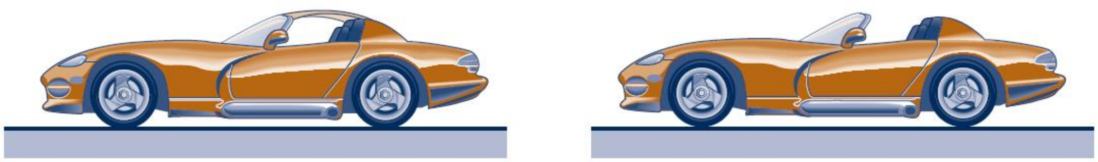
SPC 307 - Aerodynamics
Sheet 3 - Solution
Lift and Drag

1. The aerodynamic drag on a car depends on the "shape" of the car. For example, the car shown in Fig. P9.39 has a drag coefficient of 0.35 with the windows and roof closed. With the windows and roof open, the drag coefficient increases to 0.45. With the windows and roof open, at what speed is the amount of power needed to overcome aerodynamic drag the same as it is at 65 mph with the windows and roof closed? Assume the frontal area remains the same. Recall that power is force times velocity.



Windows and roof closed: $C_D = 0.35$ | Windows open; roof open: $C_D = 0.45$

$$Power = \mathcal{P} = F \cdot V$$

The force is the drag force. Let $(\,)_c$ and $(\,)_o$ denote closed and open.

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

We want to find U_o when $\mathcal{P}_o = \mathcal{P}_c$

$$\mathcal{P}_o = U_o \mathcal{D}_o = \tfrac{1}{2}\rho U_o^3 A_o C_{Do} = \mathcal{P}_c = U_c \mathcal{D}_c = \tfrac{1}{2}\rho U_c^3 A_c C_{Dc}$$

The frontal areas are the same, so $A_o = A_c$

$$U_o^3 C_{Do} = U_c^3 C_{Dc}$$

$$U_o = U_c \left(\frac{C_{Dc}}{C_{Do}}\right)^{1/3} = (65\,mph)\left(\frac{0.36}{0.45}\right)^{1/3}$$

$$\underline{\underline{U_o = 60.3\ mph}}$$

2. Two A baseball is thrown by a pitcher at 95 mph through standard air. The diameter of the baseball is 2.82 in. Estimate the drag force on the baseball.

   Left to the student.

3. If the drag on one side of a flat plate parallel to the upstream flow is D when the upstream velocity is U, what will the drag be when the upstream velocity is 2U; or U/2? Assume laminar flow.

For laminar flow $\mathscr{D} = \frac{1}{2}\rho U^2 C_{Df} A$, where $C_{Df} = \dfrac{1.328}{\sqrt{\dfrac{U\ell}{\nu}}}$

Thus,

$$\mathscr{D} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}} A = 0.664\,\rho A \frac{\sqrt{\nu}}{\sqrt{\ell}}\, U^{3/2} \sim U^{3/2}$$

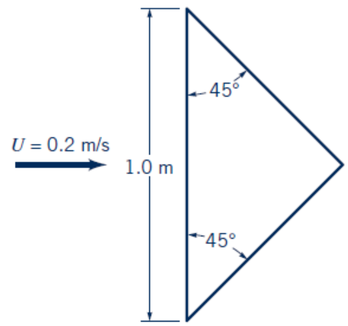
Hence,

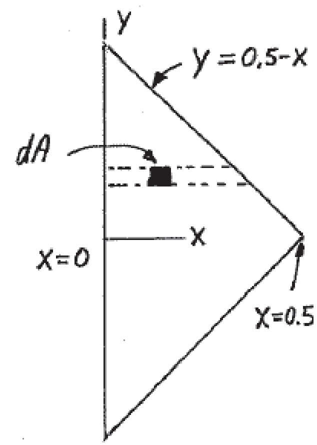
$$\frac{\mathscr{D}_U}{\mathscr{D}_{2U}} = \frac{U^{3/2}}{(2U)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathscr{D}_{2U} = 2.83\,\mathscr{D}_U}}$$

and

$$\frac{\mathscr{D}_U}{\mathscr{D}_{U/2}} \quad \frac{U^{3/2}}{\left(\frac{U}{2}\right)^{3/2}} \quad \text{or} \quad \underline{\underline{\mathscr{D}_{U/2} = 0.354\,\mathscr{D}_U}}$$

4. Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.47. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.





$$\mathcal{D} = \int \tau_w \, dA \quad \text{where} \quad \tau_w = 0.332\, U^{3/2} \sqrt{\frac{\rho \mu}{x}}$$

Thus,

$$\mathcal{D} = 0.332\, U^{3/2} \sqrt{\rho\mu} \int \frac{1}{\sqrt{x}}\, dA$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \int_{x=0}^{x=0.5} \int_{y=0}^{y=0.5-x} \frac{dy\, dx}{\sqrt{x}}$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \int_{x=0}^{0.5} \frac{0.5 - x}{\sqrt{x}}\, dx$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\, (2) \left[0.5(2) x^{\frac{1}{2}} - \frac{2}{3} x^{3/2} \right]_0^{0.5}$$

$$= 0.664 \left(0.2 \frac{m}{s}\right)^{3/2} \sqrt{999 \frac{kg}{m^3} \left(1.12 \times 10^{-3} \frac{N \cdot s}{m^2}\right)} \left[\sqrt{0.5} - \frac{2}{3} (0.5)^{3/2} \right]$$

or

$$\mathcal{D} = \underline{\underline{0.0296\, N}}$$

5. A ceiling fan consists of five blades of 0.80-m length and 0.10-m width which rotate at 100 rpm. Estimate the torque needed to overcome the friction on the blades if they act as flat plates.

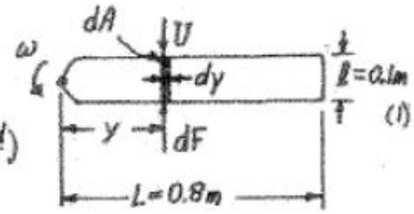
Let $d\mathcal{M}$ = torque from the drag on element dA of the blade

or

$$d\mathcal{M} = (d\mathcal{D}_{top} + d\mathcal{D}_{bottom})\,y = 2\left(\tfrac{1}{2}\rho U^2 C_{Df}\, dA\right) y$$

where $U = \omega y$ and $\omega = 100\frac{rev}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)$

or $\omega = 10.47\ \frac{rad}{s}$



The maximum Re_ℓ will occur at point (1) where $y = L$ or

$$Re_{\ell_1} = \frac{U\ell}{\nu} = \frac{\omega L \ell}{\nu} = \frac{(10.47\ \frac{rad}{s})(0.8m)(0.1m)}{1.46\times10^{-5}\ \frac{m^2}{s}} = 5.74\times10^4$$

Thus, at all points on the blade $Re_x < Re_{x_{cr}} = 5\times10^5$ and the flow is laminar.

$$C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}$$

so that from Eq. (1)

$$d\mathcal{M} = \rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}(\ell\, dy)\, y = 1.328\,\rho U^{3/2}\sqrt{\nu\ell}\ y\,dy \text{ , but with } U = \omega y$$

$$d\mathcal{M} = 1.328\,\rho\,\omega^{3/2}\sqrt{\nu\ell}\ y^{5/2}\,dy$$

$$= 1.328\,(1.23\ \tfrac{kg}{m^3})\,(10.47\ \tfrac{rad}{s})^{3/2}\left[(1.46\times10^{-5}\ \tfrac{m^2}{s})(0.1m)\right]^{1/2} y^{5/2}\,dy$$

