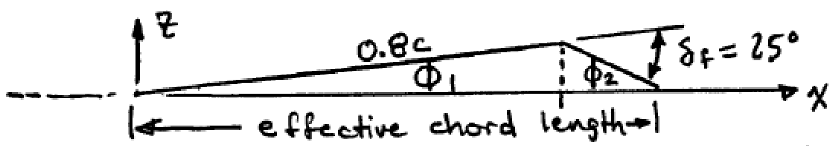
SPC 307 - Aerodynamics
Sheet 7 - Solution
INCOMPRESSIBLE FLOWS AROUND AIRFOILS OF INFINITE SPAN

1. Calculate C_l and $C_{m,\,c/4}$ for a NACA 0009 airfoil that has a plain flap whose length is 0.2c and which is deflected 25°. When the geometric angle of attack is 4°, what is the section lift coefficient? Where is the center of pressure?

The NACA 0009 airfoil is a symmetric airfoil, whose chord is a straight line. Thus, when a plain flap whose length is 0.2c is deflected 25°, we can represent the configuration by the "equivalent cambered" airfoil section.



First, we need to calculate the slopes of the two segments of the "equivalent cambered" section.

$$z_{max} = 0.8c \sin\phi_1 = 0.2c \sin\phi_2 \quad \text{and} \quad \phi_2 = 25° - \phi_1$$

Thus, $4 \sin\phi_1 = \sin(25° - \phi_1) = \sin 25° \cos\phi_1 - \cos 25° \sin\phi_1$

$4.9063 \sin\phi_1 = 0.4226 \cos\phi_1$; $\phi_1 = 4.923°$ and $\phi_2 = 20.077°$

Thus, the effective chord length (c_{eff}) is:

$$c_{eff} = 0.8c \cos\phi_1 + 0.2c \cos\phi_2 = c_1 + c_2$$

$$c_{eff} = 0.79705c + 0.18785c = 0.98490c$$

Equivalently, $c_1 = 0.80927\, c_{eff}$ and $c_2 = 0.19073\, c_{eff}$

Let us now determine the limits for the integration, i.e., what is the value of θ for which

$$0.80927\, c_{eff} = \frac{c_{eff}}{2}(1-\cos\theta)$$

Thus, $\theta = 128.21° = 2.2377$ radians

Note that:

$$\left.\frac{dz}{dx}\right)_{fore} = \tan\phi_1 = 0.08613 \; ; \; \left.\frac{dz}{dx}\right)_{aft} = -\tan\phi_2 = -0.36549$$

Thus,

$$A_0 = \alpha_{eff} - \frac{1}{\pi}\left\{\int_0^{2.2377}\left(\frac{dz}{dx}\right)_{fore} d\theta + \int_{2.2377}^{\pi}\left(\frac{dz}{dx}\right)_{aft} d\theta\right.$$

$$A_0 = \alpha_{eff} - \frac{1}{\pi}\left\{0.08613\,\theta\Big|_0^{2.2377} - 0.36549\,\theta\Big|_{2.2377}^{\pi}\right.$$

$$A_0 = \alpha_{eff} + 0.04381$$

Note that $\alpha = \alpha_{eff} - 4.923° = \alpha_{eff} - 0.08592$

$$A_1 = \frac{2}{\pi}\left\{\int_0^{128.21°}\left(\frac{dz}{dx}\right)_{fore}\cos\theta\, d\theta + \int_{128.21°}^{180°}\left(\frac{dz}{dx}\right)_{aft}\cos\theta\, d\theta\right\}$$

$$A_1 = \frac{2}{\pi}\left\{0.08613\,\sin\theta\Big|_0^{128.21°} - 0.36549\,\sin\theta\Big|_{128.21°}^{180°}\right\}$$

$$A_1 = 0.22591$$

$$A_2 = \frac{2}{\pi}\left\{\int_0^{128.21°}\left(\frac{dz}{dx}\right)_{fore}\cos 2\theta\, d\theta + \int_{128.21°}^{180°}\left(\frac{dz}{dx}\right)_{aft}\cos 2\theta\, d\theta\right\}$$

$$A_2 = \frac{2}{\pi}\left\{(0.08613)\frac{1}{2}\sin 2\theta\Big|_0^{128.21^\circ} - (0.36549)\frac{1}{2}\sin 2\theta\Big|_{128.21^\circ}^{180^\circ}\right\}$$

$$A_2 = -0.13974$$

Thus, we have calculated A_0, A_1, and A_2 for our "equivalent cambered" airfoil section.

