SPC 307 - Aerodynamics
Sheet 5
Dynamics of an incompressible, inviscid flow field

1. As illustrated in Fig. 1, a tornado can be approximated by a free vortex of strength Γ for $r > R_c$, where R_c is the radius of the core. Velocity measurements at points A and B indicate that $V_A = 125$ ft/s and $V_B = 60$ ft/s. Determine the distance from point A to the center of the tornado. Why can the free vortex model not be used to approximate the tornado throughout the flow field (r = 0)?

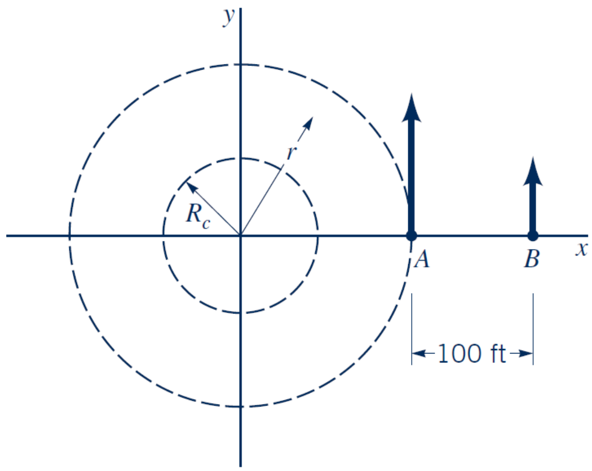

Fig.1

2. The streamlines in a particular two-dimensional flow field are all concentric circles, as shown in Fig. 2. The velocity is given by the equation $v_\theta = \omega r$ where ω is the angular velocity of the rotating mass of fluid. Determine the circulation around the path ABCD.

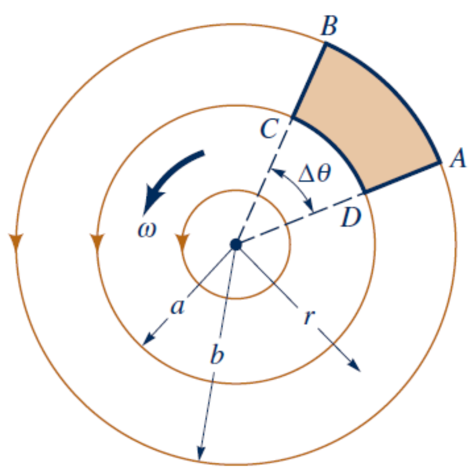

Fig.2

3. When water discharges from a tank through an opening in its bottom, a vortex may form with a curved surface profile, as shown in Fig. 3. Assume that the velocity distribution in the vortex is the same as that for a free vortex. At the same time the water is being discharged from the tank at point A, it is desired to discharge a small quantity of water through the pipe B. As the discharge through A is increased, the strength of the vortex, as indicated by its circulation, is increased. Determine the maximum strength that the vortex can have in order that no air is sucked in at B. Express your answer in terms of the circulation. Assume that the fluid level in the tank at a large distance from the opening at A remains constant and viscous effects are negligible.

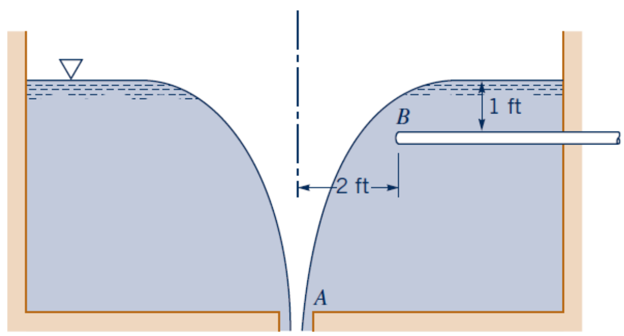

Fig. 3.

4. The Water flows over a flat surface at 4 ft/s, as shown in Fig. 4. A pump draws off water through a narrow slit at a volume rate of 0.1 ft3/s per foot length of the slit. Assume that the fluid is incompressible and inviscid and can be represented by the combination of a uniform flow and a sink. Locate the stagnation point on the wall (point A) and determine the equation for the stagnation streamline. How far above the surface, surface, H, must the fluid be so that it does not get sucked into the slit?