or

$$d\mathcal{M} = 0.0669\ y^{5/2}\,dy\ \ N\cdot m \text{ , where } y \sim m$$

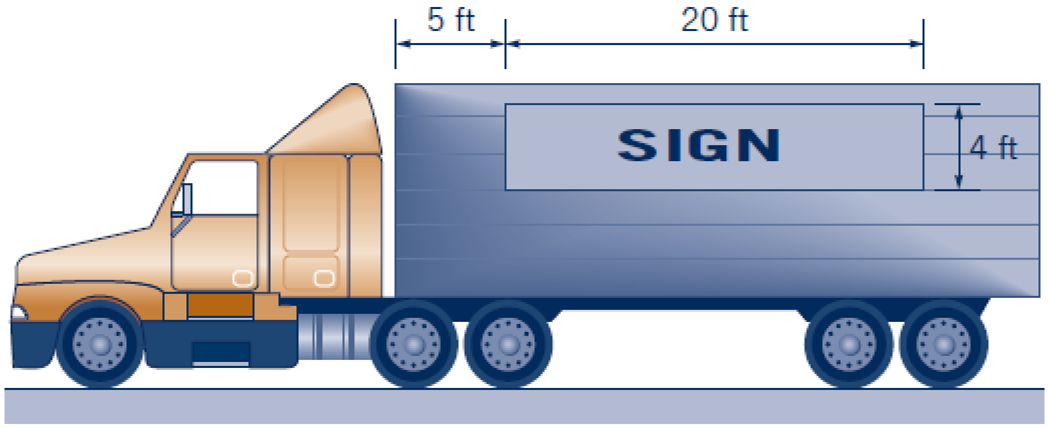
Thus, the net torque on the four blades is

$$\mathcal{M} = 5\int d\mathcal{M} = 5\int_{y=0}^{0.8m} 0.0669\ y^{5/2}\,dy = 5(0.0669)\left(\tfrac{2}{7}\right) y^{7/2}\Big|_0^{0.8}$$

or

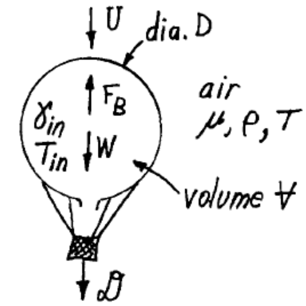
$$\mathcal{M} = \underline{\underline{0.0438\ N\cdot m}}$$

6. A thin smooth sign is attached to the side of a truck as is indicated in Fig. Estimate the friction drag on the sign when the truck is driven at 55 mph.



Left to the student.

7. A hot-air balloon roughly spherical in shape has a volume of 70,000 ft^3 and a weight of 500 lb (including passengers, basket, balloon fabric, etc.). If the outside air temperature is 80 °F and the temperature within the balloon is 165 °F, estimate the rate at which it will rise under steady-state conditions if the atmospheric pressure is 14.7 psi.



For steady rise $\Sigma F_z = 0$, or $F_B = W + \mathcal{D}$

where

$\mathcal{D} = \text{drag} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

F_B = buoyant force = $\gamma \forall$

and

W = total weight = 500 lb + $\gamma_{in} \forall$

Now $\rho = \dfrac{p}{RT} = \dfrac{(14.7 \frac{lb}{in^2})(12 \frac{in}{ft})^2}{(1715 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(460 + 80)^\circ R} = 0.00229 \frac{slugs}{ft^3}$

$\gamma = \rho g = (0.00229 \frac{slugs}{ft^3})(32.2 \frac{ft}{s^2}) = 0.0736 \frac{lb}{ft^3}$

and

$\gamma_{in} = \dfrac{p g}{R T_{in}} = \dfrac{(14.7 \frac{lb}{in^2})(12 \frac{in}{ft})^2 (32.2 \frac{ft}{s^2})}{(1715 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(460 + 165)^\circ R} = 0.0636 \frac{lb}{ft^3}$

Note: Since the balloon is open at the bottom, the pressure within the balloon is nearly the same as it is outside.

