

## Midterm 1

1/11/2016

1. Consider a flow impinging on the sharp-edged nose of an aircraft. As the Mach number increases from 0.1 to 5, can you explain what happens to the flow over the nose? (use drawings to enhance your answer).
2. Consider a Pitot static tube mounted on the nose of an experimental airplane. A Pitot tube measures the total pressure at the tip of the probe (hence sometimes called the Pitot pressure), and a Pitot static tube combines this with a simultaneous measurement of the free-stream static pressure. The Pitot and free-stream static measurements are given below for three different flight conditions. Calculate the free-stream Mach number at which the airplane is flying for each of the three different conditions:
  - a. Pitot pressure =  $1.22 \times 10^5$  Pa, static pressure =  $1.01 \times 10^5$  Pa
  - b. Pitot pressure =  $3.4579117 \times 10^5$  Pa, static pressure =  $1.0131461 \times 10^5$  Pa
  - c. Pitot pressure =  $6.2756644 \times 10^5$  Pa, static pressure =  $4.883786 \times 10^5$  Pa
3. Consider the flow through a rocket engine nozzle. Assume that the gas flow through the nozzle is an isentropic expansion of a calorically perfect gas. In the combustion chamber, the gas which results from the combustion of the rocket fuel and oxidizer is at a pressure and temperature of 15 atm and 2500 K, respectively; the molecular weight and specific heat at constant pressure of the combustion gas are 12 and 4157 J/kg.K, respectively. The gas constant of the mixture  $R = 692.8$  J/kg.K and specific heat ratio of 1.2. The gas expands to supersonic speed through the nozzle, with a temperature of 1350 K at the nozzle exit. Calculate the pressure, Mach number and density at the exit of the rocket nozzle.

Best Regards

Ahmed Elmekawy, PhD, P.E.

## Equation Sheet

### Isentropic Flow Equations

$$\frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}}$$

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{Ma}^* = \text{Ma} \sqrt{\frac{k+1}{2+(k-1)\text{Ma}^2}}$$

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left[ \left( \frac{2}{k+1} \right) \left( 1 + \frac{k-1}{2} \text{Ma}^2 \right) \right]^{0.5(k+1)/(k-1)}$$

$$\frac{P}{P_0} = \left( 1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-k/(k-1)}$$

$$\frac{\rho}{\rho_0} = \left( 1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1/(k-1)}$$

$$\frac{T}{T_0} = \left( 1 + \frac{k-1}{2} \text{Ma}^2 \right)^{-1}$$

### Normal Shock Equations

$$T_{01} = T_{02}$$

$$\text{Ma}_2 = \sqrt{\frac{(k-1)\text{Ma}_1^2 + 2}{2k\text{Ma}_1^2 - k + 1}}$$

$$\frac{P_2}{P_1} = \frac{1 + k\text{Ma}_1^2}{1 + k\text{Ma}_2^2} = \frac{2k\text{Ma}_1^2 - k + 1}{k + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\text{Ma}_1^2}{2 + (k-1)\text{Ma}_1^2} = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \frac{2 + \text{Ma}_1^2(k-1)}{2 + \text{Ma}_2^2(k-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{\text{Ma}_1 \left[ 1 + \text{Ma}_2^2(k-1)/2 \right]^{(k+1)/[2(k-1)]}}{\text{Ma}_2 \left[ 1 + \text{Ma}_1^2(k-1)/2 \right]}$$

$$\frac{P_{02}}{P_1} = \frac{(1 + k\text{Ma}_1^2) \left[ 1 + \text{Ma}_2^2(k-1)/2 \right]^{k/(k-1)}}{1 + k\text{Ma}_2^2}$$

**TABLE A-13**

One-dimensional isentropic compressible flow functions for an ideal gas with  $k = 1.4$