$$A_0 = \alpha_{eff} + 0.4381$$
$$A_1 = 0.22591$$
$$A_2 = -0.13974$$

$$C_\ell = 2\pi\left(A_0 + \frac{A_1}{2}\right) = 2\pi(\alpha_{eff} + 0.04381 + 0.11296)$$

$$C_\ell = 2\pi(\alpha_{eff} + 0.15677)$$

But this lift coefficient is based on c_{eff}. To convert to the standard reference length c,

$$\ell = C_\ell\, q_\infty\, c = 2\pi(\alpha_{eff} + 0.15677)\, q_\infty\, c_{eff}$$

$$C_\ell = 2\pi\left[\alpha + 0.08592 + 0.15677\right] 0.98490$$

Since $\frac{c_{eff}}{c} = 0.98490$

$$C_\ell = 6.18831\left[\alpha + 0.24269\right] = 6.18831\alpha + 1.50184$$

$$x_{cp} = \frac{c_{eff}}{4}\left[1 + \frac{\pi}{C_\ell}(A_1 - A_2)\right]$$

we will use C_ℓ for the effective angle of attack

$$x_{cp} = \frac{c_{eff}}{4}\left[1 + \frac{0.36565}{2(\alpha_{eff} + 0.15677)}\right]$$

$$C_{m_{0.25c}} = \frac{\pi}{4}\left(-0.13974 - 0.22591\right) = -0.28718$$

which is based on the "effective cambered" airfoil section

$$m_{0.25c} = C_{m_{0.25c}}\, q_\infty\, c\, c = \left[-0.28718\right] q_\infty\, c_{eff}\, c_{eff}$$

Solving for $C_{m_{0.25c}}$ based on the actual chord length

$$C_{m_{0.25c}} = -0.27857$$

2. The NACA 23012 wing section thus has a design lift coefficient of 0.3, has its maximum camber at 15% of the chord, and has a maximum thickness of 0.12 c. The equation for the mean camber line is

$$\frac{z}{c} = 2.6595\left[\left(\frac{x}{c}\right)^3 - 0.6075\left(\frac{x}{c}\right)^2 + 0.11471\left(\frac{x}{c}\right)\right]$$

for the region $0.0c \leq x \leq 0.2025c$ and

$$\frac{z}{c} = 0.022083\left(1 - \frac{x}{c}\right)$$

for the region $0.2025c \leq x \leq 1.000c$.
Calculate the A_0, A_1, and A_2 for this airfoil section. What is the section lift coefficient, C_l ? What is the angle of attack for zero lift, α_{0l}? What angle of attack is required to develop the design lift coefficient of 0.3? Calculate the section moment coefficient about the theoretical aerodynamic center. Compare your theoretical values with the experimental values in Fig. 1 that are reproduced from the work of Abbott and von Doenhoff (1949).
When the geometric angle of attack is 3°, what is the section lift coefficient? What is the x/c location of the center of pressure?