Thus, with $\forall = 7 \times 10^4 ft^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$, or $D = 51.1$ ft we obtain

$\mathcal{D} = C_D \frac{1}{2}(0.00229) U^2 \frac{\pi}{4}(51.1)^2$

$= 2.36\, C_D U^2$ lb, where $U \sim \frac{ft}{s}$

Also,

$W = 500\ lb + (0.0636 \frac{lb}{ft^3})(70{,}000\ ft^3) = 4952\ lb$

and

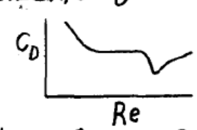
$F_B = (0.0736 \frac{lb}{ft^3})(70{,}000\ ft^3) = 5152\ lb$ Thus, $F_B = W + \mathcal{D}$ gives

$5152\ lb = 4952\ lb + 2.36\, C_D U^2$ or $C_D U^2 = 84.7$ (1)

Also, $Re = \frac{UD}{\nu}$

or $Re = \dfrac{51.1\ ft\ U}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 3.25 \times 10^5\, U$ (2)

and from Fig. 9.21 (3)



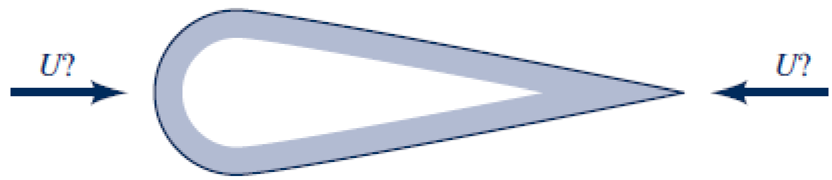
Trial and error solution: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 13.0 \frac{ft}{s} \rightarrow Re = 4.23 \times 10^6 \rightarrow C_D = 0.24 \neq 0.5$

Assume $C_D = 0.24 \rightarrow U = 18.8 \frac{ft}{s} \rightarrow Re = 6.11 \times 10^6 \rightarrow C_D = 0.30 \neq 0.24$

Assume $C_D = 0.30 \rightarrow \underline{\underline{U = 16.8 \frac{ft}{s}}} \rightarrow Re = 5.46 \times 10^6 \rightarrow C_D = 0.30$ (checks)

8. It is often assumed that "sharp objects can cut through the air better than blunt ones." Based on this assumption, the drag on the object shown in Fig. P9.56 should be less when the wind blows from right to left than when it blows from left to right. Experiments show that the opposite is true. Explain.



A significant portion of the drag on an object can be from the relatively low pressure developed in the wake region behind the object. By making the object streamlined (i.e., flow from left to right, not right to left in the above figure) boundary layer separation is avoided and a relatively thin wake with low drag is obtained. Whether the front of the object is "sharp" or "blunt" does not affect the contribution to the drag from the front part of the body—at least not as much a the width of the wake affects the drag.

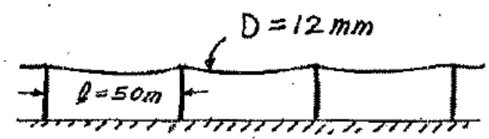
9. An object falls at a rate of 100 ft/s immediately prior to the time that the parachute attached to it opens. The final descent rate with the chute open is 10 ft/s. Calculate and plot the speed of falling as a function of time from when the chute opens. Assume that the chute opens instantly, that the drag coefficient and air density remain constant, and that the flow is quasisteady.

Left to the student.

10. Estimate the velocity with which you would contact the ground if you jumped from an airplane at an altitude of 5,000 ft and (a) air resistance is negligible, (b) air resistance is important, but you forgot your parachute, or (c) you use a 25-ft-diameter parachute.

Left to the student.

11. A 12-mm-diameter cable is strung between a series of poles that are 50 m apart. Determine the horizontal force this cable puts on each pole if the wind velocity is 30 m/s.



