SPC 307 - Aerodynamics
Sheet 2
Fundamentals of Fluid Mechanics

1. Is the following flows physically possible, that is, satisfy the continuity equation? Substitute the expressions for density and for the velocity field into the continuity equation to substantiate your answer:
   A gas is flowing at relatively low speeds (so that its density may be assumed constant) where

$$u = -\frac{2xyz}{(x^2 + y^2)^2} U_\infty L$$

$$v = \frac{(x^2 - y^2)z}{(x^2 + y^2)^2} U_\infty L$$

$$w = \frac{y}{x^2 + y^2} U_\infty L$$

   Here U_∞ and L are a reference velocity and a reference length, respectively.
2. Two of the three velocity components for an incompressible flow are:

$$u = x^3 + 3xz \qquad v = y^3 + 3yz$$

   What is the general form of the velocity component w(x,y,z) that satisfies the continuity equation?
3. The velocity components for a two-dimensional flow are

$$u = \frac{C(y^2 - x^2)}{(x^2 + y^2)^2} \qquad v = \frac{-2Cxy}{(x^2 + y^2)^2}$$

   where C is a constant. Does this flow satisfy the continuity equation?
4. For the two-dimensional flow of incompressible air near the surface of a flat plate, the steamwise (or x) component of the velocity may be approximated by the relation

$$u = a_1 \frac{y}{\sqrt{x}} - a_2 \frac{y^3}{x^{1.5}}$$

   Using the continuity equation, what is the velocity component v in the y direction? Evaluate the constant of integration by noting that $v = 0$ at y = 0.

5. Water flows through a circular pipe, as shown in Fig. 1 , at a constant volumetric flow rate of 0.5 m^3/s. Assuming that the velocities at stations 1, 2, and 3 are uniform across the cross section (i.e., the flow is one dimensional), use the integral form of the continuity equation to calculate the velocities, V1, V2, and V3. The corresponding diameters are $d_1 = 0.4$ m, $d_2 = 0.2$ m, and $d_3 = 0.6$ m.

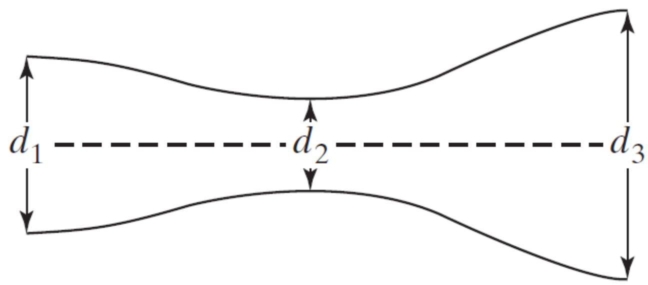

Fig. 1.

6. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of a rectangular control volume, as shown in Fig. 2. If the flow is incompressible, two dimensional, and steady, what is the total volumetric flow rate ($\iint \vec{V} \cdot \hat{n}\, dA$) across the horizontal surfaces (surfaces 3 and 4)?

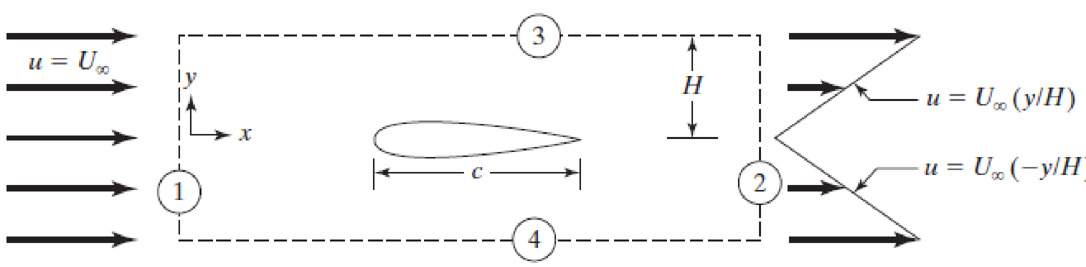

Fig. 2.

7. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of the control volume shown in Fig. 3. The flow is incompressible, two dimensional, and steady. If surfaces 3 and 4 are streamlines, what is the vertical dimension of the upstream station (H_U)?