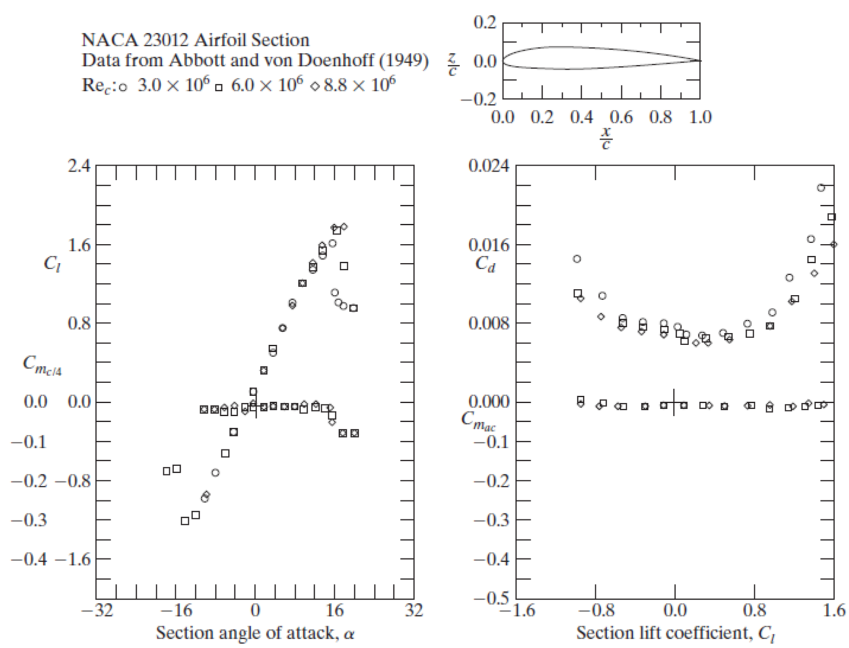


Fig. 1

For $0.0c \le x \le 0.2025c$

$$\left(\frac{z}{c}\right)_{fore} = 2.6595\left[\left(\frac{x}{c}\right)^3 - 0.6075\left(\frac{x}{c}\right)^2 + 0.11471\left(\frac{x}{c}\right)\right]$$

Thus, for $0 \le \theta \le 53.487°$ (or 0.93352 radians)

$$\left(\frac{dz}{dx}\right)_{fore} = 7.9785\left(\frac{x}{c}\right)^2 - 3.2313\left(\frac{x}{c}\right) + 0.30507$$

$$= 1.9946\cos^2\theta - 2.3736\cos\theta + 0.6840$$

For $0.2025c \le x \le 1.0000c$

$$\left(\frac{z}{c}\right)_{aft} = 0.022083\left[1 - \frac{x}{c}\right]$$

Thus, for $53.487° \le \theta \le 180°$

$$\left(\frac{dz}{dx}\right)_{aft} = -0.022083$$

$$A_0 = \alpha + \frac{1}{\pi}\left\{\int_0^{0.93352}\left[1.9946\cos^2\theta - 2.3736\cos\theta + 0.6840\right]d\theta + \int_{0.93352}^{\pi}\left[-0.022083\right]d\theta\right.$$

$$A_0 = \alpha - \frac{1}{\pi}\left\{\left[1.9946\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - 2.3736\sin\theta + 0.6840\,\theta\right|_0^{0.93352} + \left[-0.022083\,\theta\right|_{0.93352}^{\pi}\right\}$$

$$A_0 = \alpha - 0.02866$$

$$A_1 = \frac{2}{\pi}\left\{\int_0^{0.93352}\left[1.9946\cos^3\theta - 2.3736\cos^2\theta + 0.6840\cos\theta\right]d\theta + \int_{0.93352}^{\pi}\left[-0.022083\cos\theta\right]d\theta\right\}$$

$$A_1 = \left\{\left[1.9946\left(\sin\theta - \frac{1}{3}\sin^3\theta\right) - 2.3736\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) + 0.6840\sin\theta\right|_0^{0.93352} + \left[-0.022083\sin\theta\right|_{0.93352}^{\pi}\right\}$$

$$A_1 = 0.09550$$

$$A_2 = \frac{2}{\pi}\left\{\int_0^{0.93352}\left[1.9946\cos^2\theta\left(2\cos^2\theta - 1\right) - 2.3736\cos\theta\cos 2\theta + 0.6840\cos 2\theta\right]d\theta + \int_{0.93352}^{\pi}\left[-0.022083\cos 2\theta\right]d\theta\right.$$

$$A_2 = \frac{2}{\pi}\left\{\left[1.9946\left(\frac{\theta}{4} + \frac{1}{8}\sin 2\theta + \frac{\cos^3\theta\sin\theta}{2}\right) - 2.3736\left(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6}\right) + 0.6840\,\frac{\sin 2\theta}{2}\right|_0^{0.93352} + \left[-0.022083\,\frac{\sin 2\theta}{2}\right|_{0.93352}^{\pi}\right\}$$

$$A_2 = 0.07914$$

$$C_l = \pi(2A_0 + A_1) = 2\pi\alpha + 0.11992$$

when $\left\{ \begin{array}{lllll} \alpha = & -1.0936^\circ & 0.0^\circ & 1.642^\circ & 3.0^\circ \\ C_l = & 0.0 & 0.11992 & 0.30 & 0.44891 \end{array} \right.$