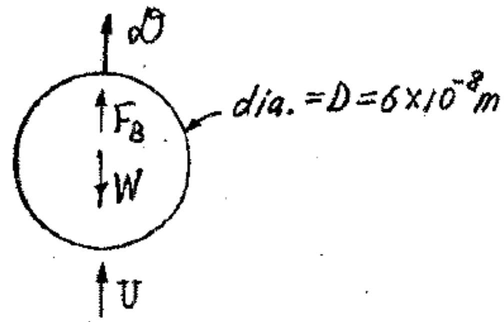
F_p = force on one pole = $\mathcal{D}$

where $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

Since $Re = \frac{UD}{\nu} = \frac{(30\frac{m}{s})(0.012m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.47\times10^4$ it follows from Fig. 9.23

that $C_D = 0.4$. Hence, $F_p = 0.4\left(\frac{1}{2}\right)(1.23\frac{kg}{m^3})(30\frac{m}{s})^2(50m)(0.012m) = \underline{\underline{133\ N}}$

12. How fast do small water droplets of 0.06 μ-mm (6 x10^{-8} m) diameter fall through the air under standard sea-level conditions? Assume the drops do not evaporate. Repeat the problem for standard conditions at 5000-m altitude.



For steady conditions, $\mathcal{D} + F_B = W$,
where if $Re = \frac{UD}{\nu} < 1$

$\mathcal{D} = \text{drag} = 3\pi D U \mu$ Also, $W = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \text{weight}$

and $F_B = \gamma_{air} \forall = \gamma_{air} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \text{buoyant force}$

Since $\gamma_{air} \ll \gamma_{H_2O}$, we can neglect the buoyant force.
That is, $\mathcal{D} = W$, or

$$3\pi D U \mu = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \quad \text{or} \quad U = \frac{\gamma_{H_2O} D^2}{18\mu} \qquad (1)$$

At sea level $\mu = 1.789 \times 10^{-5} \frac{N \cdot s}{m^2}$ so that

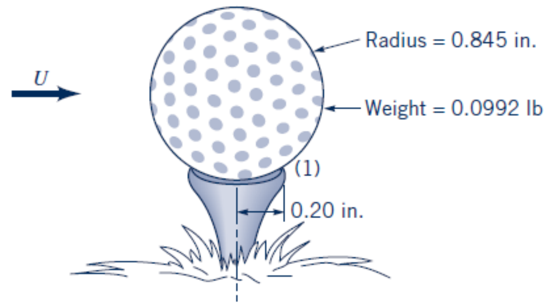
$$U = \frac{(9.80 \times 10^3 \frac{N}{m^3})(6 \times 10^{-8} m)^2}{18(1.789 \times 10^{-5} \frac{N \cdot s}{m^2})} = \underline{\underline{1.10 \times 10^{-7} \frac{m}{s}}}$$

Note that $Re = \frac{(1.10 \times 10^{-7} \frac{m}{s})(6 \times 10^{-8} m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 4.52 \times 10^{-10} \ll 1$ so the use of the low Re drag equation is valid.

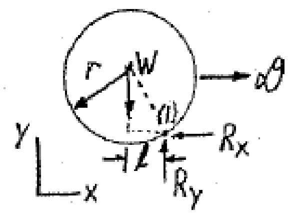
At an altitude of 5000 m, $\mu = 1.628 \times 10^{-5} \frac{N \cdot s}{m^2}$ and from Eq.(1)

$$U = \frac{(9.80 \times 10^3 \frac{N}{m^3})(6 \times 10^{-8} m)^2}{18(1.628 \times 10^{-5} \frac{N \cdot s}{m^2})} = \underline{\underline{1.20 \times 10^{-7} \frac{m}{s}}}$$

13. A strong wind can blow a golf ball off the tee by pivoting it about point 1 as shown in Fig. P9.62. Determine the wind speed necessary to do this.



