Midterm 2 - Solution

1. An atmospheric boundary layer is formed when the wind blows over the Earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. 1, typical values are n = 0.40 for urban areas, n = 0.28 for woodland or suburban areas, and n = 0.16 for flat open country.

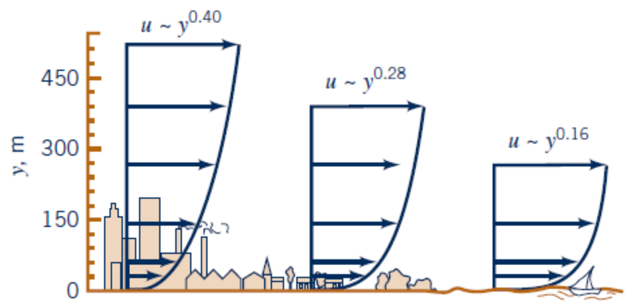


Fig.1

A 30-story office building (each story is 3.66 m tall) is built in a suburban industrial park. Plot the dynamic pressure, $\rho u^2/2$, as a function of elevation if the wind blows at 120.7 km/hr (in case of a hurricane) at the top of the building Plot four points only in the graph.

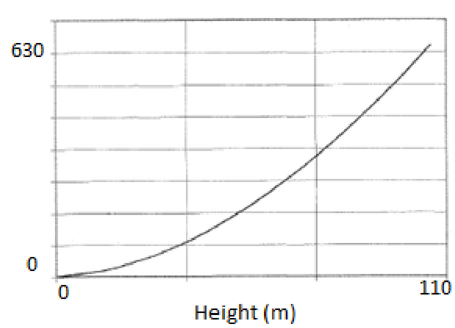
From Fig 1, the boundry layer velocity profle is given by $u = ay^{0.28}$ where a is a const.
Thus

$$\frac{u}{u_1} = \left(\frac{y}{y_1}\right)^{0.28}$$

$$u = u_1\left(\frac{y}{y_1}\right)^{0.28} = 120.7\times \frac{1000}{3600}\left(\frac{y}{30\times 3.66}\right)^{0.28}$$

$$u = 33.52\left(\frac{y}{109.8}\right)^{0.28}$$

The relation between the dynamic pressure and height is plotted below



2. One end of a pond has a shoreline that resembles a half-body as shown in Fig. 2. A vertical porous pipe is located near the end of the pond so that water can be pumped out. When water is pumped at the rate of 0.02 m^2/s (flow rate per unit length), what will be the velocity at point A in Fig. 2? Hint: Consider the flow inside a half-body. This flow is similar to the flow shown in Fig. 3.

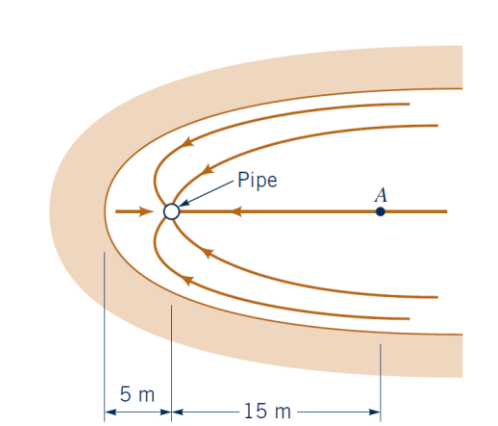


Fig.2

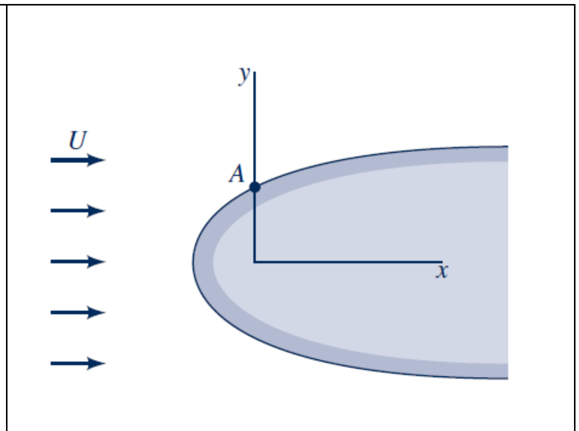


Fig.3

For a half-body,

$$\psi = U r \sin\theta + \frac{m}{2\pi}\theta$$

so that

$$v_\theta = -\frac{\partial \psi}{\partial r} = U \sin\theta$$

and

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = U \cos\theta + \frac{m}{2\pi r}$$

Thus, at point A, $\theta = 0$, $r = 15\,m$ and

$$v_\theta = 0$$

$$v_r = V_A = U + \frac{m}{2\pi(15)}$$

then with $b = 5m$

$$U = \frac{m}{2\pi b} = \frac{\left(\frac{0.06}{3}\frac{m^2}{s}\right)}{2\pi(5m)} = 6.37 \times 10^{-4}\,\frac{m}{s}$$

From Eq. (1)

$$V_A = 6.37 \times 10^{-4}\,\frac{m}{s} + \frac{\left(\frac{0.06}{3}\frac{m^2}{s}\right)}{2\pi\,(15m)}$$

$$= \underline{\underline{8.49 \times 10^{-4}\,\frac{m}{s}}}$$