These calculated values of C_l are in close agreement with the data from Abbott and von Doenhoff (1949)

The center of pressure is given by:

$$x_{cp} = \frac{c}{4}\left[1 + \frac{\pi}{4}(A_1 - A_2)\right]$$

Thus, when $\alpha = 3.0^\circ$, $C_l = 0.44891$ and

$$x_{cp} = 0.2786c$$

$$C_{m_{ac}} = \frac{\pi}{4}(A_2 - A_1) = -0.01285$$

which also is in good agreement with the experimental values of Abbott and von Doenhoff (1949)

3. The NACA 4412 airfoil has a mean camber line given by

$$\frac{z}{c} = \begin{cases} 0.25\left[0.8\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] & \text{for } 0 \le \frac{x}{c} \le 0.4 \\ 0.111\left[0.2 + 0.8\frac{x}{c} - \left(\frac{x}{c}\right)^2\right] & \text{for } 0.4 \le \frac{x}{c} \le 1 \end{cases}$$

Using thin airfoil theory, calculate

a) $\alpha_{l=0}$

b) C_l and $C_{m,\,c/4}$ at α=3°

c) compare the result with the experimental data from the following reference: Abbott, Theory of Wing Sections, Mcgraw-Hill Book 1949 (Link).

For $0 \le \frac{x}{c} \le 0.4$: $\left(\frac{dz}{dx}\right)_1 = 0.2 - 0.5\left(\frac{x}{c}\right)$

For $0.4 \le \frac{x}{c} \le 1$: $\left(\frac{dz}{dx}\right)_2 = 0.0888 - 0.2222\left(\frac{x}{c}\right)$

Since $x = \frac{c}{2}(1 - \cos\theta)$, then

$$\left(\frac{dz}{dx}\right)_1 = -0.05 + 0.25\cos\theta, \quad \text{for } 0 \le \theta \le 1.3694$$

$$\left(\frac{dz}{dx}\right)_2 = -0.0223 + 0.1111\cos\theta, \quad \text{for } 1.3694 \le \theta \le \pi$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta - 1)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{1.3694} (-0.05 + 0.25\cos\theta)(\cos\theta - 1)\, d\theta - \frac{1}{\pi}\int_{1.3694}^{\pi}$$

$$(-0.0223 + 0.1111\cos\theta)(\cos\theta - 1)\, d\theta$$

$$= -\frac{1}{\pi}\int_0^{1.3694} (0.05 - 0.3\cos\theta + 0.25\cos^2\theta)\, d\theta$$

$$-\frac{1}{\pi}\int_{1.3694}^{\pi} (0.0223 - 0.13334\cos\theta + 0.1111\cos^2\theta)\, d\theta$$

$$= -\frac{1}{\pi}\left[0.05\theta - 0.3\sin\theta + 0.25\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_0^{1.3694}$$

$$- \frac{1}{\pi} [0.0223\, \theta - 0.1334 \sin\theta + 0.111 \left(\frac{\theta}{2} + \frac{1}{4} \sin 2\theta\right)]_{1.3694}^{\pi}$$

$$= - \frac{1}{\pi} [0.06847 - 0.2939 + 0.1712 + 0.0245] - \frac{1}{\pi} [0.0701 + 0.1745]$$

$$+ \frac{1}{\pi} [0.0305 - 0.1307 + 0.0761 + 0.0109]$$

$$= \frac{-0.2281}{\pi} = -0.0726 \text{ rad} = \boxed{-4.16^{o}}$$

(b)

$c_\ell = 2\,\pi\,(\alpha + \alpha_{L=0})$ where α is in radians

$$c_\ell = \frac{2\pi}{57.3} [3 - (-4.16)] = \boxed{0.782}$$