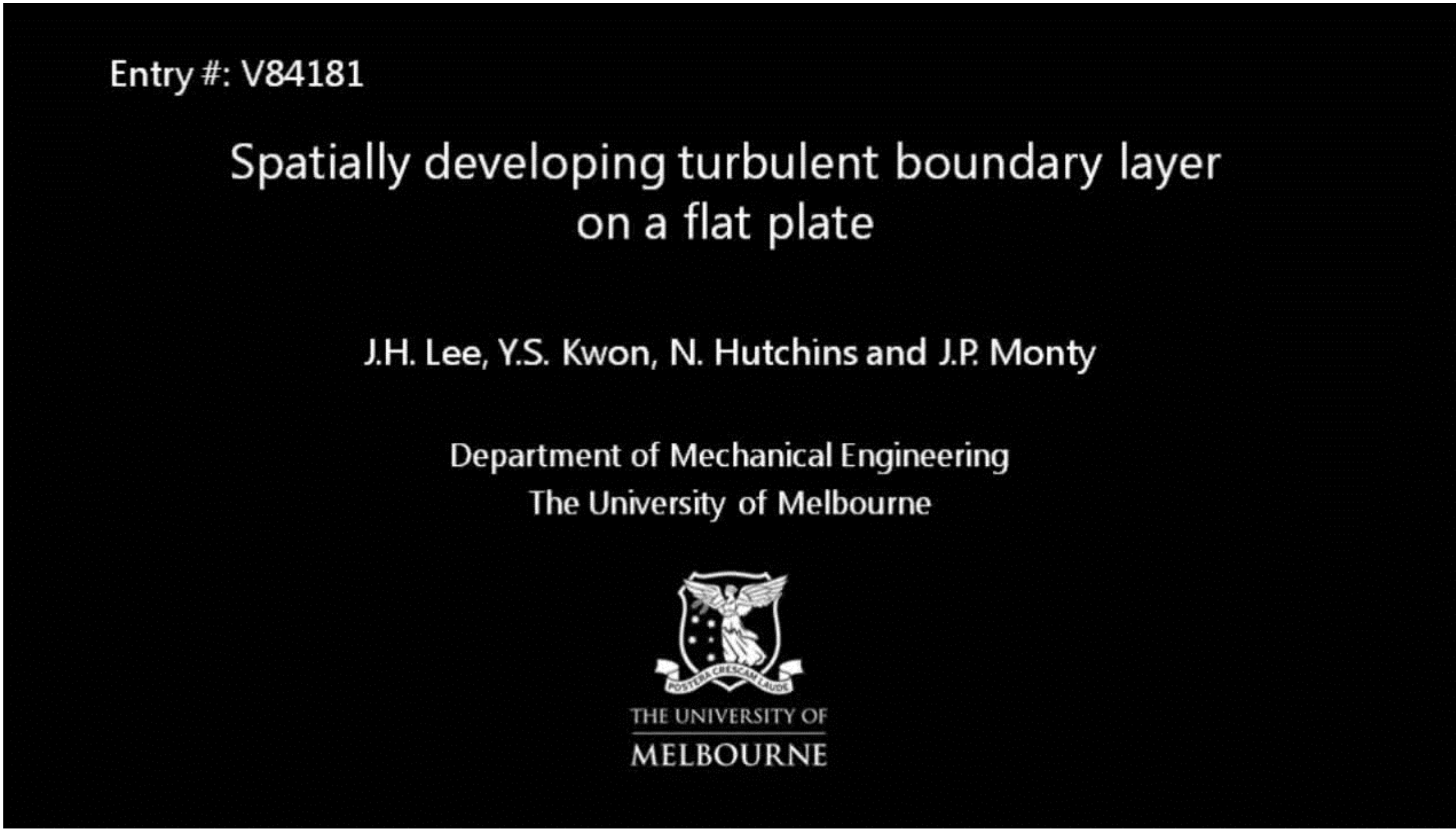


■ **Figure 9.9** **Typical characteristics of boundary layer thickness and wall shear stress for laminar and turbulent boundary layers.**

Boundary Layer transition from laminar to turbulent



Transition


Entry #: V84181
Spatially developing turbulent boundary layer on a flat plate
J.H. Lee, Y.S. Kwon, N. Hutchins and J.P. Monty
Department of Mechanical Engineering
The University of Melbourne
POSTERA CRESCAM LAUDE
THE UNIVERSITY OF
MELBOURNE

Transition

Entry #: V0056

A Computational Laboratory for the Study of Transitional and Turbulent Boundary Layers

Jin Lee & Tamer A. Zaki

JOHNS HOPKINS
WHITING SCHOOL
of ENGINEERING

Laminar Boundary Layer

Prandtl/Blasius Boundary Layer Solution

for steady, two-dimensional laminar flows with negligible gravitational effects, incompressible flow:
Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

By solving Navier–Stokes equatıons

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{1}{\rho}\,\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = -\frac{1}{\rho}\,\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Prandtl/Blasius Boundary Layer Solution

Since the boundary layer is thin, it is expected that the component of velocity normal to the plate is much smaller than that parallel to the plate and that the rate of change of any parameter across the boundary layer should be much greater than that along the flow direction. That is,

$$v \ll u \quad \text{and} \quad \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$$

With these assumptions it can be shown that the governing equations reduce to the following boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

Prandtl/Blasius Boundary Layer Solution

With these assumptions it can be shown that the governing equations reduce to the following boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

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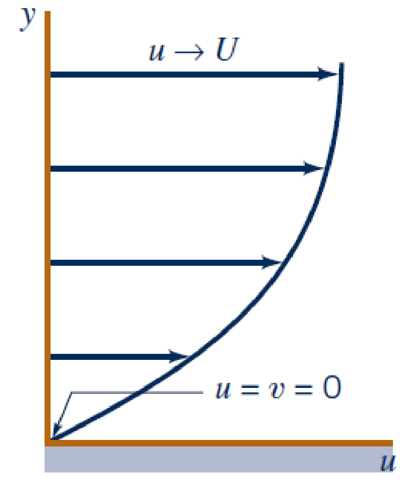
$$\frac{\partial p}{\partial y} = 0$$