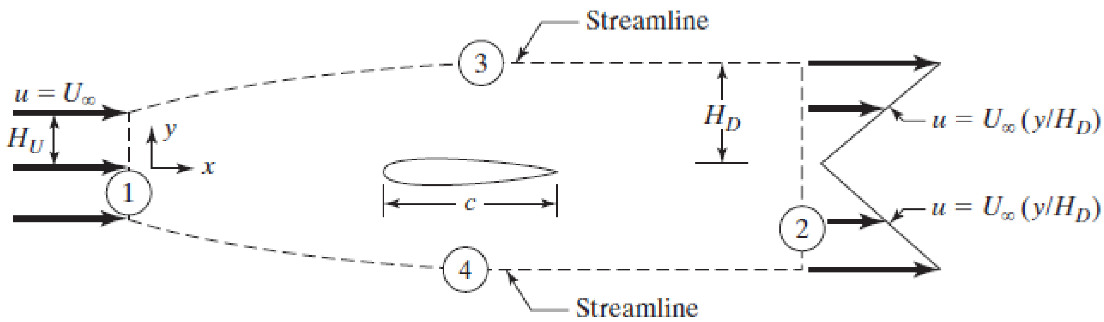

Fig. 3.

8. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of a rectangular control volume, as shown in Fig. 4. If the flow is incompressible, two dimensional, and steady, what is the total volumetric flow rate ($\iint \vec{V} \cdot \hat{n}\, dA$) across the horizontal surfaces (surfaces 3 and 4)?

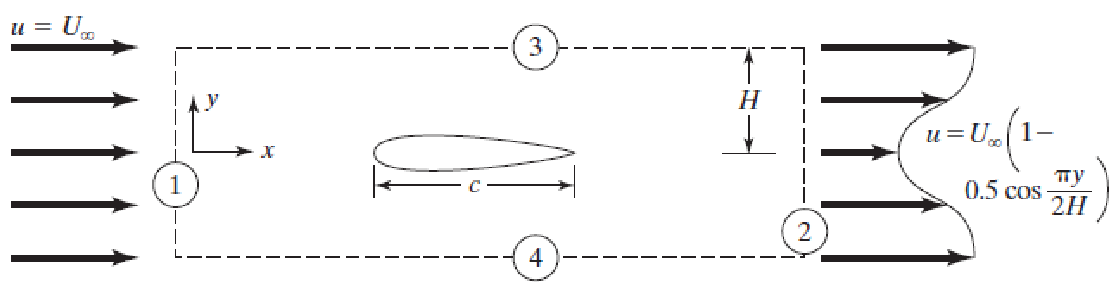

Fig.4.

9. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of the control volume shown in Fig. 5. The flow is incompressible, two dimensional, and steady. If surfaces 3 and 4 are streamlines, what is the vertical dimension of the upstream station (H_U)?

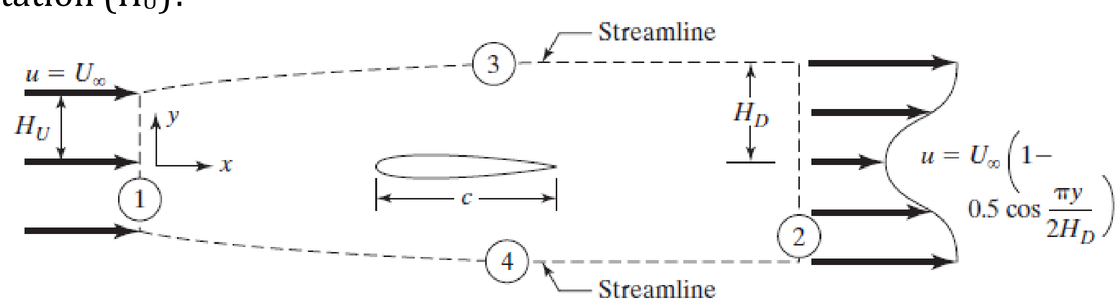

Fig. 5.

10. Given the velocity field

$$\vec{V} = (6 + 2xy + t^2)\hat{i} - (xy^2 + 10t)\hat{j} + 25\hat{k}$$

what is the acceleration of a particle at (3, 0, 2) at time t = 1?