Ma	Ma*	$A/A^*$	$P/P_0$	$\rho/\rho_0$	$T/T_0$
0	0	$\infty$	1.0000	1.0000	1.0000
0.1	0.1094	5.8218	0.9930	0.9950	0.9980
0.2	0.2182	2.9635	0.9725	0.9803	0.9921
0.3	0.3257	2.0351	0.9395	0.9564	0.9823
0.4	0.4313	1.5901	0.8956	0.9243	0.9690
0.5	0.5345	1.3398	0.8430	0.8852	0.9524
0.6	0.6348	1.1882	0.7840	0.8405	0.9328
0.7	0.7318	1.0944	0.7209	0.7916	0.9107
0.8	0.8251	1.0382	0.6560	0.7400	0.8865
0.9	0.9146	1.0089	0.5913	0.6870	0.8606
1.0	1.0000	1.0000	0.5283	0.6339	0.8333
1.2	1.1583	1.0304	0.4124	0.5311	0.7764
1.4	1.2999	1.1149	0.3142	0.4374	0.7184
1.6	1.4254	1.2502	0.2353	0.3557	0.6614
1.8	1.5360	1.4390	0.1740	0.2868	0.6068
2.0	1.6330	1.6875	0.1278	0.2300	0.5556
2.2	1.7179	2.0050	0.0935	0.1841	0.5081
2.4	1.7922	2.4031	0.0684	0.1472	0.4647
2.6	1.8571	2.8960	0.0501	0.1179	0.4252
2.8	1.9140	3.5001	0.0368	0.0946	0.3894
3.0	1.9640	4.2346	0.0272	0.0760	0.3571
5.0	2.2361	25.000	0.0019	0.0113	0.1667
$\infty$	2.2495	$\infty$	0	0	0

**TABLE A-14**

One-dimensional normal shock functions for an ideal gas with  $k = 1.4$

Ma <sub>1</sub>	Ma <sub>2</sub>	$P_2/P_1$	$\rho_2/\rho_1$	$T_2/T_1$	$P_{02}/P_{01}$	$P_{02}^*/P_1$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.8929
1.1	0.9118	1.2450	1.1691	1.0649	0.9989	2.1328
1.2	0.8422	1.5133	1.3416	1.1280	0.9928	2.4075
1.3	0.7860	1.8050	1.5157	1.1909	0.9794	2.7136
1.4	0.7397	2.1200	1.6897	1.2547	0.9582	3.0492
1.5	0.7011	2.4583	1.8621	1.3202	0.9298	3.4133
1.6	0.6684	2.8200	2.0317	1.3880	0.8952	3.8050
1.7	0.6405	3.2050	2.1977	1.4583	0.8557	4.2238
1.8	0.6165	3.6133	2.3592	1.5316	0.8127	4.6695
1.9	0.5956	4.0450	2.5157	1.6079	0.7674	5.1418
2.0	0.5774	4.5000	2.6667	1.6875	0.7209	5.6404
2.1	0.5613	4.9783	2.8119	1.7705	0.6742	6.1654
2.2	0.5471	5.4800	2.9512	1.8569	0.6281	6.7165
2.3	0.5344	6.0050	3.0845	1.9468	0.5833	7.2937
2.4	0.5231	6.5533	3.2119	2.0403	0.5401	7.8969
2.5	0.5130	7.1250	3.3333	2.1375	0.4990	8.5261
2.6	0.5039	7.7200	3.4490	2.2383	0.4601	9.1813
2.7	0.4956	8.3383	3.5590	2.3429	0.4236	9.8624
2.8	0.4882	8.9800	3.6636	2.4512	0.3895	10.5694
2.9	0.4814	9.6450	3.7629	2.5632	0.3577	11.3022
3.0	0.4752	10.3333	3.8571	2.6790	0.3283	12.0610
4.0	0.4350	18.5000	4.5714	4.0469	0.1388	21.0681
5.0	0.4152	29.000	5.0000	5.8000	0.0617	32.6335
$\infty$	0.3780	$\infty$	6.0000	$\infty$	0	$\infty$

End of Exam.