Prandtl/Blasius Boundary Layer Solution

Boundary conditions



$$u = v = 0 \quad \text{on} \quad y = 0$$

$$u \to U \quad \text{as} \quad y \to \infty$$

No Exact Solution is available for those equations

Prandtl/Blasius Boundary Layer Solution

For Compressible B. L.

$$\boxed{\textit{Continuity:} \qquad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0} \tag{17.28}$$

$$\boxed{\textit{x momentum:} \qquad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)} \tag{17.29}$$

$$\boxed{\textit{y momentum:} \qquad \frac{\partial p}{\partial y} = 0} \tag{17.30}$$

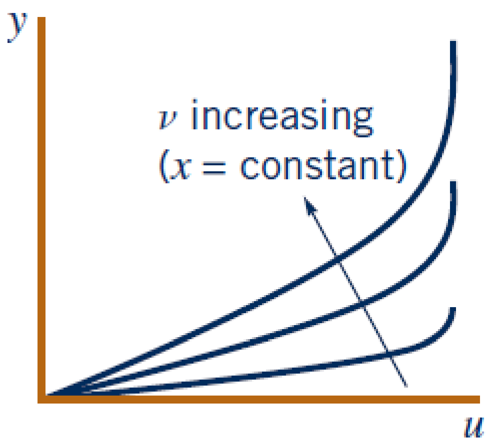
$$\boxed{\textit{Energy:} \qquad \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{dp_e}{dx} + \mu \left(\frac{\partial u}{\partial y}\right)^2} \tag{17.31}$$

Prandtl/Blasius Boundary Layer Solution

It can be argued that in dimensionless form the boundary layer velocity profiles on a flat plate should be similar regardless of the location along the plate. That is,

$$\frac{u}{U} = g\left(\frac{y}{\delta}\right)$$

$$\delta \sim \left(\frac{\nu x}{U}\right)^{1/2}$$



Prandtl/Blasius Boundary Layer Solution

The final solution is

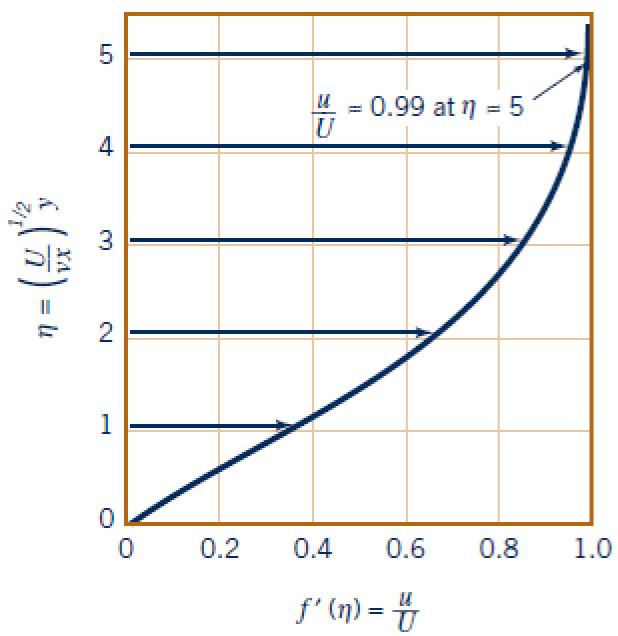
$$\frac{\delta}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

$$\frac{\delta^*}{x} = \frac{1.721}{\sqrt{\mathrm{Re}_x}} \qquad \frac{\Theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

$$\tau_w = 0.332 U^{3/2} \sqrt{\frac{\rho\mu}{x}}$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \qquad c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

Prandtl/Blasius Boundary Layer Solution



Laminar Flow along a Flat Plate (the Blasius Solution)

$\eta = y(U/\nu x)^{1/2}$	$f'(\eta) = u/U$	η	$f'(\eta)$
0	0	3.6	0.9233
0.4	0.1328	4.0	0.9555
0.8	0.2647	4.4	0.9759
1.2	0.3938	4.8	0.9878
1.6	0.5168	5.0	0.9916
2.0	0.6298	5.2	0.9943
2.4	0.7290	5.6	0.9975
2.8	0.8115	6.0	0.9990
3.2	0.8761	∞	1.0000