11. You are relaxing on an international flight when a terrorist leaps up and tries to take over the airplane. The crew refuses the demands of the terrorist and he fires his pistol, shooting a small hole in the airplane. Panic strikes the crew and other passengers. But you leap up and shout, "Do not worry! I am an engineering student and I know that it will take ______ seconds for the cabin pressure to drop from $0.5*10^5$ N/m^2 to $0.25*10^5$ N/m^2. " Calculate how long it will take the cabin pressure to drop. Make the following assumptions:

(i) The air in the cabin behaves as a perfect gas:

$$\rho_c = \frac{p_c}{RT_c}$$

where the subscript c stands for the cabin. R = 287.05 N.m/kg.K. Furthermore, Tc = 22 C and is constant for the whole time.

(ii) The volume of air in the cabin is 71.0 m^3. The bullet hole is 0.75 cm in diameter.

(iii) Air escapes through the bullet hole according to the equation:

$$\dot{m_c} = -0.040415\frac{p_c}{\sqrt{T_c}}A_{hole}$$

where p_c is in N/m^2, T_c is in K , A_{hole} is in m^2, and $\dot{m_c}$ is in kg/s.

12. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of a rectangular control volume, as shown in Fig. 6. If the flow is incompressible, two dimensional, and steady, what is the drag coefficient for the airfoil?

The vertical dimension H is 0.025c and

$$C_d = \frac{d}{\frac{1}{2}\rho_\infty U_\infty^2 c}$$

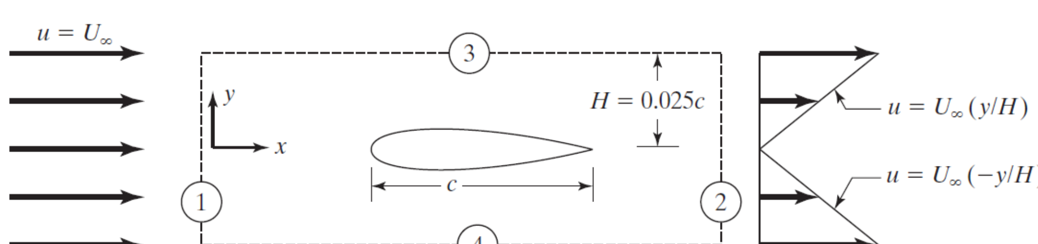

Fig. 6.

The pressure is p_∞ (a constant) over the entire surface of the control volume. (This problem is an extension of Problem 2.10.)

13. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of the control volume shown in Fig. 7. Surfaces 3 and 4 are streamlines. If the flow is incompressible, two dimensional, and steady, what is the drag coefficient for the airfoil? The vertical dimension H_D is 0.025c. You will need to calculate the vertical dimension of the upstream station (H_U). The pressure is p_∞ (a constant) over the entire surface of the control volume. (This problem is an extension of Problem 2.11.)

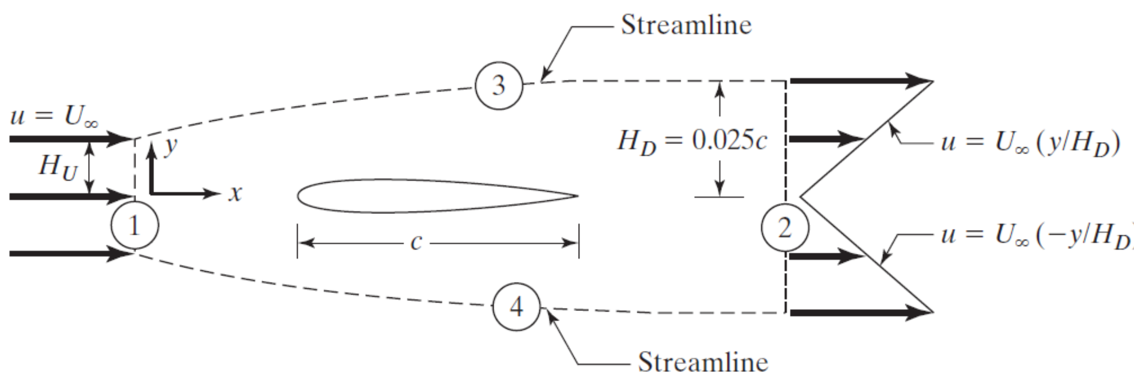

Fig. 7.

14. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of a rectangular control volume, as shown in Fig.8. If the flow is incompressible, two dimensional, and steady, what is the drag coefficient for the airfoil? The vertical dimension H is 0.025c. The pressure is p_∞ (a constant) over the entire surface of the control volume. (This problem is an extension of Problem 2.12.)

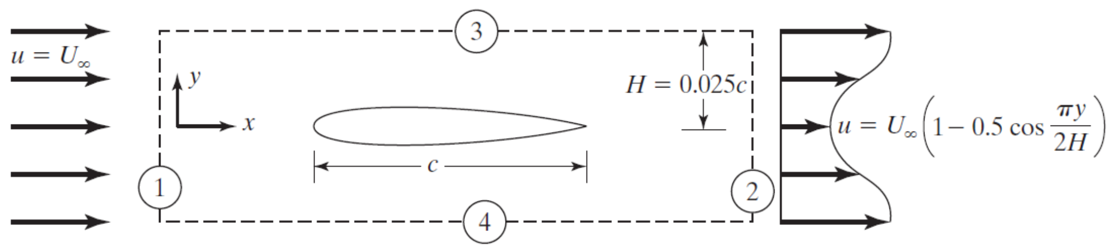

Fig. 8.

15. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of the control volume shown in Fig. 9. Surfaces 3 and 4 are streamlines. If the flow is incompressible, two dimensional, and steady, what is the drag coefficient for the airfoil? The vertical dimension at the downstream station (station 2) is H_D =

0.025c. The pressure is p_∞ (a constant) over the entire surface of the control volume. (This problem is an extension of Problem 2.13.)

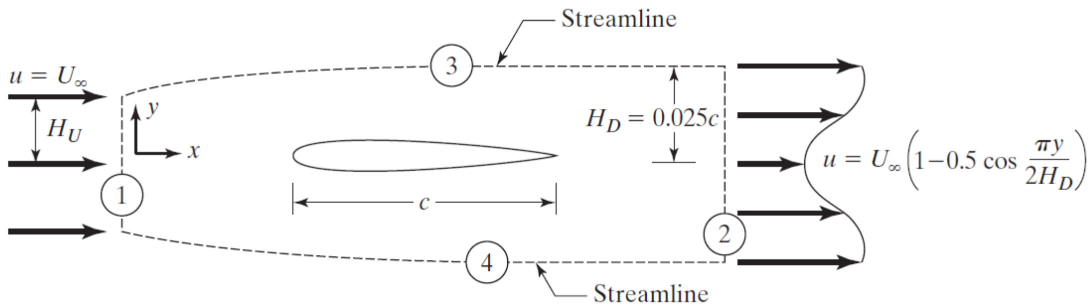

Fig. 9.

16. What are the free-stream Reynolds number [as given by equation (2.20)] and the freestream Mach number [as given by equation (2.19)] for the following flows?
    (a) A golf ball, whose characteristic length (i.e., its diameter) is 4.5 cm, moves through the standard sea level atmosphere at 60 m/s.
    (b) Boeing 747 whose characteristic length is 70.6 m flies at an altitude of 10 km. with a speed of 250 m/s.

17. (a) An airplane has a characteristic chord length of 10.4 m. What is the free-stream Reynolds number for the Mach 3 flight at an altitude of 20 km?
    (b) What is the characteristic free-stream Reynolds number of an airplane flying 160 mi/h in a standard sea-level environment? The characteristic chord length is 4.0 ft.

18. Consider the wing-leading edge of a Cessna 172 flying at 60 m/s through the standard atmosphere at 3 km altitude. Use the Mach number and total temperature relationship

$$\frac{T_t}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2$$

For air $\gamma = 1.4$, M_∞ is the free stream Mach number, T_∞ is the free stream temperature and T_t is the total temperature.
Mach number M_∞ is defined as

$$M_\infty = \frac{u_\infty}{a_\infty}$$

where a_∞ is the speed of sound, which is given as $\sqrt{\gamma R T_\infty}$. Find the total temperature and compare with the free-stream static temperature for this flow. Is convective heating likely to be a problem for this aircraft?