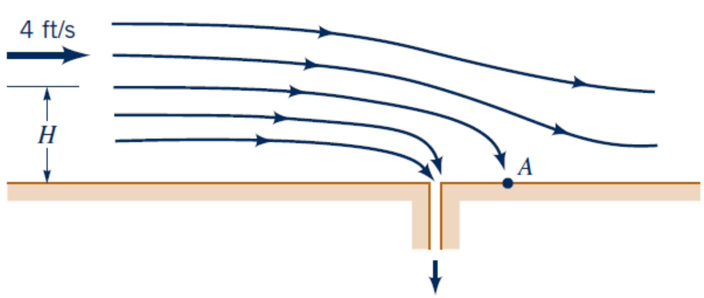

Fig. 4.

5. The Two sources, one of strength m and the other with strength 3m, are located on the x axis as shown in Fig. 5. Determine the location of the stagnation point in the flow produced by these sources.

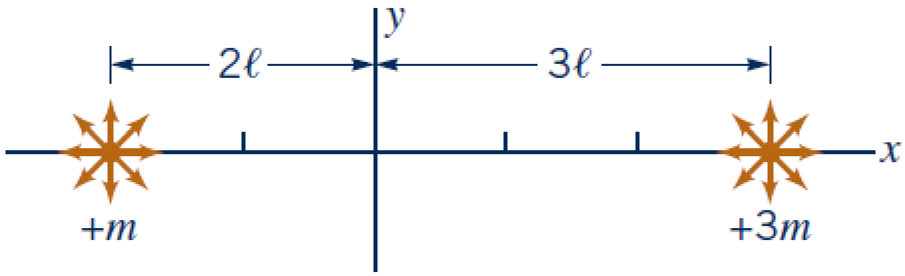

Fig. 5.

6. The velocity potential for a spiral vortex flow is given by
$$\phi = (\Gamma/2\pi)\theta - (m/2\pi)\ln r$$
where Γ and m are constants. Show that the angle, α, between the velocity vector and the radial direction is constant throughout the flow field (see Fig. 6).

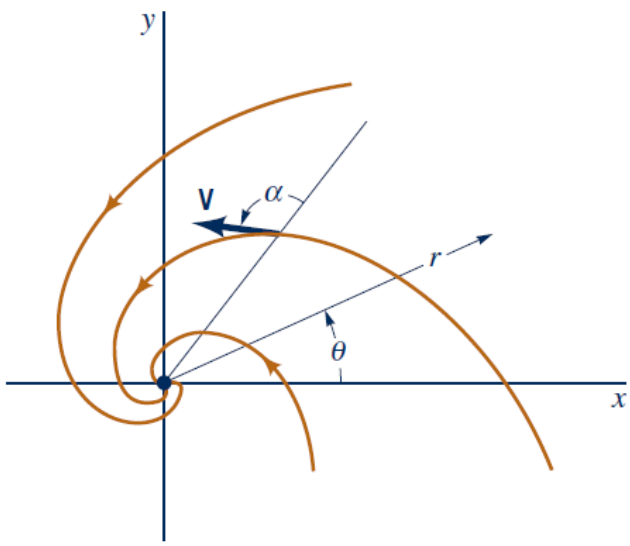

Fig. 6.

7. Consider The combination of a uniform flow and a source can be used to describe flow around a streamlined body called a half-body. Assume that a certain body has the shape of a half-body with a thickness of 0.5 m. If this body is placed in an airstream moving at 15 m/s, what source strength is required to simulate flow around the body?

8. The One end of a pond has a shoreline that resembles a half-body as shown in Fig. 7. A vertical porous pipe is located near the end of the pond so that water can be pumped out. When water is pumped at the rate of 0.08 m^3/s through a 3-m-long pipe, what will be the velocity at point A? Hint: Consider the flow inside a half-body.

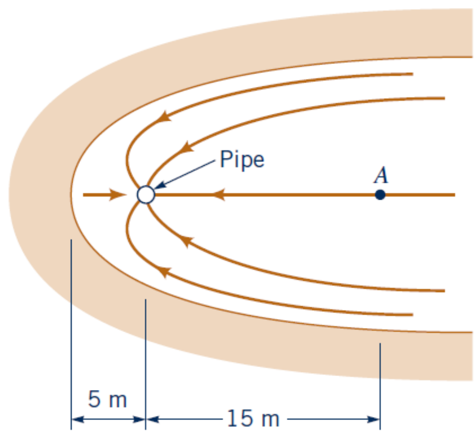

Fig. 7.