Prandtl/Blasius Boundary Layer Solution

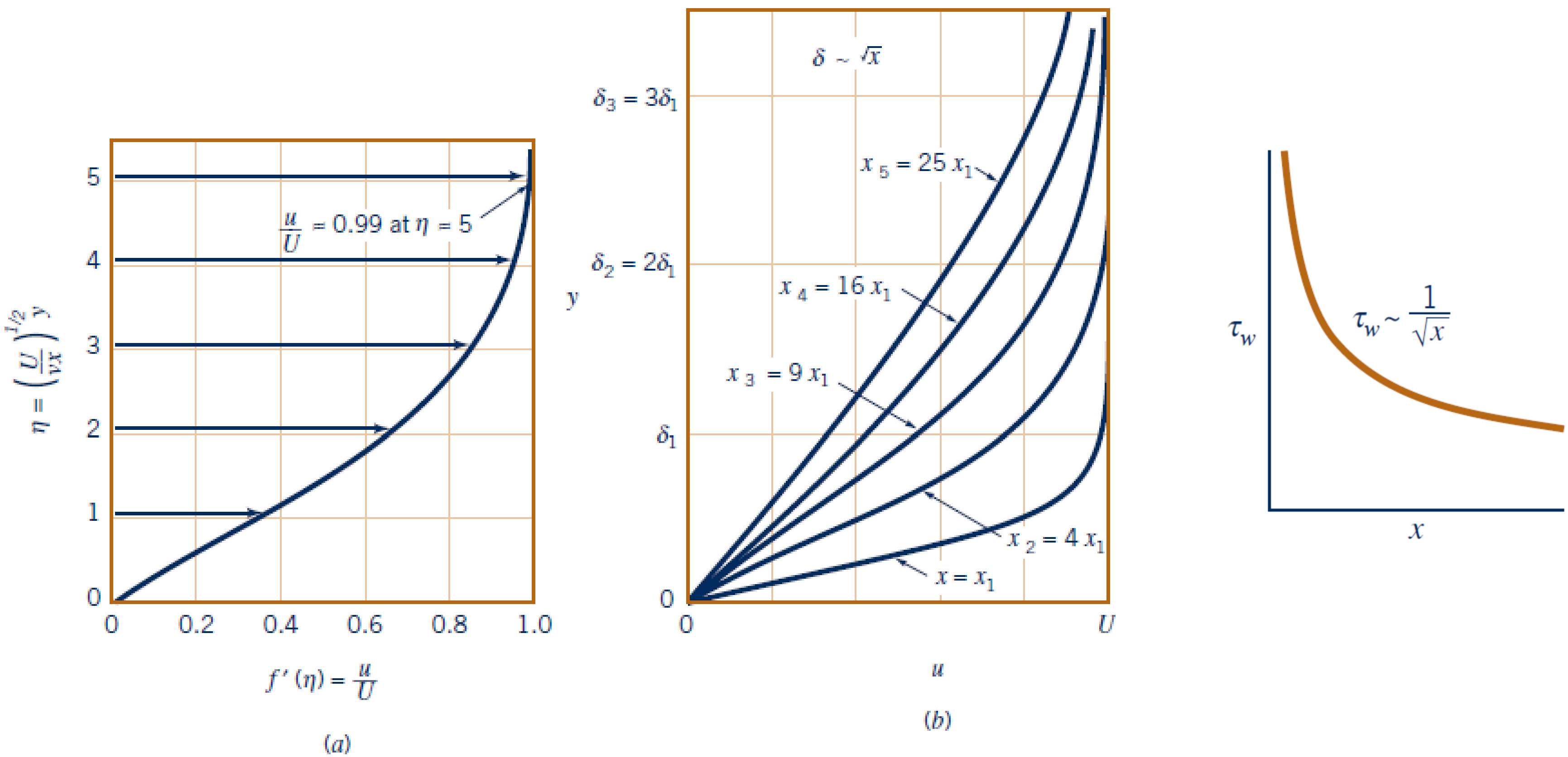


Figure 9.10 Blasius boundary layer profile: (*a*) boundary layer profile in dimensionless form using the similarity variable η, (*b*) similar boundary layer profiles at different locations along the flat plate.

Prandtl/Blasius Boundary Layer Solution

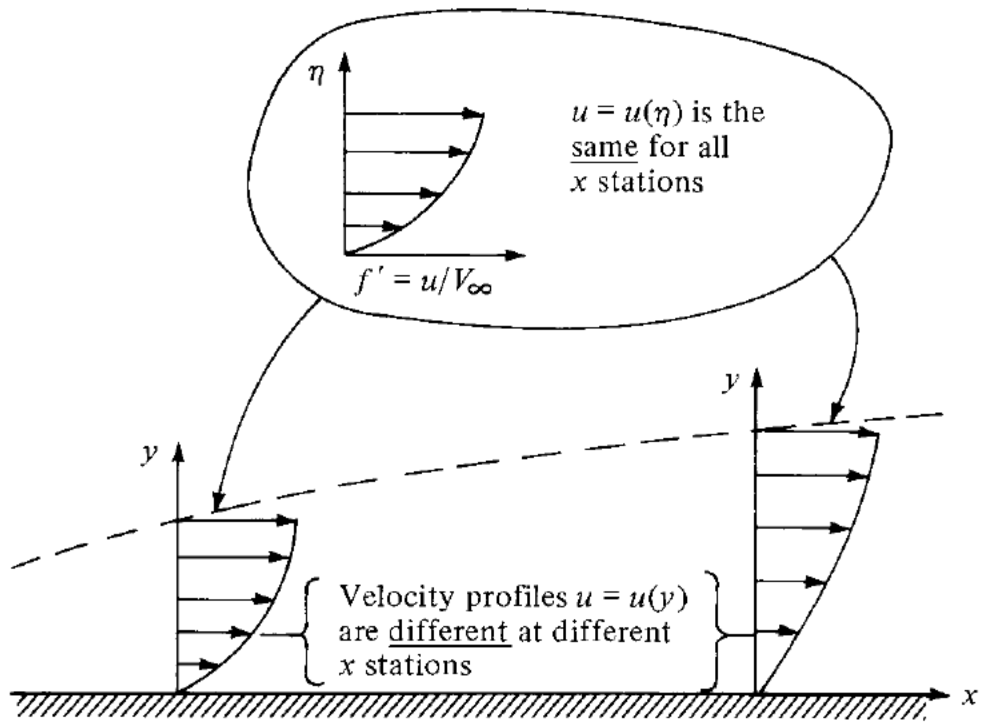


Figure 18.3 Velocity profiles in physical and transformed space, demonstrating the meaning of self-similar solutions.