When the ball is about to be blown from the tee the free body diagram is as shown. Hence, by summing moments about (1):



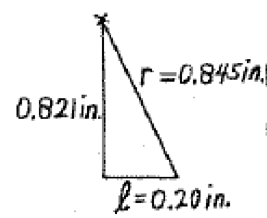
$\Sigma M_1 = 0$, or $W\ell = \mathcal{D} r$

Thus,

$(0.0992\ lb)(0.20 in.) = \mathcal{D}(0.821\ in.)$

or

$\mathcal{D} = 0.0242\ lb$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \pi r^2$



Thus,

$0.0242\ lb = C_D \frac{1}{2}(0.00238 \frac{slugs}{ft^3}) U^2 \pi \left(\frac{0.845\ in.}{12 \frac{in.}{ft}}\right)^2$

or

$C_D U^2 = 1305$, where $U \sim \frac{ft}{s}$ (1)

For a sphere* $C_D = C_D(Re)$ (see Fig. 9.18) where (2)

$Re = \frac{\rho U D}{\mu} = \frac{(0.00238\ slugs/ft^3)\ U\ (2(0.845)/12\ ft)}{3.47 \times 10^{-7} (lb \cdot s/ft^2)}$

or

$Re = 966\ U$, where $U \sim \frac{ft}{s}$ (3)

Trial and error solution:

Assume $C_D = 0.4$ so that from Eq.(1), $U = 57.1 \frac{ft}{s}$ and from Eq.(3), $Re = 966(57.1) = 5.52 \times 10^4$. Thus, from Fig. 9.18, $C_D \approx 0.25 \neq 0.40$ Try again.

Assume $C_D = 0.22$ so that $U = 77.0 \frac{ft}{s}$ and $Re = 7.44 \times 10^4$

Thus, from Fig. 9.18, $C_D \approx 0.22$ Checks.

Hence, $\underline{\underline{U \approx 77.0 \frac{ft}{s}}}$

* golf ball (i.e. with dimples)

14. How much more power is required to pedal a bicycle at 15 mph into a 20-mph head-wind than at 15 mph through still air? Assume a frontal area of 3.9 ft2 and a drag coefficient of C_D = 0.88.

$\mathcal{P}$ = power = $U_B \mathcal{D}$ and $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$, where U_B = speed of the bike $= 15 \frac{mi}{hr}\left(\frac{88 \frac{ft}{s}}{60 \frac{mi}{hr}}\right) = 22 \frac{ft}{s}$

and U = wind speed relative to bike.

Thus,

$$\mathcal{P} = \left(22 \frac{ft}{s}\right)(0.88)\left(\frac{1}{2}\right)\left(0.00238 \frac{slugs}{ft^3}\right)U^2 (3.9\, ft^2) = 0.0898\, U^2 \; \frac{ft \cdot lb}{s} \quad (1)$$

with $U \sim \frac{ft}{s}$

a) With a 20 mph headwind, $U = (15+20)\frac{mi}{hr}\left(\frac{88 \frac{ft}{s}}{60 \frac{mi}{hr}}\right) = 51.3 \frac{ft}{s}$

Thus,

$$\mathcal{P}_a = 0.0898\,(51.3)^2 = 236 \;\frac{ft \cdot lb}{s}$$

b) With still air, $U = 15\, mph = 22 \frac{ft}{s}$

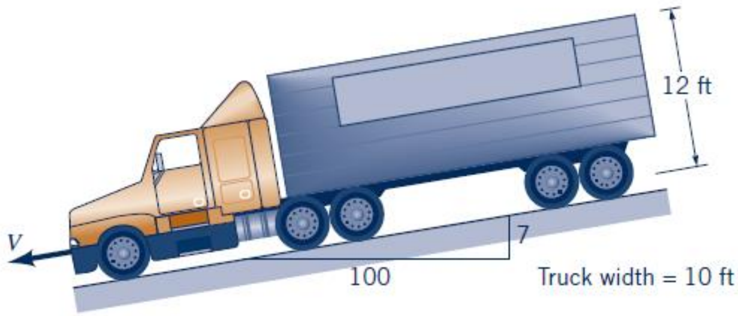
Thus,

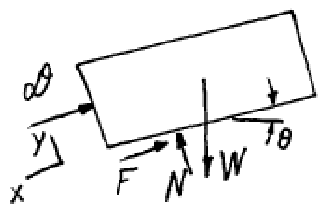
$$\mathcal{P}_b = 0.0898\,(22)^2 = 43.5 \;\frac{ft \cdot lb}{s}$$

