1. Water flows past a flat plate that is oriented parallel to the flow with an upstream velocity of 0.5 m/s. Determine the approximate location downstream from the leading edge where the boundary layer becomes turbulent. What is the boundary layer thickness at this location?

\[
\frac{R_e}{\nu} = \frac{5 \times 10^5}{\nu} = \frac{U x_c}{\nu} = 5 \times 10^5 \frac{1.12 \times 10^{-6} \text{ m/s}}{0.5 \text{ m/s}} = 1.12 \text{ m}
\]

\[
\delta = 5 \sqrt{\frac{\nu}{U}} = 5 \sqrt{\frac{1.12 \times 10^{-6} \text{ m/s}}{0.5 \text{ m/s}^2}} = 7.92 \times 10^{-3} \text{ m}
\]

2. A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow, \( \delta = C \sqrt{x} \), where \( C \) is a constant.

Thus,

\[
C = \frac{\delta}{\sqrt{x}} = \frac{12 \times 10^{-3} \text{ m}}{\sqrt{1.3 \text{ m}}} = 0.0105 \quad \text{or} \quad \delta = 0.0105 \sqrt{x} \quad \text{where} \quad x \sim \text{m}, \delta \sim \text{m}
\]

<table>
<thead>
<tr>
<th>( x, \text{m} )</th>
<th>( \delta, \text{m} )</th>
<th>( \delta, \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.00470</td>
<td>4.70</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0148</td>
<td>14.8</td>
</tr>
<tr>
<td>20.0</td>
<td>0.0470</td>
<td>47.0</td>
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</table>
3. Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re = 1)? Explain. Repeat for a 0.004 in. diameter hair and a 6-ft-diameter smokestack.

\[ Re = \frac{UD}{\nu} < 1 \text{ or } U < \frac{\nu}{D} \text{ if viscous effects are to be important throughout the flow.} \]

For standard air, \( \nu = 1.57 \times 10^{-4} \text{ ft}^2/\text{s} \)

Thus,

\[ U < \frac{1.57 \times 10^{-4}}{D}, \text{ where } D \text{ is the diameter in feet.} \]

<table>
<thead>
<tr>
<th>Object</th>
<th>( D, \text{ ft} )</th>
<th>( U, \text{ ft/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>twig</td>
<td>( 2.08 \times 10^{-2} )</td>
<td>( 7.54 \times 10^{-3} )</td>
</tr>
<tr>
<td>hair</td>
<td>( 3.33 \times 10^{-4} )</td>
<td>0.471</td>
</tr>
<tr>
<td>smokestack</td>
<td>6</td>
<td>( 2.62 \times 10^{-5} )</td>
</tr>
</tbody>
</table>
4. Air enters a square duct through a 1-ft opening as is shown in Fig. 1. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant \( U = 2 \) ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, \( d \), as a function of \( x \) for \( 0 \leq x \leq 10 \) ft if \( U \) is to remain constant. Assume laminar flow.

**Fig. 1**

For incompressible flow \( Q_0 = Q(x) \) where \( Q_0 = \) flowrate into the duct and

\[
Q(x) = U A, \quad \text{where} \quad A = (d - 2 \delta^*)^2 \quad \text{is the effective area of the duct (allowing for the decreased flowrate in the boundary layer)}.
\]

Thus,

\[
Q_0 = U (d - 2 \delta^*)^2 \quad \text{or} \quad d = 1 + 2 \delta^*.
\]

where

\[
\delta^* = 1.72 \sqrt{\frac{2x}{U}} = 1.72 \left[ \frac{(5.7 \times 10^{-3} \text{ ft}) x}{2 \text{ ft}^3} \right]^{1/2} = 0.0152 \sqrt{x} \text{ ft}, \quad \text{where} \ x \text{ ft}
\]

Hence, from Eq. (1)

\[
d = 1 + 0.0304 \sqrt{x} \text{ ft}
\]

For example, \( d = 1 \) ft at \( x = 0 \) and \( d = 1.096 \) ft at \( x = 10 \) ft.
5. An atmospheric boundary layer is formed when the wind blows over the Earth’s surface. Typically, such velocity profiles can be written as a power law: \( u = ay^n \), where the constants \( a \) and \( n \) depend on the roughness of the terrain. As is indicated in Fig. 2, typical values are \( n = 0.40 \) for urban areas, \( n = 0.28 \) for woodland or suburban areas, and \( n = 0.16 \) for flat open country.

(a) If the velocity is 20 ft/s at the bottom of the sail on your boat \((y = 4\text{ ft})\), what is the velocity at the top of the mast \((y = 30\text{ ft})\)?

(b) If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

\[ \begin{align*}
\text{Fig. 2}
\end{align*} \]

\[ \begin{align*}
\text{(a) } U &= Cy^{0.16}, \text{ where } C \text{ is a constant} \\
\text{Thus, } \frac{U_2}{U_1} &= \left( \frac{y_2}{y_1} \right)^{0.16} \text{ or } U_2 = 20 \frac{\text{f/s}}{\text{f/s}} \left( \frac{30\text{ f}}{4\text{ f}} \right)^{0.16} = 27.6 \frac{\text{f/s}}{\text{s}} \\
\text{(b) } U &= C' y^{0.40}, \text{ where } C' \text{ is a constant} \\
\text{Thus, } \frac{U_2}{U_1} &= \left( \frac{y_2}{y_1} \right)^{0.40} \text{ or } U_2 = 10 \frac{\text{mph}}{\text{mph}} \left( \frac{60}{10} \right)^{0.4} = 20.5 \frac{\text{mph}}{\text{m}}
\end{align*} \]