Momentum Integral Boundary Layer Equation for a Flat Plate

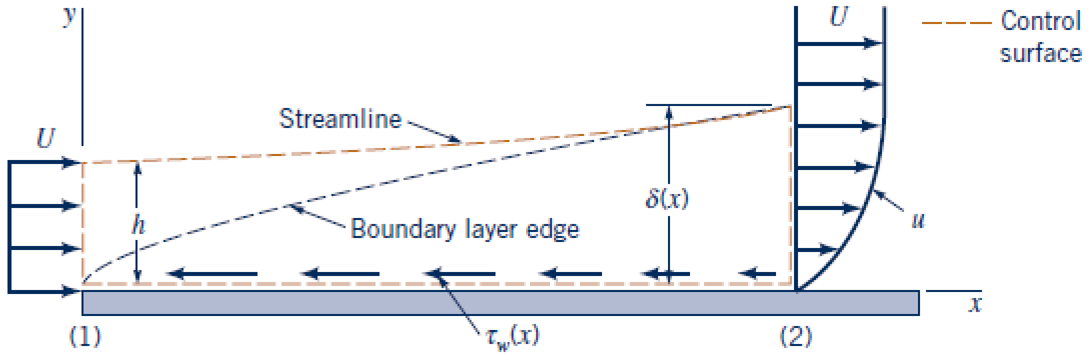


Figure 9.11 Control volume used in the derivation of the momentum integral equation for boundary layer flow.

$$\sum F_x = -\mathfrak{D} = -\int_{\text{plate}} \tau_w \, dA = -b \int_{\text{plate}} \tau_w \, dx$$

$$-\mathfrak{D} = \rho \int_{(1)} U(-U)\, dA + \rho \int_{(2)} u^2 \, dA$$

$$\mathfrak{D} = \rho U^2 b h - \rho b \int_0^\delta u^2 \, dy$$

Momentum Integral Boundary Layer Equation for a Flat Plate

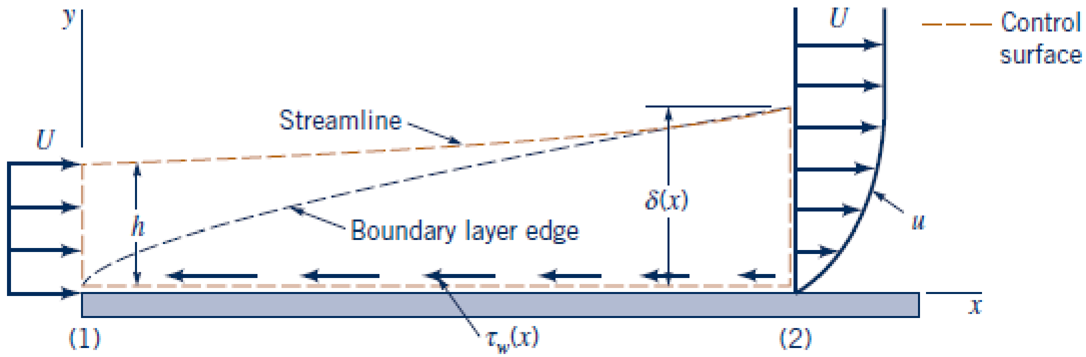


Figure 9.11 Control volume used in the derivation of the momentum integral equation for boundary layer flow.

$$\mathfrak{D} = \rho b \int_0^\delta u(U - u)\, dy$$

$$\mathfrak{D} = \rho b U^2\, \Theta$$

$$\tau_w = \rho U^2 \frac{d\Theta}{dx}$$

Momentum Integral Boundary Layer Equation for a Flat Plate

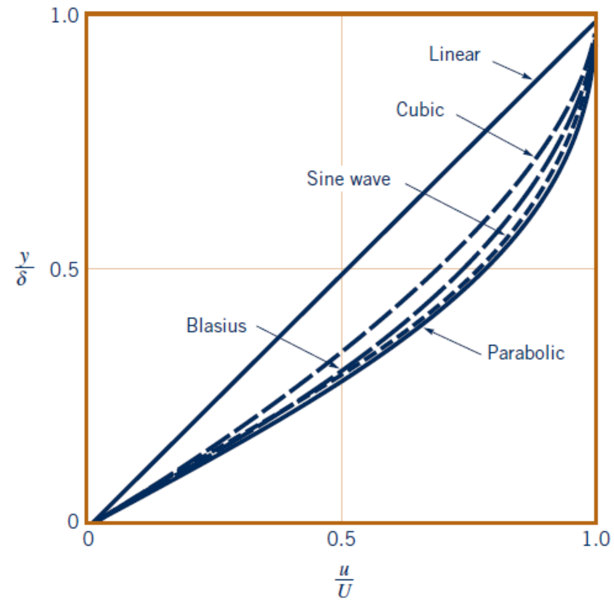


Figure 9.12 Typical approximate boundary layer profiles used in the momentum integral equation.