Hence, need an additional power of $\mathcal{P}_a - \mathcal{P}_b = (236 - 43.5)\frac{ft \cdot lb}{s}\left(\frac{1\, hp}{550 \frac{ft \cdot lb}{s}}\right)$

$$= \underline{\underline{0.350\, hp}}$$

15. A 25-ton (50,000-lb) truck coasts down a steep 7% mountain grade without brakes, as shown in Fig. The truck's ultimate steady-state speed, V, is determined by a balance between weight, rolling resistance, and aerodynamic drag. Determine V if the rolling resistance for a truck on concrete is 1.2% of the weight and the drag coefficient based on frontal area is 0.76.





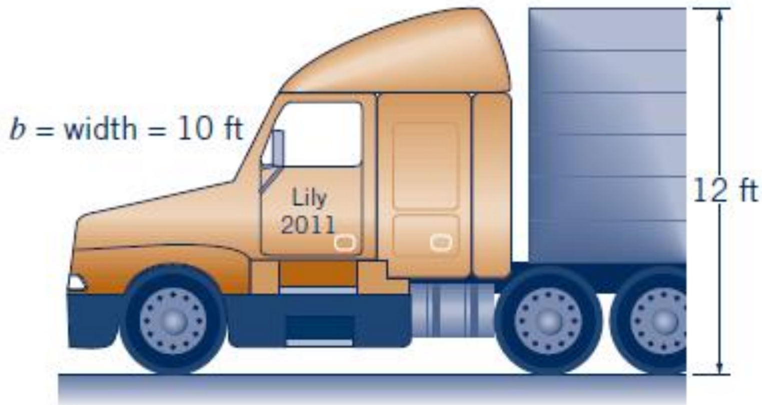
For constant speed, $\Sigma F_x = m a_x = 0$
or
$W \sin\theta = \mathcal{D} + F$
where $\theta = \arctan\left(\frac{7}{100}\right) = 4.00\ \text{deg}$, $\mathcal{D} = \frac{1}{2}\rho V^2 C_D A$

Thus,
$50{,}000\ \text{lb}\,(\sin 4.00\text{deg}) = \frac{1}{2}\left(0.00238\ \frac{\text{slugs}}{\text{ft}^3}\right) V^2 C_D\,(12\ \text{ft} \times 10\ \text{ft}) + 0.012\,(50{,}000\ \text{lb})$
or
$3488\ \text{lb} = 0.143\, V^2 C_D + 600\ \text{lb}$

(a) If $C_D = 0.96$, then $V = 145\ \frac{\text{ft}}{\text{s}} = \underline{\underline{98.9\ \text{mph}}}$

(b) If $C_D = 0.70$, then $V = 170\ \frac{\text{ft}}{\text{s}} = \underline{\underline{116\ \text{mph}}}$

16. As shown in Fig., the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from CD = 0.96 to CD = 0.70 corresponds to a reduction of how many horsepower needed at a highway speed of 65 mph?



(a) $C_D = 0.70$

(b) $C_D = 0.96$

$\mathcal{P}$ = power = $\mathcal{D}U$ where

$\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$

Thus, $\Delta\mathcal{P}$ = reduction in power

$= \mathcal{P}_b - \mathcal{P}_a$

$= \frac{1}{2}\rho U^3 A\left[C_{D_b} - C_{D_a}\right]$

With $U = 65\ mph = 95.3\ fps$,

$\Delta\mathcal{P} = \frac{1}{2}\left(0.00238\ \frac{slugs}{ft^3}\right)\left(95.3\ \frac{ft}{s}\right)^3(10\ ft)(12\ ft)\left[0.96 - 0.70\right]$

$= 32{,}100\ \frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}}\right) = \underline{\underline{58.4\ hp}}$