9. Rankine oval is formed by combining a source–sink pair, each having a strength of 36 ft^2/s and separated by a distance of 12 ft along the x axis, with a uniform velocity of 10 ft/s (in the positive x direction). Determine the length and thickness of the oval.

10. An ideal fluid flows past an infinitely long, semicircular "hump" located along a plane boundary, as shown in Fig. 8. Far from the hump the velocity field is uniform, and the pressure is p_0. (a) Determine expressions for the maximum and minimum values of the pressure along the hump, and indicate where these points are located. Express your answer in terms of r, U, and p_0. (b) If the solid surface is the streamline, determine the equation of the streamline passing through the point $\theta = \pi/2$, r = 2a.

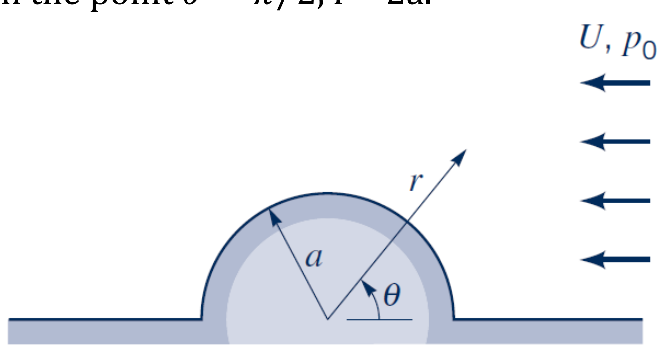

Fig. 8.

11. Water flows around a 6-ft-diameter bridge pier with a velocity of 12 ft/s. Estimate the force (per unit length) that the water exerts on the pier. Assume that the flow can be approximated as an ideal fluid flow around the front half of the cylinder, but due to flow separation, the average pressure on the rear half is constant and approximately equal to 1/2 the pressure at point A (see Fig. 9).

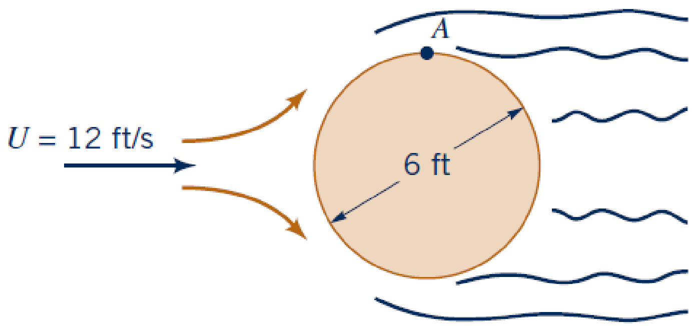

Fig. 9.

12. Consider the steady potential flow around the circular cylinder shown in Fig. 10. On a plot show the variation of the magnitude of the dimensionless fluid velocity, V/U, along the positive y axis. At what distance, y/a (along the y axis), is the velocity within 1% of the free-stream velocity?

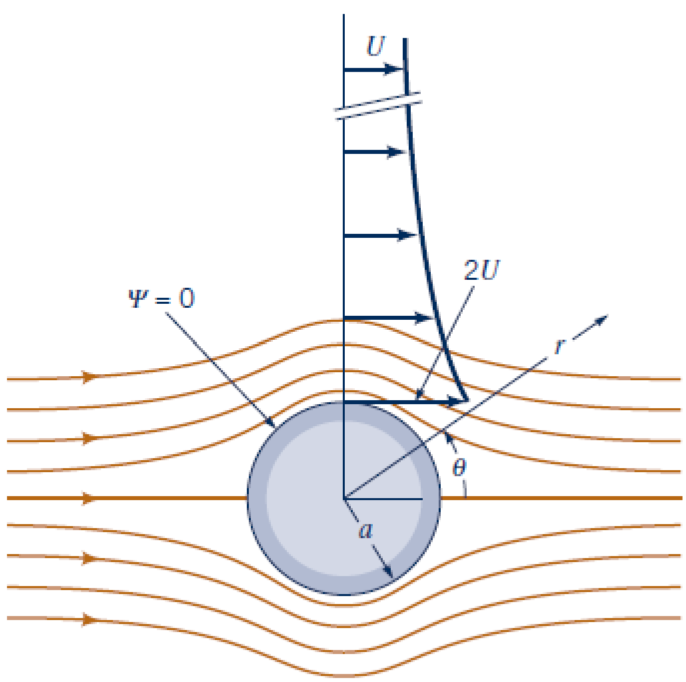

Fig. 10.