Turbulent Boundary Layer

Turbulent Boundary Layer

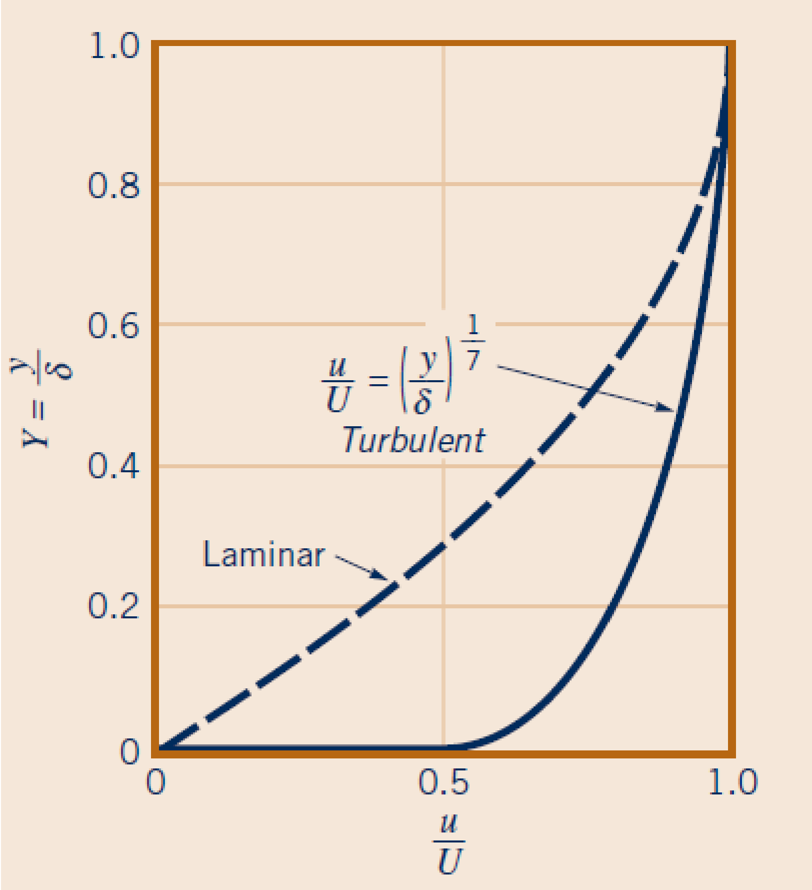
Experimental measurements have shown that the time-averaged velocity for a turbulent boundary layer on a flat plate may be represented by the 1/7th power law:

$$\frac{u}{u_e} = \left(\frac{y}{\delta}\right)^{1/7}$$

$$\delta = \frac{0.37x}{\mathrm{Re}_x^{1/5}}$$

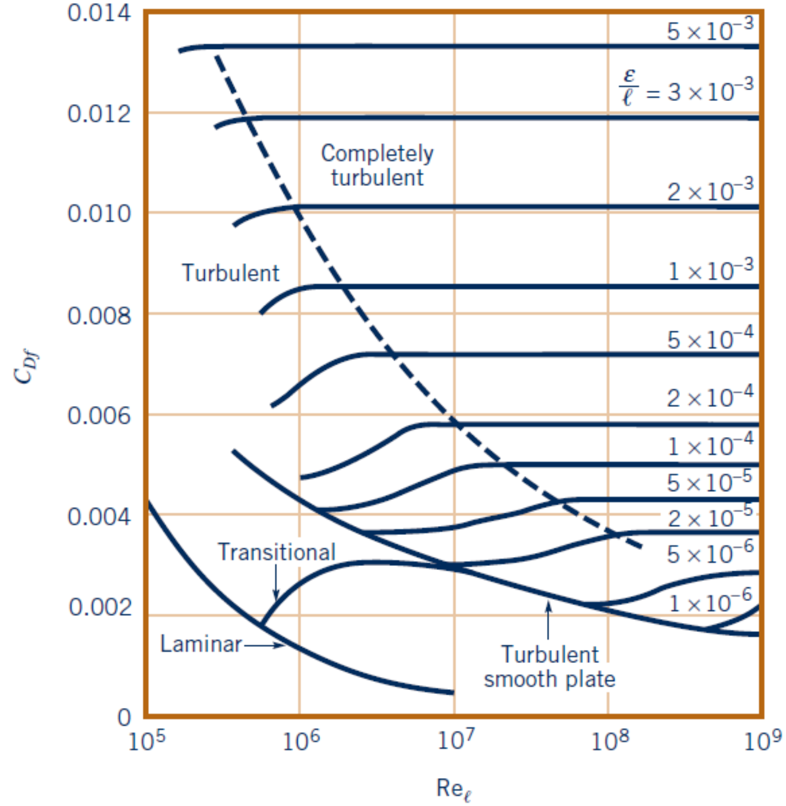
$$C_f = \frac{0.074}{\mathrm{Re}_c^{1/5}}$$

Turbulent Boundary Layer



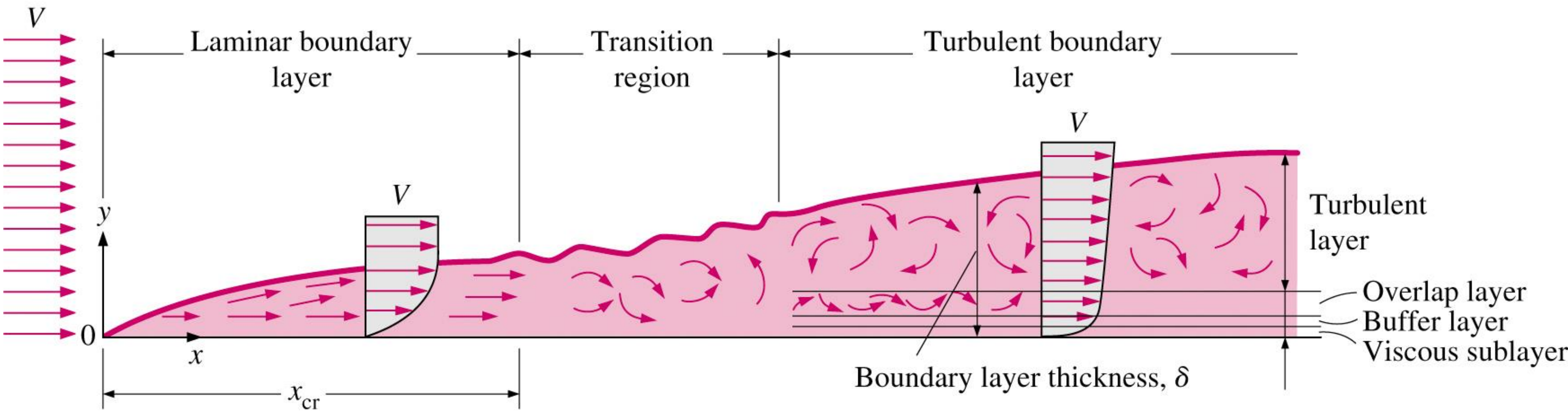
$$\frac{u}{u_e} = \left(\frac{y}{\delta}\right)^{1/7}$$

Turbulent Boundary Layer



■ **Figure 9.15** **Friction drag coefficient for a flat plate parallel to the upstream flow (Ref. 18, with permission).**

Flat Plate Drag

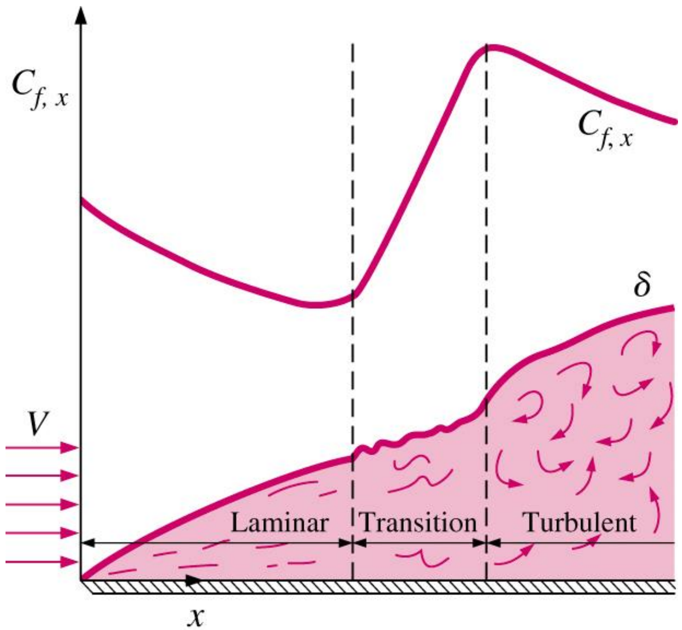


v = 0.99 V

- Drag on flat plate is solely due to friction ($F_D = F_{Dfriction}$) created by laminar, transitional, and turbulent boundary layers.
- BL thickness, δ, is the distance from the plate at which v = 0.99 V

Flat Plate Drag

$C_{D,\text{ friction}} = C_f$



- Local friction coefficient
  - Laminar: $$C_{f,x} = \frac{0.664}{Re_x^{1/2}}$$
  - Turbulent: $$C_{f,x} = \frac{0.059}{Re_x^{1/5}}$$
- Average friction coefficient

$$C_f = \frac{1}{L}\int_0^L C_{f,x}\,dx$$

For some cases, plate is long enough for turbulent flow, but not long enough to neglect laminar portion

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,lam}\,dx + \int_{x_{cr}}^{L} C_{f,x,turb}\,dx\right)$$

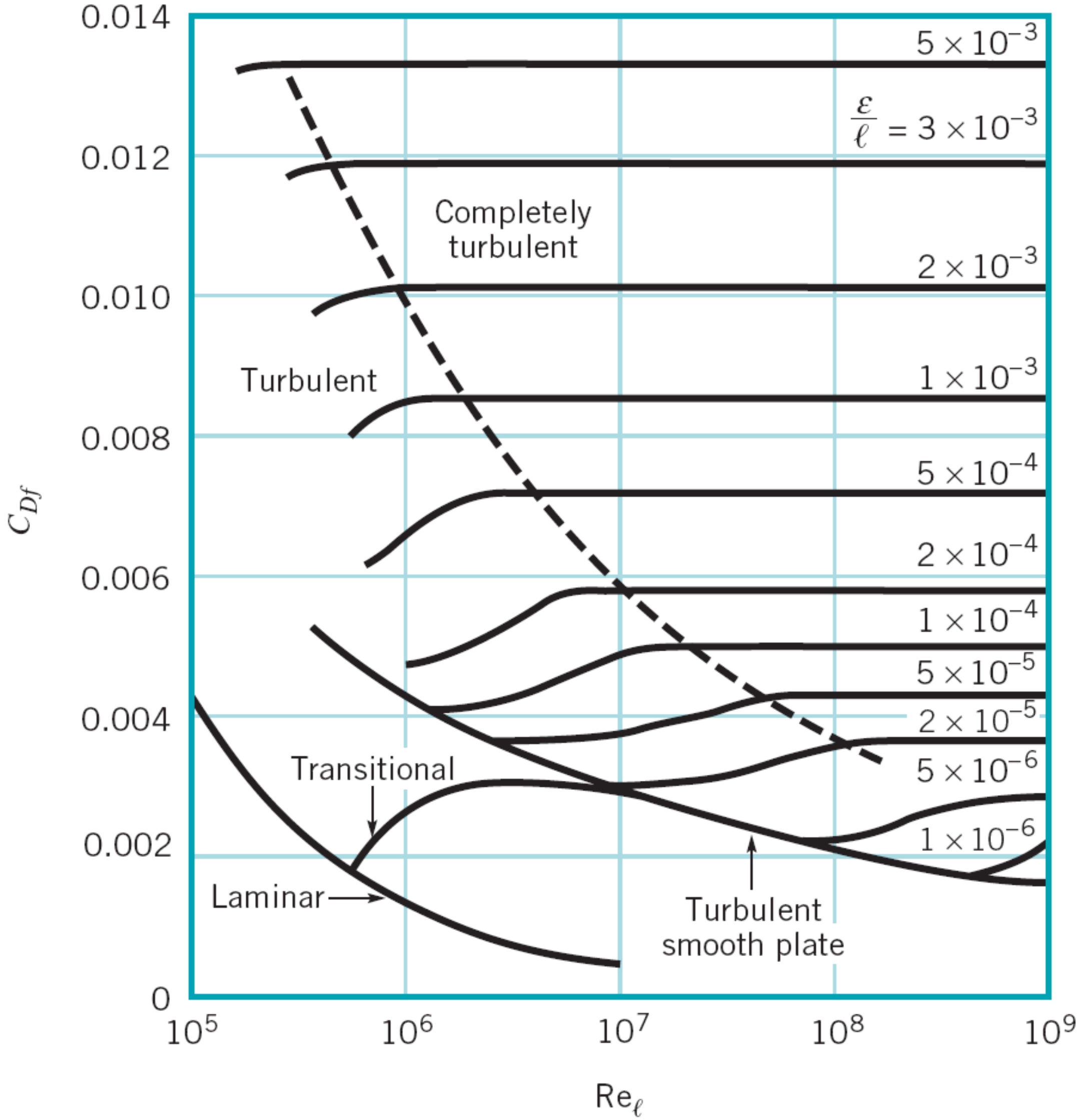
Flat Plate Drag

Empirical Equations for the Flat Plate Drag Coefficient

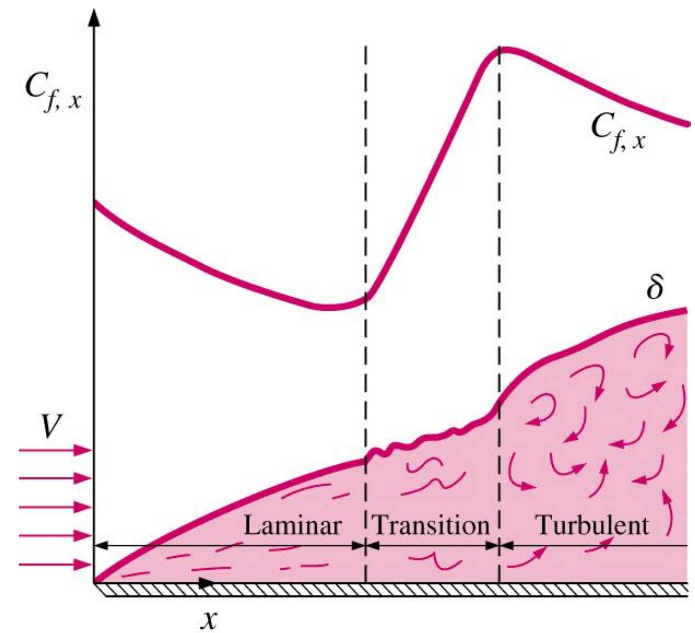
Equation	Flow Conditions
$C_{Df} = 1.328/(\mathrm{Re}_\ell)^{0.5}$	Laminar flow
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58} - 1700/\mathrm{Re}_\ell$	Transitional with $\mathrm{Re}_{xcr} = 5 \times 10^5$
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58}$	Turbulent, smooth plate
$C_{Df} = [1.89 - 1.62 \log(\varepsilon/\ell)]^{-2.5}$	Completely turbulent

Transition takes place at a distance x given by:
$Re_{xcr}=2X10^5$ to $3X10^6$ – We will use $Re_{xcr}= 5X10^5$

Drag coefficient may also be obtained from charts such as those on the next slides



Friction drag coefficient for a flat plate parallel to the upstream flow.



Laminar: $C_{Df} = f\ (Re)$ Turbulent: $C_{Df} = f\ (Re, \varepsilon/L)$

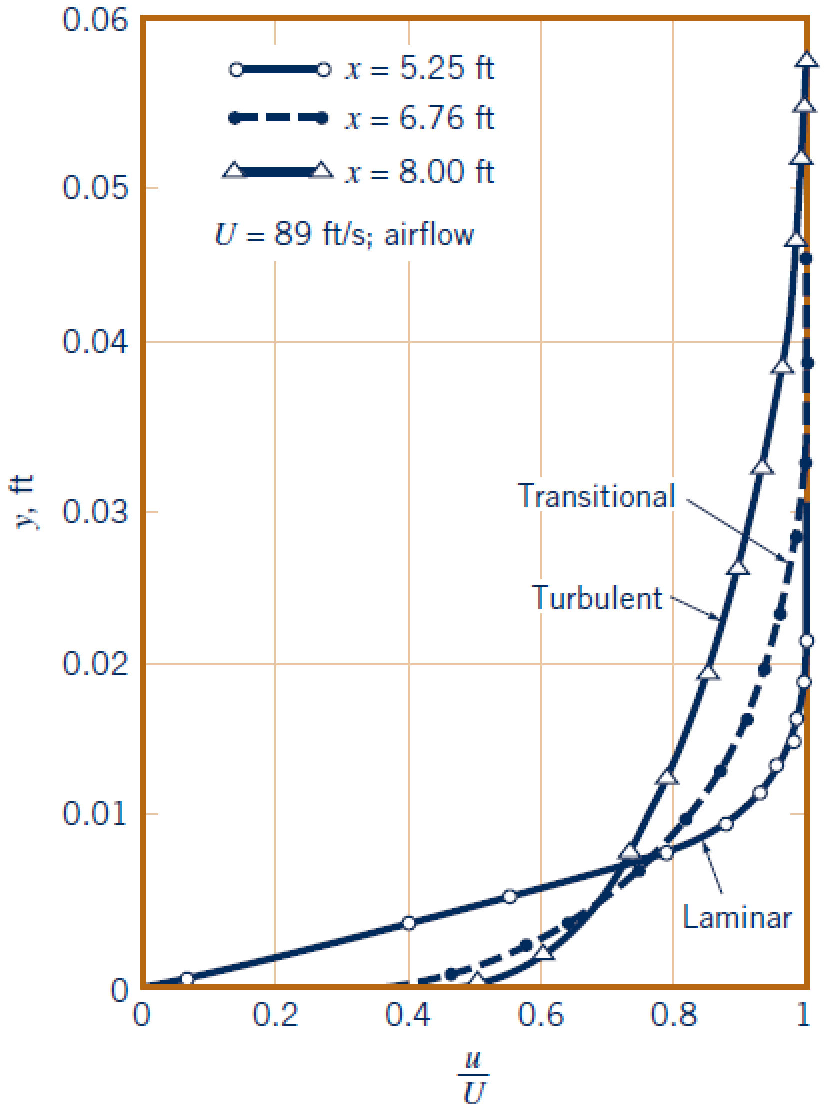
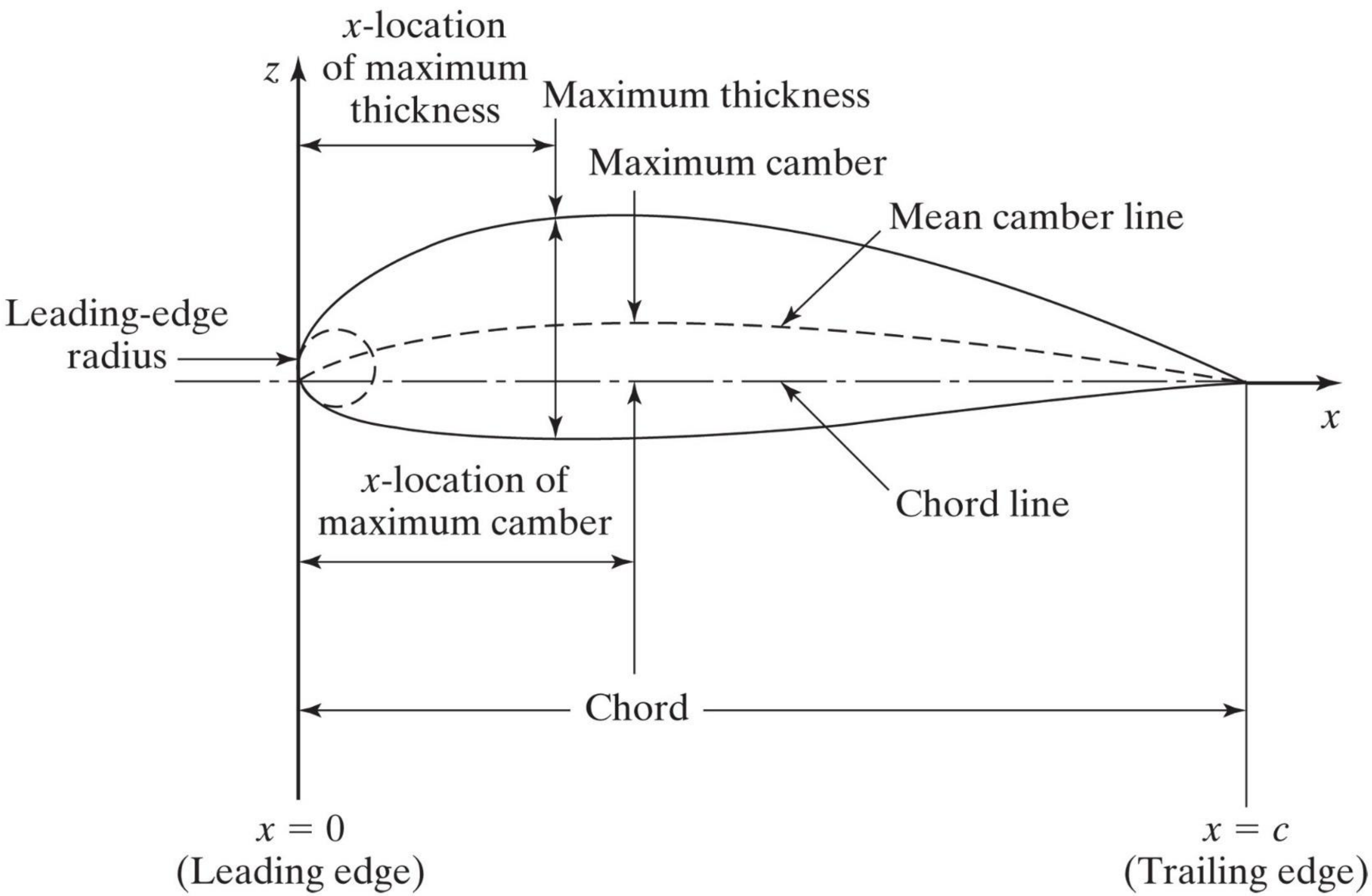


Figure 9.14 Typical boundary layer profiles on a flat plate for laminar, transitional, and turbulent flow (Ref. 1).

AIRFOIL GEOMETRY PARAMETERS



AIRFOIL GEOMETRY PARAMETERS

- If a horizontal wing is cut by a vertical plane, the resultant section is called the *airfoil section* .
- The generated lift and stall characteristics of the wing depend strongly on the geometry of the airfoil sections that make up the wing.
- Geometric parameters that have an important effect on the aerodynamic characteristics of an airfoil section include
  - (1) the leading-edge radius,
  - (2) the mean camber line,
  - (3) the maximum thickness and the thickness distribution of the profile, and
  - (4) the trailing-edge angle.
- The effect of these geometric parameters, will be discussed after an introduction to airfoil-section nomenclature.

Parameters used to describe the airfoil

- Some of the basic parameters to describe the airfoil geometry are:
  1. Leading edge—the forward most point on the airfoil (typically placed at the origin for convenience)
  2. Trailing edge—the aft most point on the airfoil (typically placed on the *x* axis for convenience)
  3. Chord line—a straight line between the leading and trailing edges (the *x* axis for our convention)
  4. Mean camber line—a line midway between the upper and lower surfaces at each chord-wise position
  5. Maximum camber—the largest value of the distance between the mean camber line and the chord line, which quantifies the camber of an airfoil
  6. Maximum thickness—the largest value of the distance between the upper and lower surfaces, which quantifies the thickness of the airfoil
  7. Leading-edge radius—the radius of a circle that fits the leading-edge curvature
- These geometric parameters are used to determine certain aerodynamic characteristics of an airfoil.